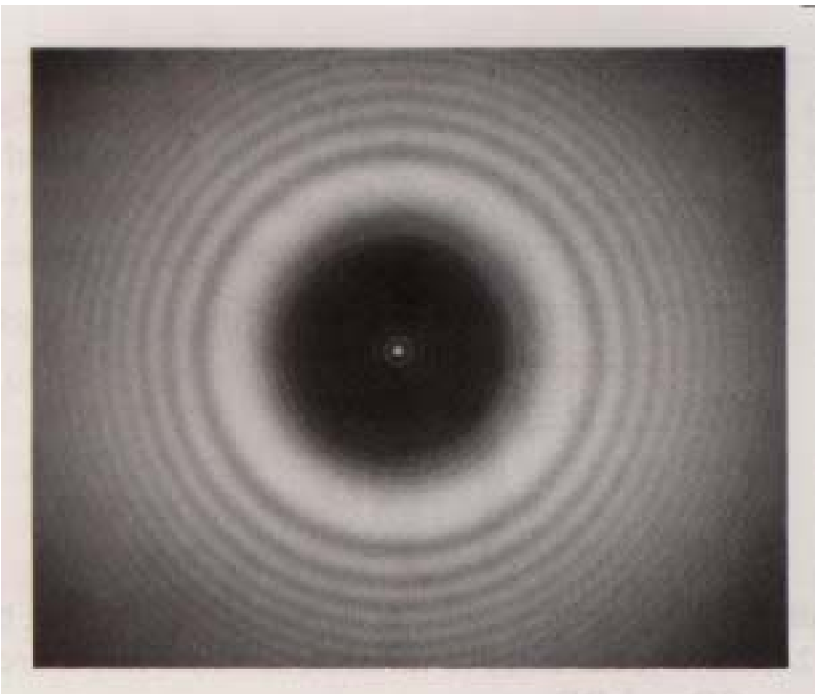
Chapter 38 Wave Optics (II)

Initiation: Young's ideas on light were daring and imaginative, but he did not provide rigorous mathematical theory and, more importantly, he is arrogant.

Progress: Fresnel, independently performed double-slits experiment and did a mathematical theory of interference, diffraction, and other phenomena. He found that the size of the aperture or obstacle relative to the wavelength is significant in determining the amount of diffraction.

Acceptance: Poisson pointed out an "absurd" consequence of Fresnel's wave theory. All the diffracted wave from the edge of a circular obstacle should arrive in phase at the center of the shadow. Soon after, Fresnel demonstrated the existence of the "Poisson spot".

The Poisson Spot



38.1 Fraunhofer and Fresnel Diffraction

Diffraction patterns are usually classified into two categories depending on the source and screen are placed.

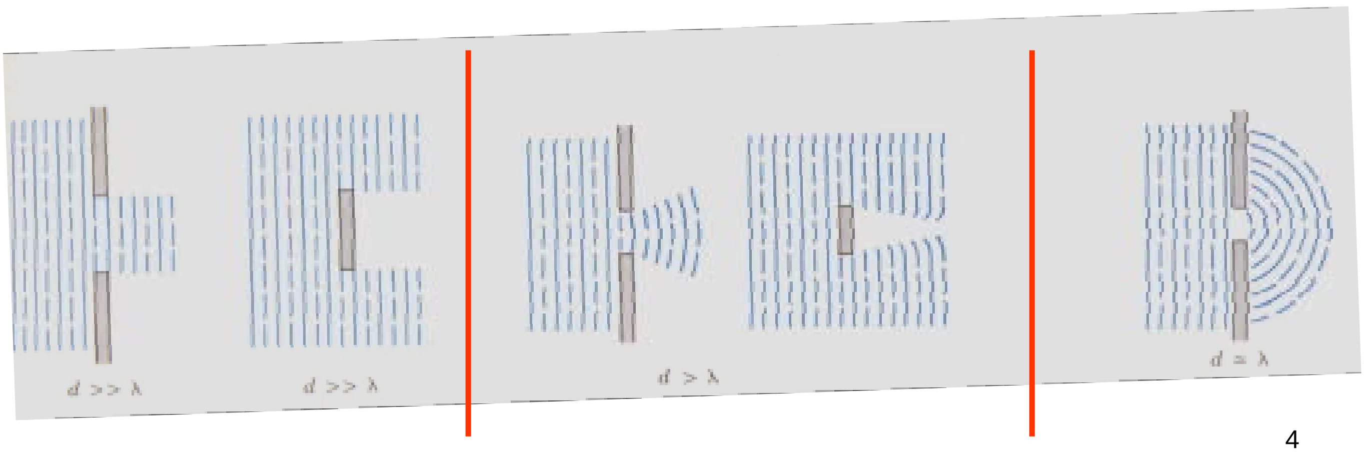
Fresnel diffraction: When either the source or the screen is near the aperture or obstruction, the wavefronts are *spherical* and the pattern is quite complex. (*near-field*)

