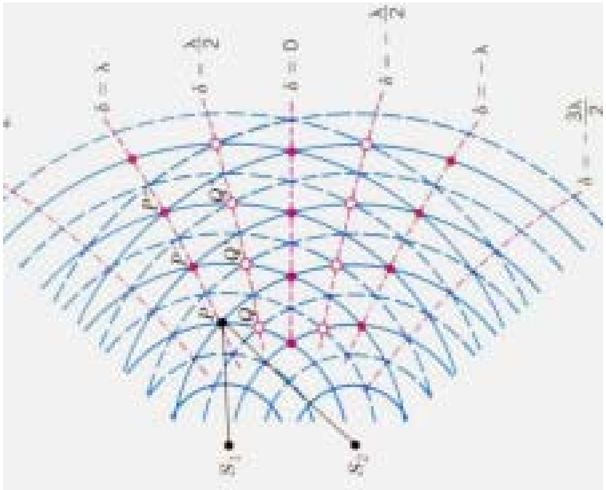
Chapter 37 Wave Optics (I)

Destructive interference: When a crest and a trough overlap, they momentarily cancel to produce zero displacement.

Constructive interference: When two crests overlap, they reinforce each other so that the amplitude of the resonant is double that of either pulse.

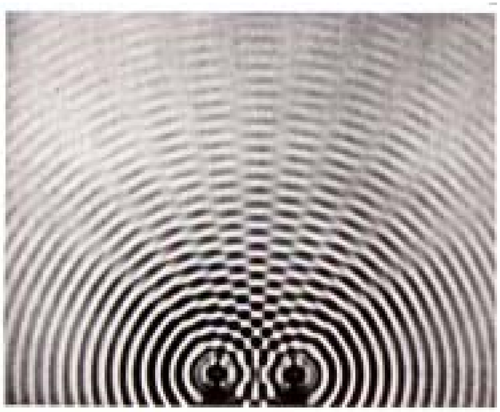


37.1 Interference

The interference condition of two or more waves depends strongly on their **phase relation** $\Delta\phi$.

Maxima: If two waves are **in phase** ($\delta=m\lambda$), they will have constructive interference.

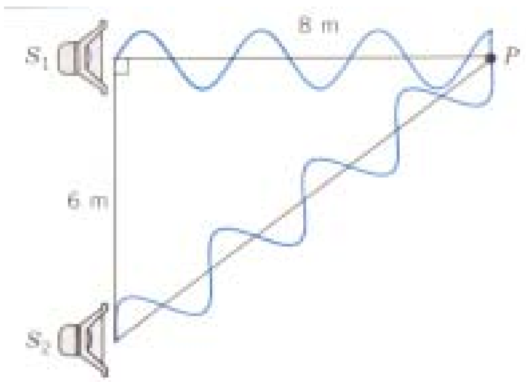
Minima: On the contrary, if two waves are **out of phase** ($\delta=(m+1/2)\lambda$), they will experience destructive interference.



Example 37.1

Two speakers S_1 and S_2 are separated by 6 m and emit sound waves in phase. Point *P* in Fig. 37.4 is 8 m from S_1. What is the minimum frequency at which the intensity at *P* is (a) a minimum, (b) a maximum? Take the speed of sound to be 340 m/s.

Solution:



