Chapter 31 Electromagnetic Induction

The link between electricity and magnetism:

Electric current could produce magnetism: 1. The magnetic effect (compass fluctuation) produced by an electric current; 2. An ion bar was magnetized when placed inside a current-carry solenoid.

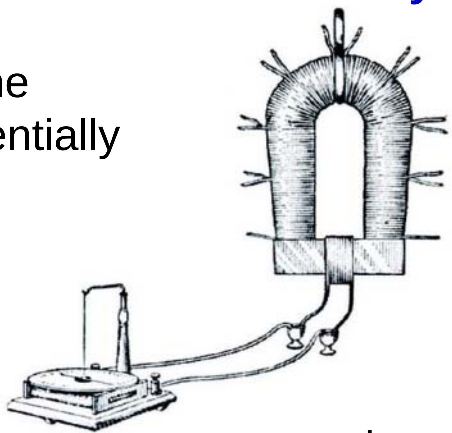
Can we find the inverse effect---an electric current produced by magnetism? Yes.

In 1830 Henry placed a bar across the poles of an electromagnet and wrapped an insulated coil around the bar. When the current in the electromagnet was turned on, he discovered that a current is induced in the coil when the magnetic field through it changes.

The results were not published due to Henry's heavy teaching load.

Convert Magnetism into Electricity

Farady independently made the same discovery– through essentially the same experiment.

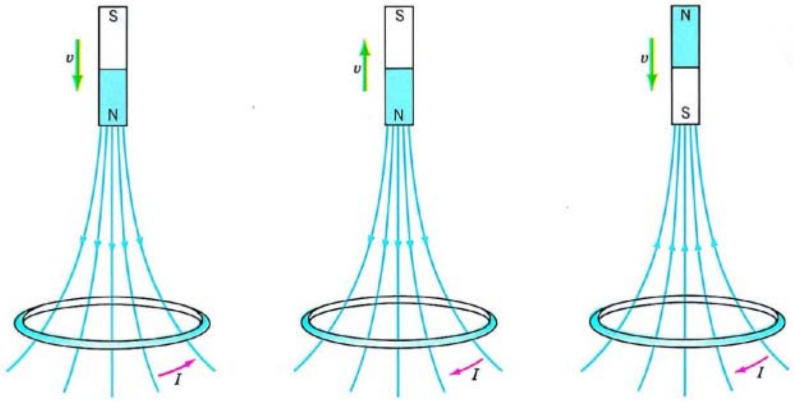


Electromagnetic induction encompasses two phenomena. The first involves a current that is induced in a conductor moving relative to magnetic field lines. The second involves the generation of an electric field associated with a time-dependent magnetic field.

Electromagnetic induction governs the operation of generators and transformers, and many other applications.

31.1 Electromagnetic Induction
(i) Change in Field Strength

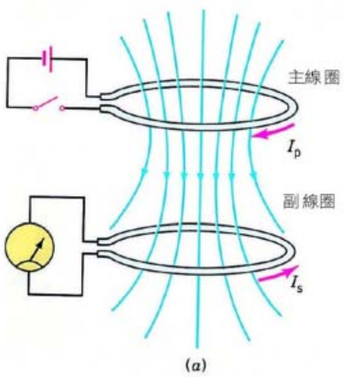
A simple way to generate an electric current with a magnet and a loop of wire.
When the magnet and loop are stationary, nothing happens.
When the north pole moves toward/away, there is counterclockwise/clockwise current.
These result are unchanged when the magnet is kept at rest and the lop is moving instead.



31.1 Electromagnetic Induction
(i) Change in Field Strength

Now consider two coils at rest. The “primary” coil is connected in series wit a battery and a switch, whereas the “secondary’ coil is connected to a galvanometer.

When the switch is closed, the galvanometer deflects for an instant. When the switch is opened, the meter again has a momentary deflection, but in the opposite sense.

