# Chapter 33 Alternating Current Circuits

We have discussed only circuit with *direct current* (dc)--- which flows in only one direction. However, many instruments or appliances plugged into a wall outlet are powered by ac sources of emf.

How to deal with emf sources that produce an *alternating current* (ac) that changes direction periodically?

We study the response of resistors, inductors, and capacitors to an applied alternating emf. We first treat each of these circuit elements individually and then study a series combination. The series RLC circuit is particularly important because the current in it exhibits resonance as the frequency of ac source is varied.



# 33.1 Some Preliminaries

*How do the ac emf and the current produced vary in time? Sinusoidal in time.*

Assume that the instantaneous current *i* always has the form

$$i = i_0 \sin(\omega t)$$

Where *f* is the frequency in hertz (Hz) and $\omega=2\pi f$ is the angular frequency. The amplitude $i_0$ is called the peak value of the current.

The instantaneous potential difference across the terminals of the source is written as

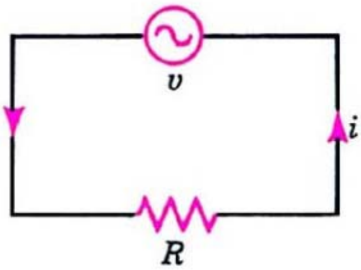
$$v = v_0 \sin(\omega t + \phi)$$

Where v0 is the peak value and $\phi$ is the phase difference between the current and the potential difference.

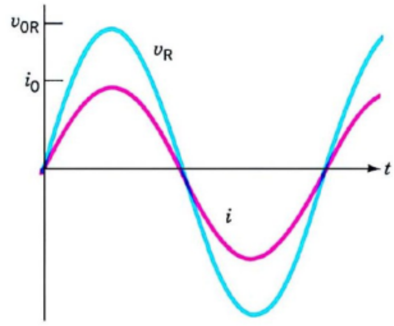
Note: The lowercase letters are used for the instantaneous values of current and the potential difference.

# 33.2 A Resistor in an AC Circuit; Root Mean Square Values

A resistor is connected to an ideal ac source of emf. According to Kirchhoff's loop rule, we find that the instantaneous potential difference across the resistor is




$$v_R = iR = i_0 R \sin(\omega t) = v_{0R} \sin(\omega t)$$




From above equation, we see that the current and potential difference are **in phase** (ϕ=0).



# The Instantaneous and RMS Powers

The instantaneous power, *p*, dissipated in the resistor is

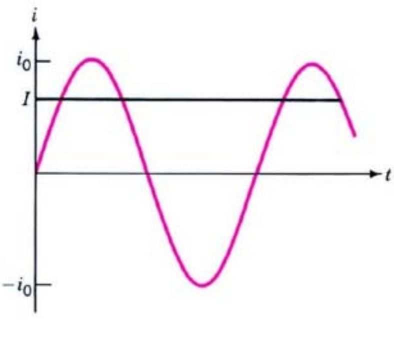
$$p = iv_R = i^2 R = i_0^2 R \sin^2(\omega t)$$

$$= i_0^2 R \frac{1}{2}(1 - \cos 2\omega t)$$

The average values of power and current square are

$$(p)_{avg} = i_0^2 R \frac{1}{2}\int_0^{2\pi/\omega}(1-\cos 2\omega t)dt = \frac{i_0^2 R}{2}$$

$$(i^2)_{avg} = \frac{i_0^2}{2} \;\Rightarrow\; I = \sqrt{(i^2)_{avg}} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$



## The Instantaneous and RMS Powers (II)

The uppercase letters *I*, *V*, and *P* will be used for **root mean square** (rms) values of current, potential difference, and power.

$$I = \sqrt{(i^2)_{avg}} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$

$$V = \sqrt{(v^2)_{avg}} = \frac{v_0}{\sqrt{2}} = 0.707 v_0$$

$$P = IV_R = I^2 R = V_R^2 / R$$

We retain the usual (dc) form of the equation for electrical power provided the rms values of current or potential difference are used.