L1 (S1-P)=8 m

L2 (S2-P)=10 m

δ=2 m

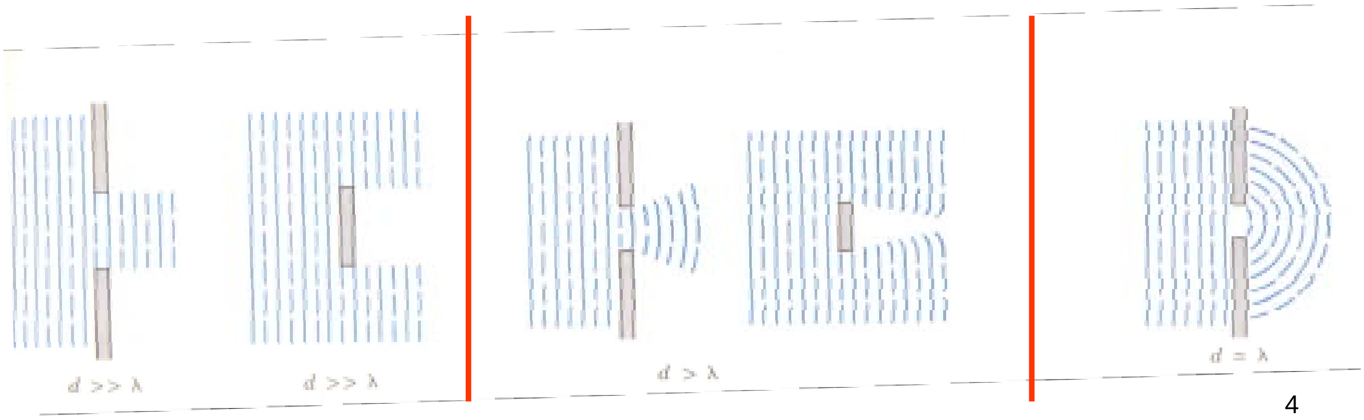
Minimum: $\delta=1/2\lambda$, λ=4 m, f=340/4=85 Hz

Maximum: $\delta=\lambda$, λ=2 m, f=340/2=170 Hz

37.2 Diffraction

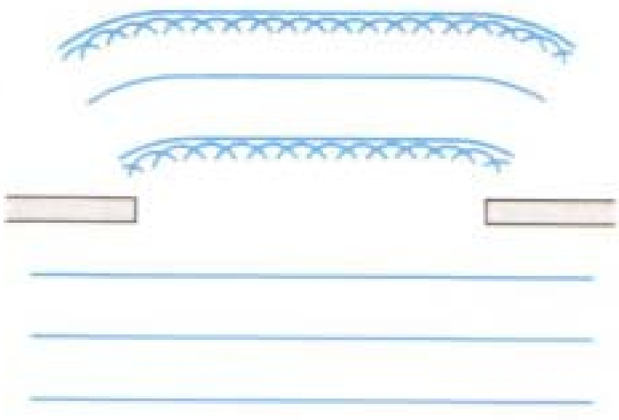
Diffraction: Rays or wavefronts bend at the edge of an opening, or an obstruction.

Size effect: The extent to which diffraction modifies the rectilinear propagation of waves depends on the relative size of the wavelength and the opening (or obstruction).



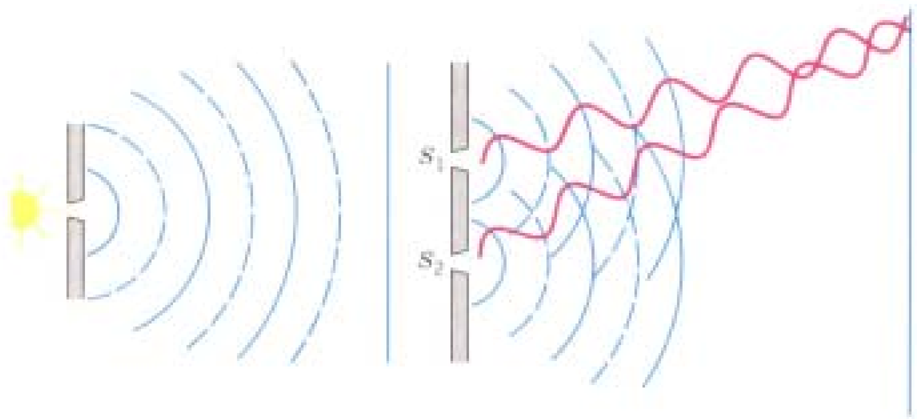
37.2 Diffraction (II)

Huygens' construction: Each point on the approaching wavefronts acts as a source of secondary wavelets. When the fronts reach the aperture or obstruction, only the wavelets from the unobstructed region can contribute to the wavefronts on the right side.