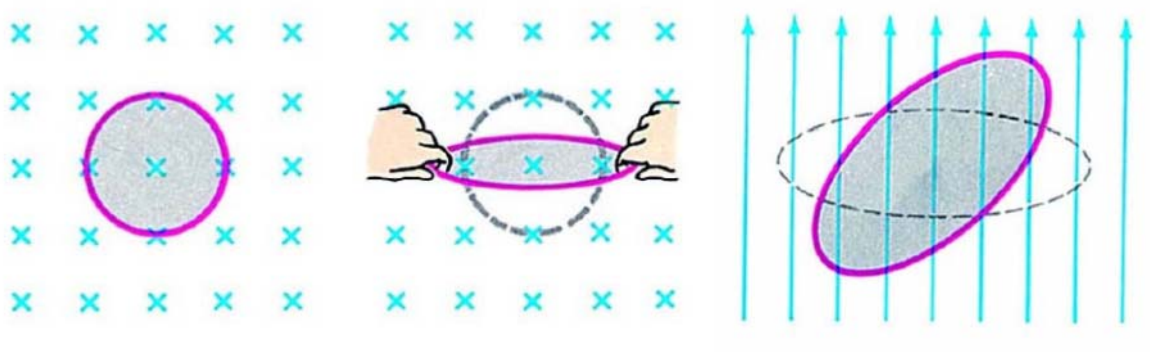


(a)


(b)

31.1 Electromagnetic Induction
(ii) Change in Area & (iii) Change in Orientation

Changing the shape and orientation of a loop in a uniform magnetic field could induce electric current.

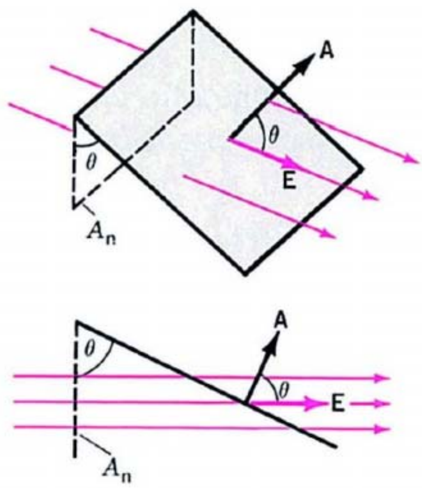


Although many electromagnetic inductions are demonstrated here, what principle governs these behavior?

24.1 Electric Flux

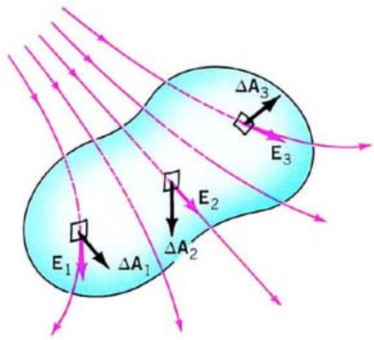
The electric flux Φ_E through this surface is defined as

$$\Phi_E = EA\cos\theta$$
$$= \mathbf{E}\cdot\mathbf{A}$$



For a nonuniform electric field

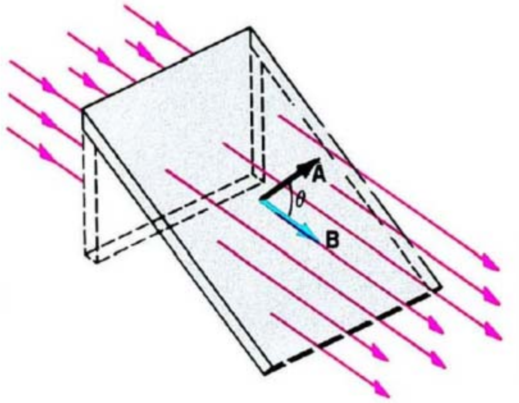
$$\Phi_E = \int \mathbf{E}\cdot\hat{\mathbf{n}}da$$



31.2 Magnetic Flux

The magnetic flux Φ_B is defined in the same way as electric flux.

$$\Phi_B = BA\cos\theta$$
$$= \mathbf{B}\cdot\mathbf{A}$$

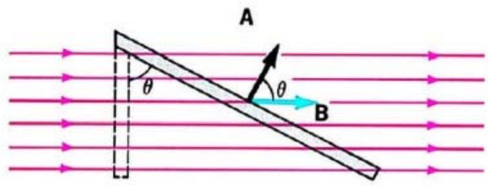


The SI unit of magnetic flux is the weber (Wb)