## Example 33.1

A light bulb is rated at 100 W rms when connected to a 120-V rms wall outlet. Find: (a) the resistance of the bulb; (b) the peak potential difference of the source; (c) the rms current through the bulb.

Solution:

$$\text{(a)}\ \ \mathrm{R} = V_{rms}^2 / P_{avg} = 120^2 / 100 = 144\ \Omega$$

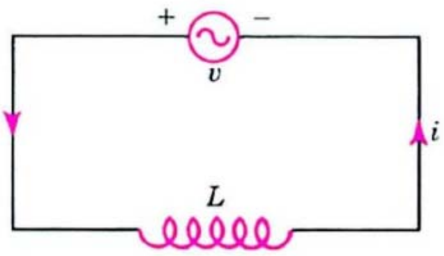
$$\text{(b)}\ \ v_{0R} = \sqrt{2} V_{rms} = 1.414 \times 120 = 170\ \mathrm{V}$$

$$\text{(c)}\ \ I_{rms} = P_{avg} / V_{rms} = 0.833\ \mathrm{A}$$

# 33.3 An Inductor in an AC Circuit

The right figure shows an inductor connected to an ac source.

From the loop rule, we know




$$v - v_L = 0$$

where $v_L = L\dfrac{di}{dt}$ is the instantaneous potential difference across the inductor.

Note: We use $v_L$ rather than the induced emf ($V_{emf}$=-Ldi/dt), so that the equations for R, L, and C will have the same forms. How can we do this?



# 33.3 An Inductor in an AC Circuit (II)

The rate of change of the current is

$$\frac{di}{dt} = i_0 \omega \cos(\omega t) \quad \Rightarrow \quad v_L = L\frac{di}{dt} = i_0 \omega L \cos(\omega t) = v_{0L} \sin(\omega t + \frac{\pi}{2})$$

The phase angle is $\phi$=+90°, which means that $v_L$ leads $i$ by 90°.

The peak value is

$$v_{0L} = i_0 \omega L \quad \Rightarrow \quad v_{0L} = i_0 X_L$$

where the quantity $X_L = \omega L$ is called the reactance of the inductor.

The SI unit of reactance is the ohm.

The instantaneous power supplied to the inductor is

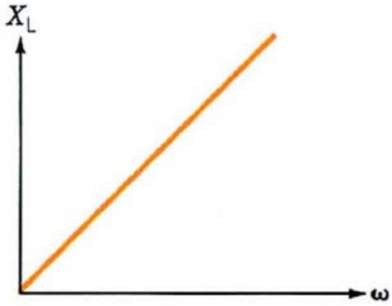
$$p = i v_L = i_0^2 \omega L \sin(\omega t) \cos(\omega t)$$

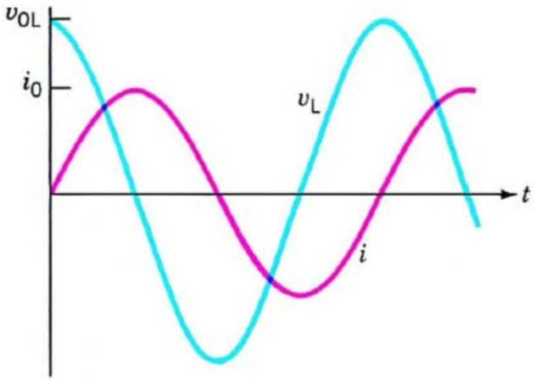
## 33.3 An Inductor in an AC Circuit (III)