37.3 Young's Experiment

In 1802, Young demonstrated **the wave nature of light** through a double slits experiment.

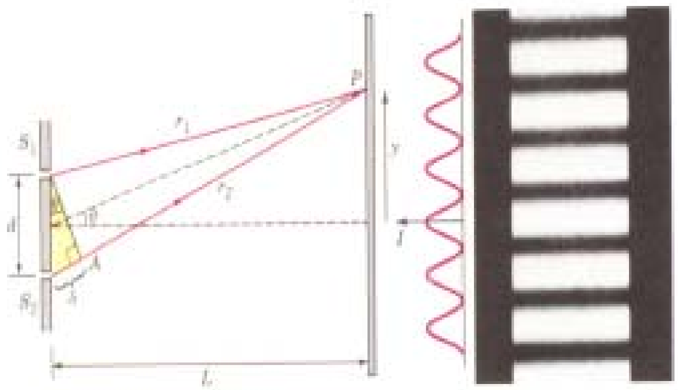


Such fringes cannot be explained by **the particle model of light**, which was advocated by Newton and generally accepted during the 18th century.

Young's Experiment (II)

To derive an expression for the position of the fringes, assume the light has a **single wavelength**, λ, and that the separation between the slits is d. If the screen is far away, the outgoing rays are almost **parallel**, and the path difference, δ (S2-A), is

$$\delta \pm = r_2 - r_1 \approx d \sin\theta$$



$$\text{Maxima}: \; d \sin\theta = m\lambda$$

$$\text{Minima}: \; d \sin\theta = (m + \tfrac{1}{2})\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Young's Experiment (III)

Condition for the interference pattern to be observed:

1. Single frequency source,
2. Constant phase relationship.

➔ Sources that emit waves of the same frequency and have a constant phase relationship are said to be **coherent**.

How to generate such condition if the only available source is the sun light?

Example 37.2

Calculate the spacing between the bright fringes of yellow light of wavelength 600 nm. The slit separation is 0.8 mm, and the screen is 2 m from the slits.

Solution:

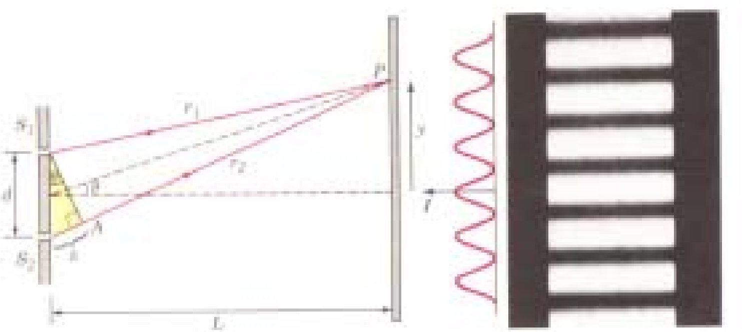
$d=8\text{x}10^{-4}$ m

$\lambda=6\text{x}10^{-7}$ m

$d \sin\theta = m\lambda$

$\sin\theta = y/2$

$\Delta y = 2\lambda/d = 1.5\text{x}10^{-3}$ m=1.5 mm



37.4 Intensity of the Double-Slit Pattern

The traveling wave equations are:

$$E_1(t,s_1)=E_0\sin(\omega t-ks_1)$$
$$E_2(t,s_2)=E_0\sin(\omega t-ks_2)$$

For a given position, ks_1 and ks_2 can be treated as phase constant. Only the **phase difference** will contribute to the intensity.

$$E_1(t)=E_0\sin(\omega t)$$
$$E_2(t)=E_0\sin(\omega t+\phi)$$

A path difference δ corresponds to a phase change ϕ through the following relation:

$$\phi=\underbrace{k}_{\frac{2\pi}{\lambda}}\underbrace{(S_1-S_2)}_{\delta}=\frac{2\pi\delta}{\lambda}$$