1 T=1 Wb/m^2

For a nonuniform magnetic field

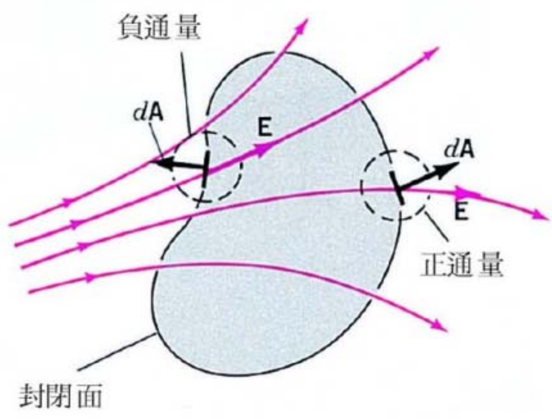
$$\Phi_B = \int \mathbf{B}\cdot\hat{\mathbf{n}}da$$



24.1 Electric Flux (II)

Flux leaving a closed surface is positive, whereas flux entering a closed surface is negative.

The net flux through the surface is zero since the number of lines that enter the surface is equal to the number that leave.



31.3 Faraday's Law and Lenz's Law

The generation of an electric current in a circuit implies the existence of an emf. Faraday's statement is nowadays expressed in terms of magnetic flux:

$$V_{EMF} \propto \frac{d\Phi}{dt}$$

The induced emf along any closed path is proportional to the rate of change of magnetic flux through the area bounded by the path.

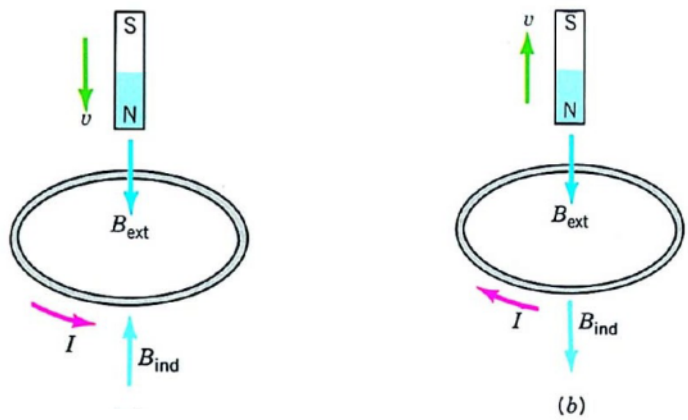
The derivative of magnetic flux is

$$\frac{d\Phi}{dt} = \frac{dB}{dt} A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt}$$

Lenz's Law (I)

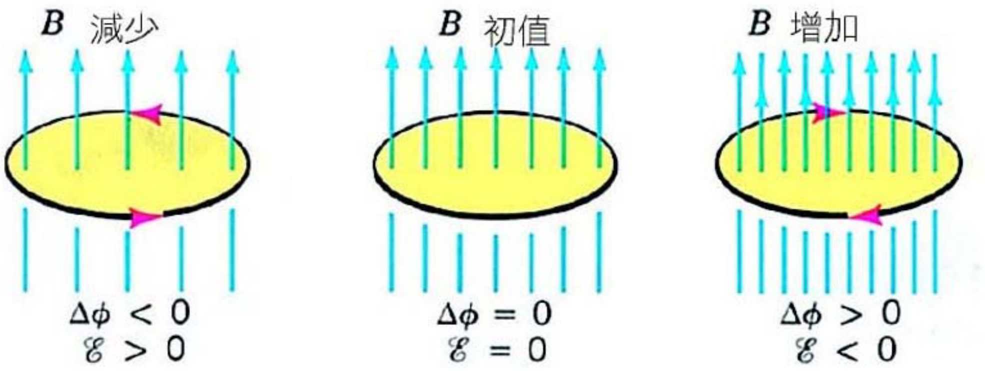
Maxwell restated Lenz's rule in a more general way:

The effect of the induced emf is such as to oppose the change in flux that produces it.