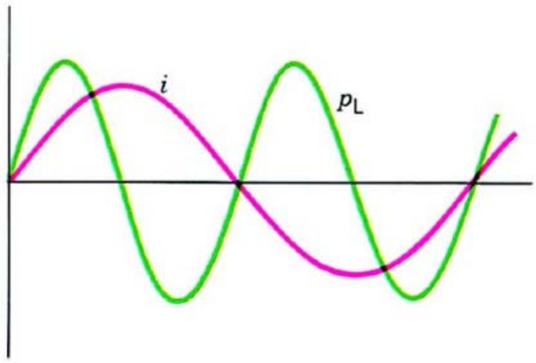
$$i = i_0 \sin(\omega t)$$

$$v_L = i_0 \omega L \cos(\omega t) = v_{0L} \sin(\omega t + \frac{\pi}{2})$$

$$p = i v_L = i_0^2 \omega L \sin(\omega t) \cos(\omega t)$$



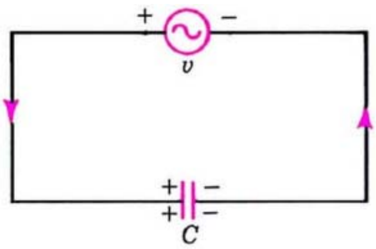



## 33.4 A Capacitor in an AC Circuit

The right figure shows a capacitor connected to an ac source.

From the loop rule, we know

$$v - v_C = 0 \quad \text{where } v_C = \frac{q}{C} = \frac{1}{C}\int i\,dt.$$




Note: The current in the circuit (not through the capacitor itself) is charging the plate, so *i*=+*dq/dt* or *dq*=*idt*. Thus,

$$q = \int i\,dt = \int i_0 \sin(\omega t)\,dt = -\frac{i_0}{\omega}\cos(\omega t) + \text{constant}$$

The constant depends on the initial conditions and we take it to be zero. Why?

# 33.4 An Capacitor in an AC Circuit (II)

According to the loop rule, we have

$$v_C = -\frac{i_0}{\omega C}\cos(\omega t) = -v_{0C}\cos(\omega t) = v_{0C}\sin(\omega t - \frac{\pi}{2})$$

The phase angle is $\phi$=-90°, which means that $v_C$ lags $i$ by 90°.

The peak value is

$$v_{0C} = i_0 \frac{1}{\omega C} \quad \Rightarrow \quad v_{0C} = i_0 X_C$$

where the quantity $X_C = 1/\omega C$ is called the reactance of the capacitor.

The SI unit of reactance is the ohm.

The instantaneous power supplied to the capacitor is

$$p = i v_C = -\frac{i_0^2}{\omega C}\sin(\omega t)\cos(\omega t)$$

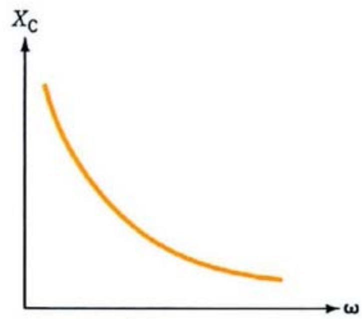


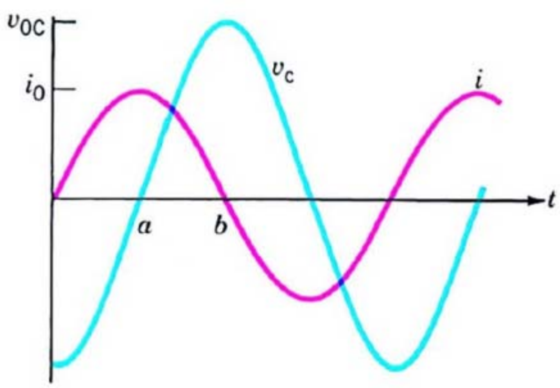
# 33.4 An Capacitor in an AC Circuit (III)

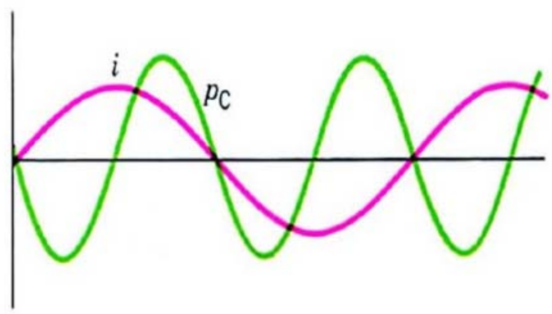
$$i = i_0 \sin(\omega t)$$

$$v_C = -\frac{i_0}{\omega C}\cos(\omega t) = v_{0C}\sin(\omega t - \frac{\pi}{2})$$