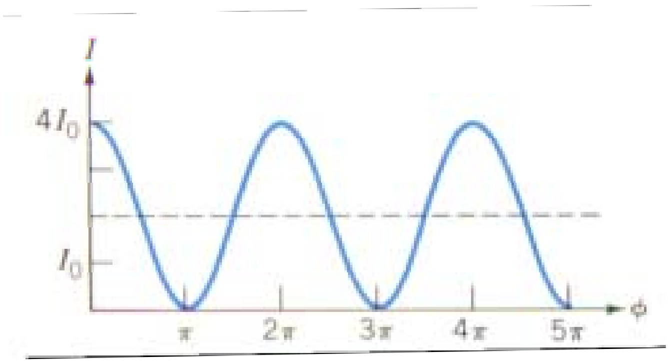
Intensity of the Double-Slit Pattern (II)

The resonant field is found from the principle of superposition:

$$
\begin{aligned}
E &= E_1 + E_2 \\
&= E_0(\sin(\omega t) + \sin(\omega t + \phi)) \\
&= \left[2E_0 \cos(\frac{\phi}{2})\right] \sin(\omega t + \frac{\phi}{2})
\end{aligned}
$$

The intensity of a wave is proportional to the square of the amplitude, so we have

$$I = 4I_0 \cos^2(\frac{\phi}{2})$$

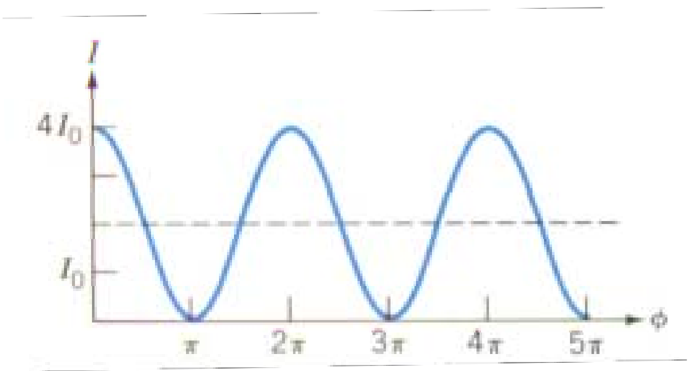


Intensity of the Double-Slit Pattern (III)

$$\text{Maxima}: \ \phi = 2m\pi, \qquad d\sin\theta = m\lambda$$

$$\text{Minima}: \ \phi = (2m+1)\pi, \ d\sin\theta = (m + \tfrac{1}{2})\lambda$$

$$m = 0, \pm 1, \pm 2, \ldots$$



37.5 Thin Films

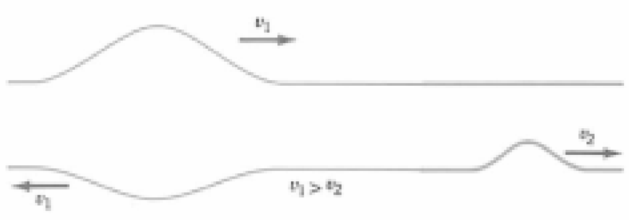
The **colors** in soap bubbles, in oil patches on the road, and in peacock feathers are due to the **interference of light waves** that have been reflected from the **two surfaces** of a thin film.

When light encounters a different reflective index, it will partly **transmit** and partly **reflect**.

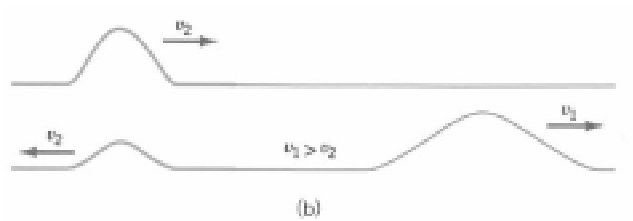
When light encounters a medium of **higher refractive index**, the reflected wave suffers **a phase change of** π.