Lenz's Law (II)

A sign convention for the induced emf. First we choose the direction of the vector area to make the initial flux positive. The right-hand rule, with the thumb along B and the fingers curled around the loop, tells us whether clockwise or counterclockwise is the positive sense.



Faraday's Law

The emf is always opposite to the sign of the change in flux $\Delta\Phi$. This feature can be incorporated into Faraday's law by including a negative sign.

The modern statement of Faraday's law of electromagnetic induction is

$$V_{EMF} = -\frac{d\Phi}{dt}$$

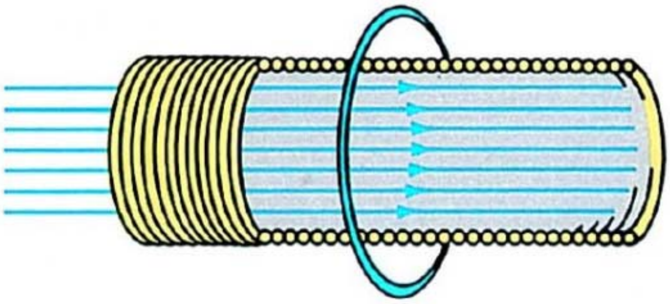
Suppose that the loop is replaced by a coil with *N* turn. The net emf induced in a coil with *N* turns is

$$V_{EMF} = -N\frac{d\Phi}{dt}$$

Example 31.2

An infinite solenoid has 10 turns/cm and a radius of 2 cm. A flat circular coil, of radius 4 cm and 15 turns, is placed around the solenoid with its plane perpendicular to the axis of the solenoid. If the current in the solenoid drops steadily from 3 A to 2 A in 0.05 s, what is the emf induced in the coil?

Solution:



$$\Phi = BA = \mu_0 nIA$$

$$V_{\text{emf}} = -N\frac{d\Phi}{dt}$$

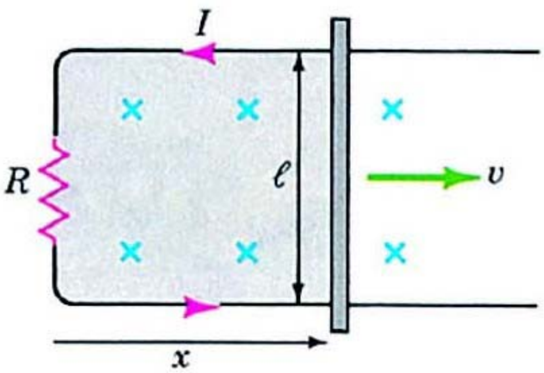
$$= -N\mu_0 nA\frac{dI}{dt}$$

Hint: Capital *N* is different from lower case *n*.

Example 31.3

A metal rod of length L sides at constant velocity v on conducting rails that terminate in a resistor R. There is a uniform and constant magnetic field perpendicular to the plane of the rails. Find: (a) the current in the resistor; (b) the power dissipated in the resistor; (c) the mechanical power needed to pull the rod.

Solution:



(a) $$|V_{\text{emf}}| = \frac{d\Phi}{dt} = B\frac{dA}{dt} = Blv$$

$$\text{I} = \frac{|V_{\text{emf}}|}{R} = \frac{Blv}{R}$$

b) $$P_{elec} = I^2 R = \frac{(Blv)^2}{R}$$

c) $$P_{mech} = \mathbf{F}_{ext} \cdot \mathbf{v} = \frac{(Blv)^2}{R}$$

Example 31.4

A metal bar is moving at 2 cm/s over a U-shape metal rail. At t=0, the external field is 0.2 T out of the page and is increasing at the rate of 0.1 T/s. Take l=5 cm, and at t=-, x=5. Find the induced emf.