$$p = i v_C = -\frac{i_0^2}{\omega C}\sin(\omega t)\cos(\omega t)$$

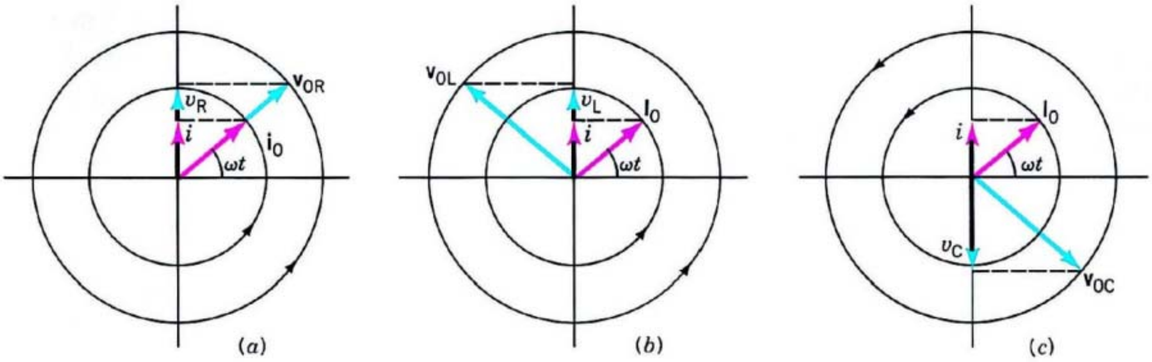


# 33.5 Phasors

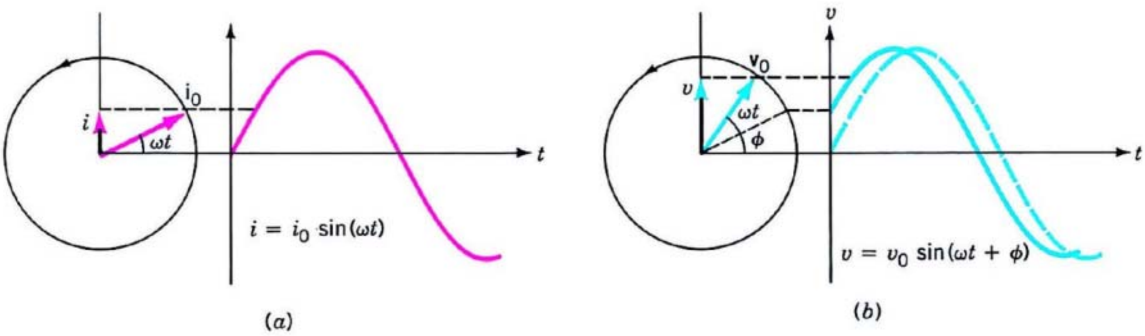
The phase relation between current and potential difference are easily determined for a single capacitor or inductor. However, when several such elements are combined in a circuit, the relation becomes much more complicate. Thus, we have to develop more powerful analytical tools.





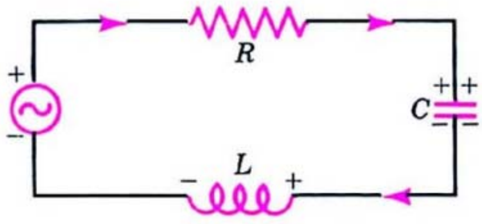

# 33.5 Phasors (II)

A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents and which rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.



# 33.6 *RLC* Series Circuit

Consider a circuit consisting of a resistor, an inductor, and a capacitor in series with an ac source, as shown in Fig. 33.15.




The instantaneous current is the same at all points of the circuit. Why? due to conservation of charge.