When light encounters a medium of **lower refractive index**, the reflective wave does **not change** its phase.

Between the Extreme Cases of a Fixed End or a Free End



from **light** string to **heavy** string

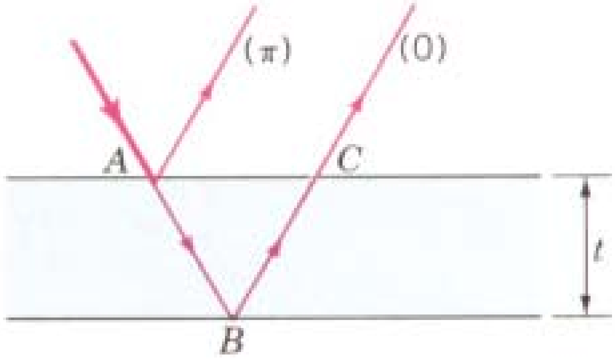


from **heavy** string to **light** string

This results in **partial** reflection and transmission.

Since the tensions are the same, the relative magnitudes of the wave velocities are determined by **mass densities**.

Thin Films (II)



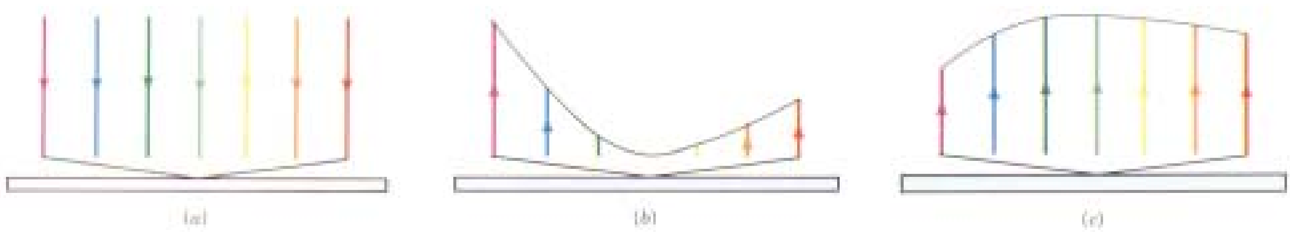
Note: the speed of light in the medium (v=c/n) is different from that in the vacuum. Therefore, the wavelength in the film is shorter by a factor of refractive index n, $\lambda_F=\lambda/n$.

$$\text{Maxima}: \ 2t = (m + \tfrac{1}{2})\lambda_F$$

$$\text{Minima}: \ 2t = m\lambda_F$$

$$m = 0, \pm 1, \pm 2, \ldots$$

The Nature of Color in Thin Films



Subtractive Colors:

When the green is missing, the apparent color of the film is magenta (品紅色).

When the red is missing, the apparent color of the film is cyan (青綠色).

When the blue is missing, the apparent color of the film is yellow.

Lens Coating

When light is incident normally on the boundary between air (n=1) and glass (n=1.5), about 4% of the energy is reflected and 96% is transmitted. Thus, a camera with 6 lenses has 12 air-glass interfaces, which means that only $(0.96)^{12}=0.61$ or 61% of the incident energy is transmitted.

How to optimize the transmission of signal intensity? **Lens coating**.

The **loss due to the reflection** is minimized by coating each lens surface with a thin film.

The **thickness** and **refractive index** of the thin film are chosen so that the reflected yellow light is destructive interference.

MgF_2 (n=1.38) is often used for its *durability*.