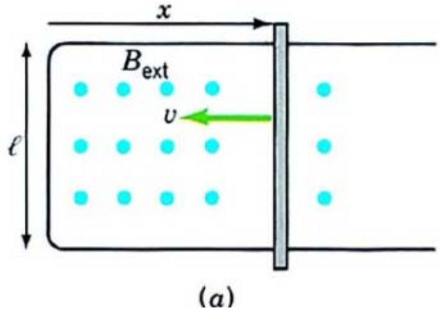
Solution:

$$\left|V_{\text{emf}}\right| = \frac{d\Phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt}$$

$$= 0.2\times 0.05\times 0.02$$

$$+0.1\times 0.0025$$

$$= 5\times 10^{-5}\ \text{V}$$

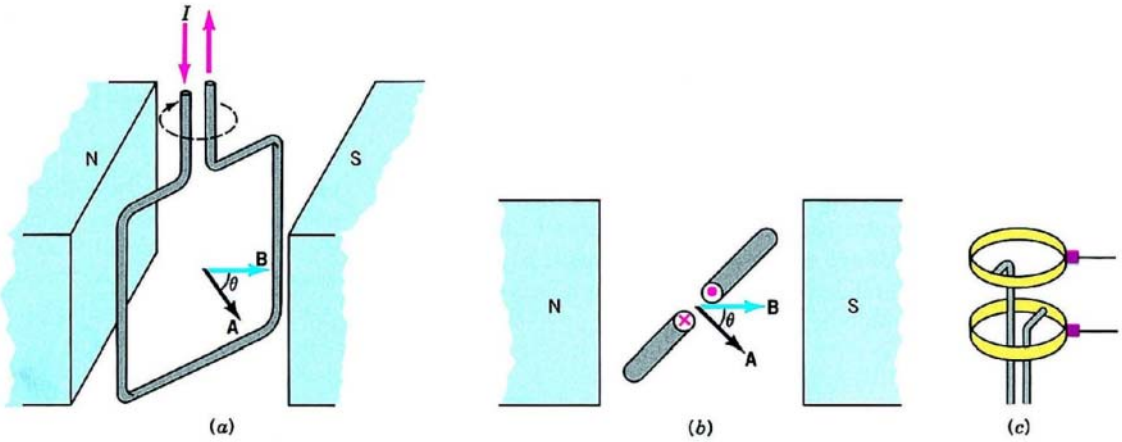


(a)

31.4 Generators

An important application of electromagnetic induction is found in the generator.

It consists of a coil with N turns rotating at angular velocity ω in a uniform external magnetic field.

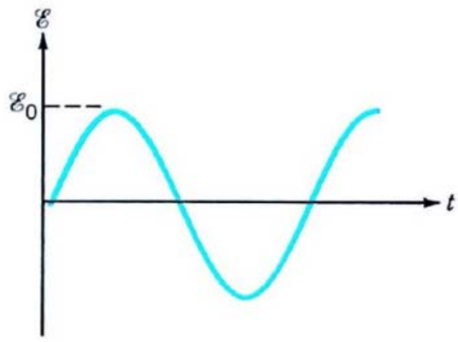


31.4 Generators (II)

The magnetic flux and the induced emf is

$$\Phi = BA\cos(\omega t)$$

$$V_{\text{emf}} = -N\frac{d\Phi}{dt} = NAB\omega\sin(\omega t)$$



As the coil rotates, the emf varies in a sinusoidal fashion; it alternates in sign.

Note that the peak emf occurs at the instant when the flux through the coil is zero.

Example 31.5

A square coil has 25 turns and sides of length 50 cm. It rotates at 120 rpm in a feld of 400 G. at t=0 the plane of the coil is normal to the lines. Find: (a) the peak value of emf; (b) the emf at 1/24 s.

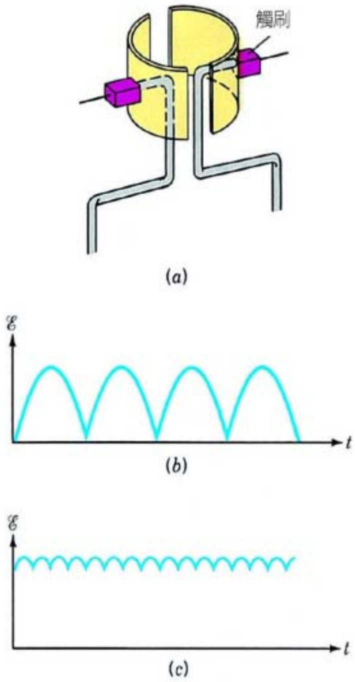
Solution:

$$\left|V_{\text{emf}}\right| = N\frac{d\Phi}{dt} = NBA\omega\sin(\omega t)$$
$$= 25\times 0.04\times 0.25\times 4\pi\sin(\omega t)$$
$$= 3.14\sin(4\pi t)$$

Hint: The initial condition determines the time relation.