According to the loop rule, the instantaneous potential differences are related by

$$v - v_R - v_L - v_C = 0$$

$$v = v_R + v_L + v_C$$



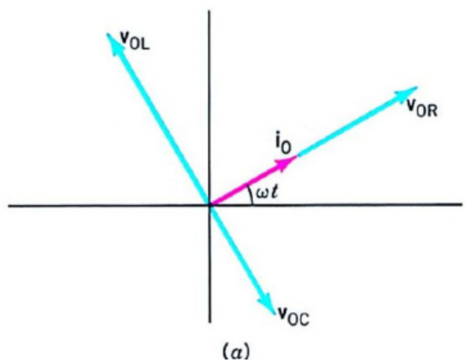
# 33.6 *RLC* Series Circuit (II)

$$v = v_R + v_L + v_C = i_0 R \sin \omega t + i_0 L \omega \cos(\omega t) + \frac{\int i_0 \sin(\omega t) dt}{C}$$

Can we write above relation in a simpler form? Yes, phasor.

To relate *v* to *i*, we must find the vector sum of the potential difference phasors:

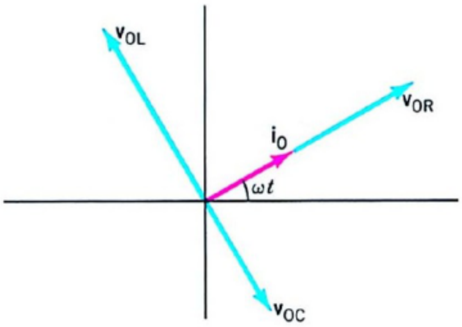
$$\mathbf{v}_0 = \mathbf{v}_{0R} + \mathbf{v}_{0L} + \mathbf{v}_{0C}$$



# 33.6 *RLC* Series Circuit (III)

By Pythagoras's theorem, the magnitude of the sum is given by

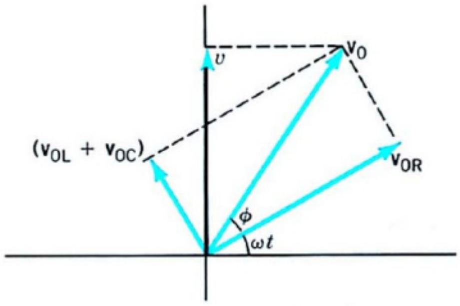
$$v_0^2 = \mathbf{v}_{0R}^2 + \mathbf{v}_{0L}^2 + \mathbf{v}_{0C}^2$$
$$= i_0^2\left[R^2 + (X_L - X_C)^2\right]$$



(a)

This may be written in the form of Ohm's law,

$$v_0 = i_0 Z$$
$$Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}$$



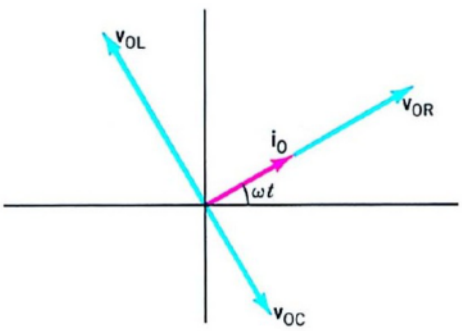
(b)



# 33.6 *RLC* Series Circuit (III)

$$Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}$$

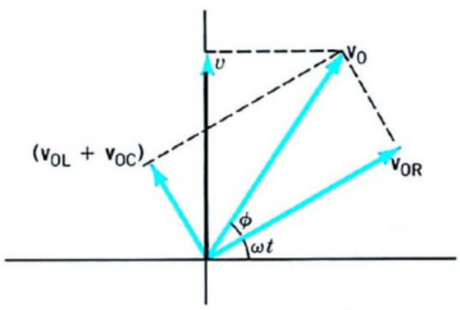
Is the impedance of the series circuit. The SI unit of impedance is the ohm.



(a)

$$\tan\phi = \frac{X_L - X_C}{R}$$

A positive phase angle indicates that the driving potential difference v is ahead of the current i by $\phi$



(b)

## Example 33.2

An ac source of emf with frequency 50 Hz and a peak potential difference of 100  V is in an RLC series circuit with R=9 ohm, L=0.04 H, and C=100 uF. Find: (a) the impedance; (b) the phase angle; (c) the peak potential difference across each element.