Example 37.3

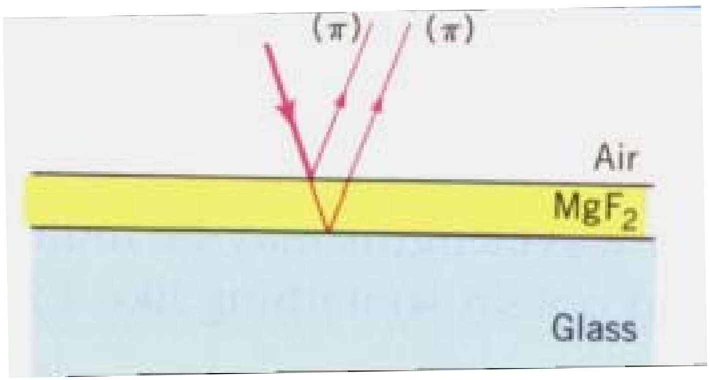
White light is incident normally on a lens (n=1.52) that is coated with a film of MgF2 (n_F=1.38). For what minimum thickness of the film will yellow light (λ=550 nm in air) be missing in the reflected light?

Solution:

$2t=(m+1/2)\lambda/n_F$, m=0

$t_{min}= \lambda/4n_F=5.5\times10^{-7}/(4\times1.38)$

$\approx$100 nm

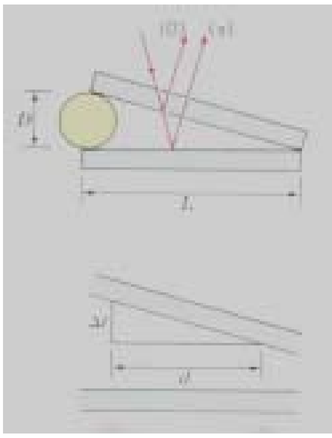


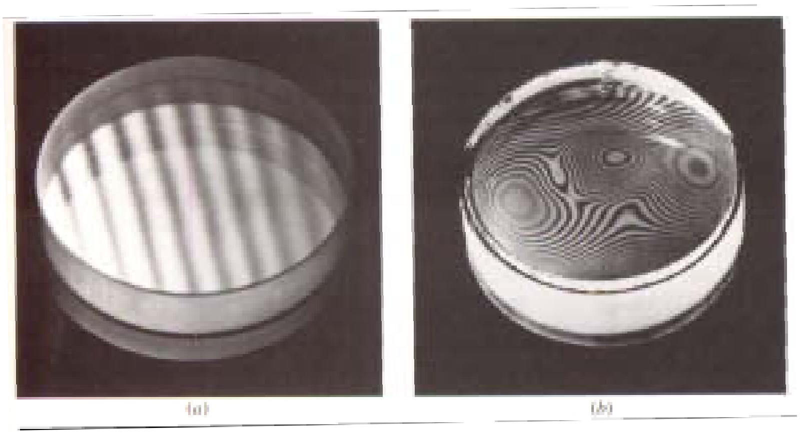
Is it possible for the medium (e.g. glass) to absorb the light and convert it into heat?

Fringes of Equal Thickness

A wedge-shaped film of air may be produced by placing a sheet of paper or a hair between the ends of two glass plates, as shown in Fig. 37.17.

With flat plates, one sees **a series of bright and dark bands**, each characteristic of a particular thickness.

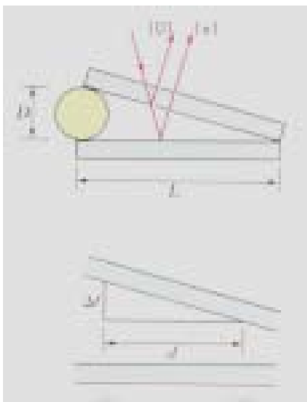




Example 37.4

A wedge-shaped film of air is produced by placing a fine wire of diameter D between the ends of two flat glass plates of length L=20 cm, as in Fig.37.17. When the air film is illuminated with light of wavelength 550 nm, there are 12 dark fringes per centimeter. Find D.