31.4 Generators (III)

Is it possible to generate direct current (dc) with the generator?



The first generators produced alternating current (ac) which was unsuitable for many types of experiment.

Sturgeon invented a simple device called a commutator that prevents the direction of the current from changing.

Wheatstone used multiple coils wrapped around a cylindrical form and a multielement commutator. The output of each coil was "tapped" only as it reached its peak emf.

31.5 The Origin of the Induced EMF

Which mechanism generates the "induced emf"?

In Chapter 28, emf was defined as the work done per unit charge by a nonelectrostatic source as the charg moves around a closed loop.

$$V_{\mathrm{emf}} = \frac{W_{\mathrm{ne}}}{q} = \frac{1}{q}\oint \mathbf{F}\cdot d\ell$$

$$= \oint (\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot d\ell$$

- The first term involves an **induced electric field**, which is associated with a time-dependent magnetic field.
- The second term involves motion relative to a magnetic field and is called a **motional emf**.

31.6 Induced Electric Fields

When there is no relative motion between the source of the magnetic field and the boundary of the path around which the emf is evaluated, only the induced electric field is present.

$$V_{\text{emf}} = \oint \mathbf{E} \cdot d\ell = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} \quad \Rightarrow \quad \nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

There is an induced electric field in any closed path, whether in matter or in empty space, through which the magnetic field is changing.

The induced electric field differs from an electrostatic field in two ways. First, the induced electric field are closed loops. Second, the induced electric field is nonconservative field.

Example 31.6

The current in an ideal solenoid of radius *R* varies as a function of time, Find the induced electric field at points (a) inside, and (b) outside the solenoid. Express the results in terms of *dB/dt*.

Solution:

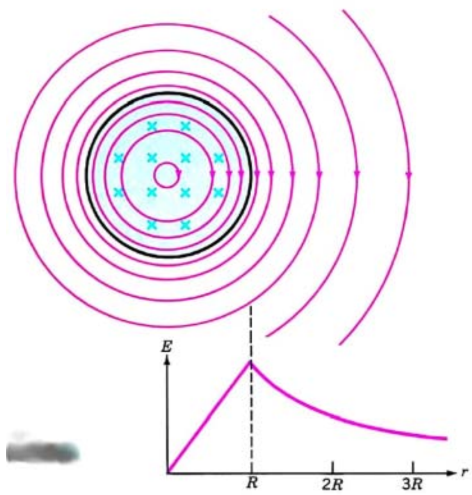
$$\oint E \cdot d\ell = E(2\pi r)$$

$$\text{(a)} \;\; E(2\pi r) = -(\pi r^2)\frac{dB}{dt}$$

$$E = -\frac{r}{2}\frac{dB}{dt} \quad (r < R)$$

$$\text{(b)} \;\; E(2\pi r) = -(\pi R^2)\frac{dB}{dt}$$

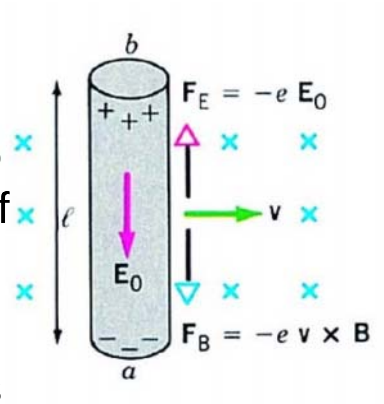
$$E = -\frac{R^2}{2r}\frac{dB}{dt} \quad (r > R)$$



31.7 Motional EMF

When the magnetic field is constant in time, there is no induced electric field.

$$V_{\text{emf}} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\ell = -\frac{d\Phi}{dt}$$



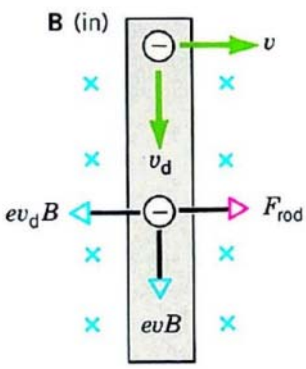
When a mental rod moving perpendicular to magnetic field lines. There is a separation of charge and an associated electrostatic potential difference set up.