Solution:

$$\text{(a)}\ \omega = 2\pi f = 100\pi \text{ rad/s},\ X_L = \omega L = 4\pi;\ X_C = \frac{1}{\omega C} = \frac{100}{\pi}$$

$$\text{(b)}\ \tan\phi = \frac{X_L - X_C}{R} = \frac{-19.2}{9},\ \phi = \tan^{-1}(\frac{-19.2}{9})$$

$$\text{(c) The peak current through the circuit } i_0 = v_0 / Z = 100/21.2 = 4.72 \text{ A}$$

$$v_{0R} = i_0 R = 42.5 \text{ V},\quad v_{0L} = i_0 X_L = 59.5 \text{ V},\quad v_{0C} = i_0 X_C = 150 \text{ V}$$

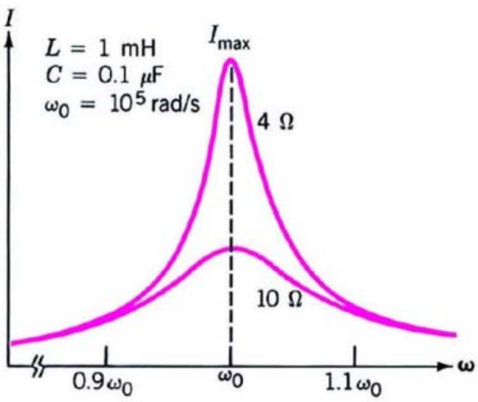


## 33.7 *RLC* Series resonance

At what condition we can obtain the optimum current. The mimimum impedance.

$$i_0 = v_0 / Z,\ \text{where } Z = \sqrt{\left[R^2 + (X_L - X_C)^2\right]}$$

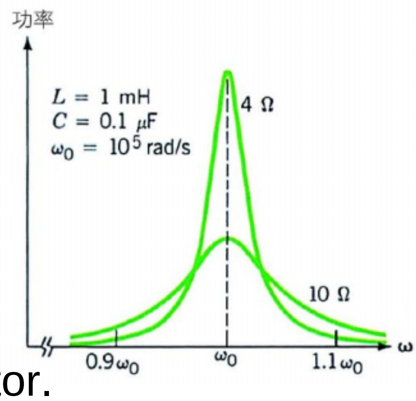
$$Z_{min} @ X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \text{ i.e. } \omega = \frac{1}{\sqrt{LC}}$$



# 33.8 Power in AC Circuit

The instantaneous power delivered by the source of emf is

$$p = iv = i_0 v_0 \sin(\omega t)\sin(\omega t + \phi)$$
$$= i_0 v_0 \sin^2(\omega t)\cos\phi + \sin\omega t\cos\omega t\sin\phi$$
$$p_{av} = \langle i_0 v_0 \sin(\omega t)\sin(\omega t + \phi)\rangle = \frac{1}{2} i_0 v_0 \cos\phi$$




The quantity cosϕ is called the power factor.

The **rms power**, P=$p_{av}$, delivered by the source is

$$P = p_{av} = \frac{1}{2} i_0 v_0 \cos\phi = \langle i\rangle\langle v\rangle\cos\phi$$
$$= IV\cos\phi = I^2 R$$



# Example 33.3

In an RLC series circuit R=50 ohm, C=80 uF, and L=30 mH. The 60-Hz source has an rms potential difference of 120 V. Find: (a) the rms current and potential difference for each element; (b) the power factor; (c) the rms power delivered by the source; (d) the resonance frequency; (e) the peak values of current and potential difference for each element at the resonance frequency.

Solution:

$$\text{(a)}\ \omega = 2\pi f = 120\pi \text{ rad/s},\ X_L = \omega L = 3.6\pi;\ X_C = \frac{1}{\omega C} = 33.2\,\Omega$$

$$\text{(b)}\ \tan\phi = \frac{11.3 - 33.2}{50} = -0.438,\ \phi = -23.6°,\ \cos\text{-}23.6° = 0.916$$

$$\text{(c)}\ P = IV\cos\phi = I^2 R$$

$$\text{(d)}\ f_0 = \frac{\omega_0}{2\pi} = 103\,\text{Hz}$$

# Exercises and Problems

Ch.33:

Ex. 23, 30, 32, 35

Prob. 4, 5, 9, 11