Solution:



$2t=m\lambda$ m=0, 1, 2 …

$\Delta t= \lambda/2$; d=1/12 cm

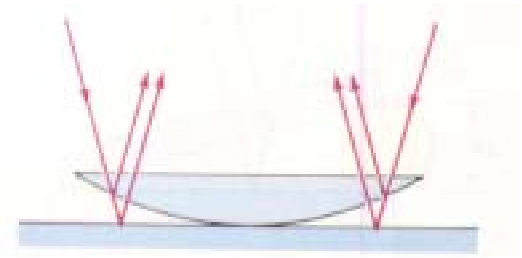
$D/L=\Delta t/d$

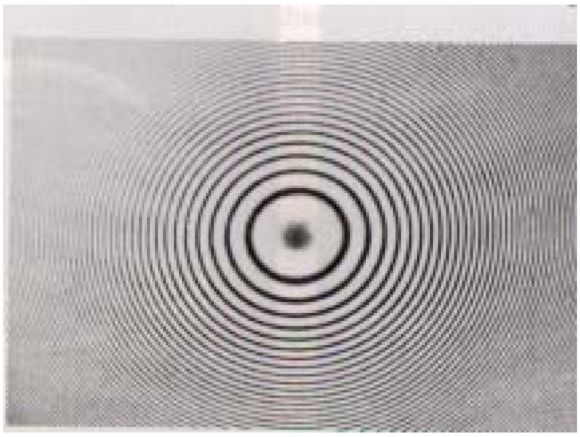
$D=\lambda L/2d=66$ um

What is the difference between “geometrically flat” and “optically flat”?

Newton's Ring

When a lens with a large radius of curvature is place on a flat plate, as in Fig. 37.19, a **thin film of air** is formed. When Newton is illuminated with **mono-chronomatic** light, **circular fringes**, called **Newton's Rings**, can be seen.





Why the center spot is dark? It implies the wave nature.

Example 37.5

In an experiment on Newton's rings the light has a wavelength of 600 nm. The lens has a refractive index of 1.5 and a radius of curvature of 2.5 m. Find the radius of the 5th bright fringe.

Solution:

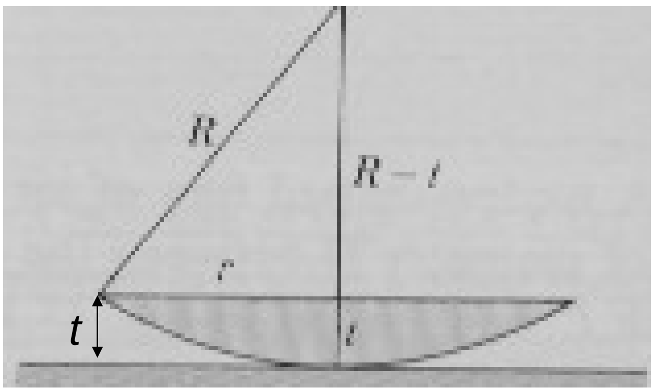
$r^2 = 2Rt$, if $R >> t$

$2t = (m+1/2)\lambda_{air}$, m=4

$t = 4.5*600$ nm /2

$= 1.35$ um

$r = \text{sqrt}(2Rt) = 2.6$ mm

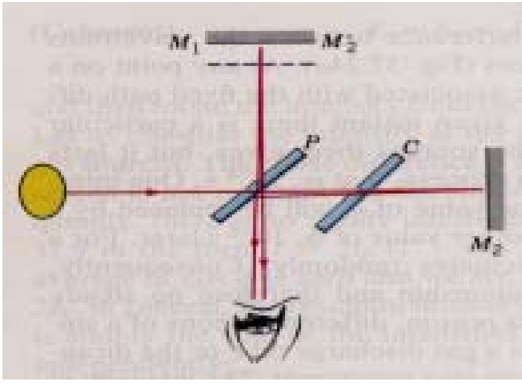


37.6 Michelson Interferometer

An interferometer is a device that uses **interference** to make precise measurements of **distances** in terms of the **wavelength** of the light.

The system is equivalent to an **air film**.

Michelson's interferometer is useful because one mirror may be **moved** on a finely threaded screw, so that the **thickness** of the film is **continuously adjustable**.



Exercises and Problems

Ch.37:

Ex. 11, 18, 21, 24, 37, 67, 69, 72

Prob. 6, 11,15