The potential difference associated with this electrostatic field is given by $V_b\text{-}V_a\text{=}E_0L\text{=}BLv$.

Since there is no current flowing, the "terminal potential difference" is equal to the **motional emf**.

31.7 Motional EMF (II)

In the previous viewgraph, we find a source of emf converts some form of energy into electrostatic energy and does work on charges. Can magnetic forces do work? No.



The magnetic field acts, in a sense, as an intermediary in the transfer of the energy from the external agent to the rod. To be explained in class.

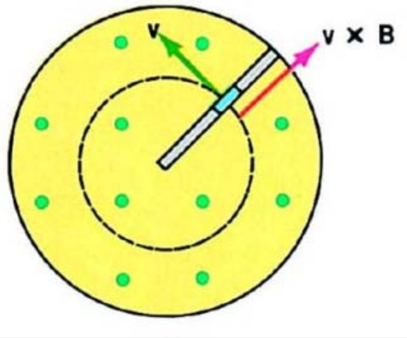
Example 31.7

In a homopolar generator a conducting disk of radius R rotates at angular velocity ω rad/s. Its plane is perpendicular to a uniform and constant magnetic field **B**. What is the emf generated between the center and the rim?

Solution:

$$V_{\text{emf}} = \oint (v \times B) \cdot d\ell = \int_0^R vBdr$$

$$= \int_0^R \omega r B dr = \frac{1}{2}\omega B R^2$$

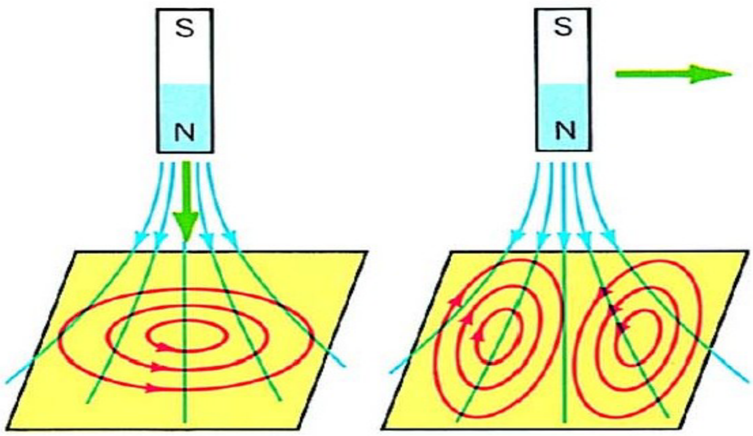


Hint1: How to choose a proper closed loop?

Hint2: The total magnetic flux passing through the disk is constant in time. Where is the induced emf coming from? (Extra bonus, Ref. Benson & Feyman)

31.8 Eddy Currents (I)

What happens when a bar magnet approaches or moves parallel to a conducting plate? It induces eddy current.

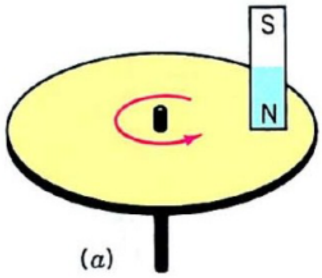


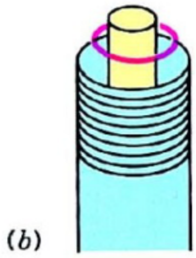
The eddy current is distributed throughout the plate.

31.8 Eddy Currents (II)

Applications of the eddy current:

1. The braking system of a train.
2. Eddy current generated in copper pots can also be used for “inductive cooking”.
3. Project a metal ring. The ring gets very hot when projected.





Exercises and Problems

Ch.31:

Ex. 14, 15, 22, 25

Prob. 3, 6, 7, 9, 10