Fraunhofer diffraction: When both the source and the screen are at a great distance from the aperture or obstruction, the incident light is in the form of *plane wave* and the pattern is simpler to analyze. (*far-field*)

37.1 Interference

Diffraction: Rays or wavefronts bend at the edge of an opening, or an obstruction.

Size effect: The extent to which diffraction modifies the rectilinear propagation of waves depends on the relative size of the wavelength and the opening (or obstruction).



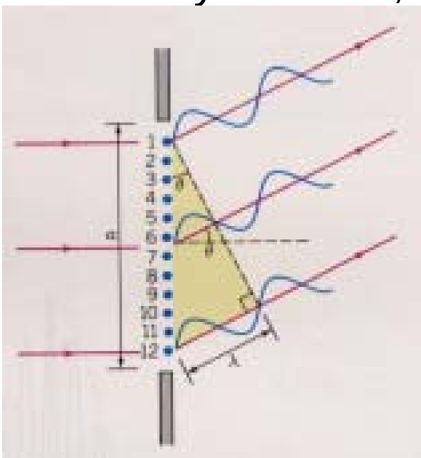
38.2 Single-Slit Diffraction (I)

Diffraction: When plane wavefronts (parallel rays) incident on a slit of width a, it will form a diffraction pattern.

Central maximum: In the forward direction, all the secondary wavelets will be in phase, so there is a central bright region at θ =0.

Minima: When the path difference between two rays is mλ/2, they interfere destructively.

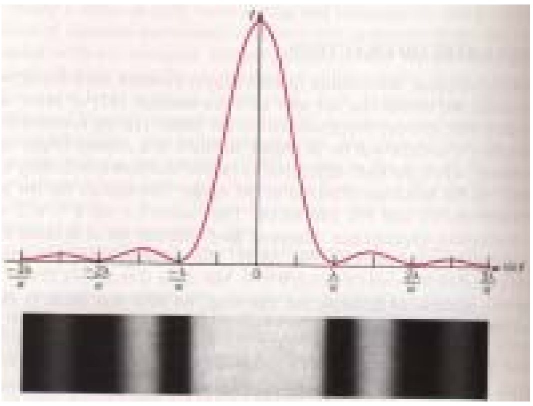
$$\text{Minima}: \ a\sin\theta = m\lambda \qquad m = 1, 2, 3, \ldots$$



Single-Slit Diffraction (II)

Secondary maximum: The positions are approximately given by a sin θ =(m+1/2)λ, where they interfere constructively.

$$\text{Mixima}: \ a\sin\theta = (m + \tfrac{1}{2})\lambda \qquad m = 1, 2, 3, \ldots$$



How Young's double-slits experiment works by using single silt as the light source?

Example 38.1

Light of wavelength 600 nm is incident normally on a slit of width 0.1 mm. (a) What is the angular position of the first minimum? (b) What is the position of the second-order minimum on a screen 3 m from the slit?

Solution:

(a) Minima: $a\sin\theta = m\lambda \qquad m = 1;\ a = 1\times10^{-4}\,\text{m};\ \lambda = 6\times10^{-7}\,\text{m}$

$$\sin\theta = 6\times10^{-3};\ \frac{\theta}{360}2\pi = 6\times10^{-3};\ \theta = 0.34^\circ$$

(b) Second-order minima: $a\dfrac{y}{L} = 2\lambda$

$$L = 3\,\text{m};\ a = 1\times10^{-4}\,\text{m};\ \lambda = 6\times10^{-7}\,\text{m}$$

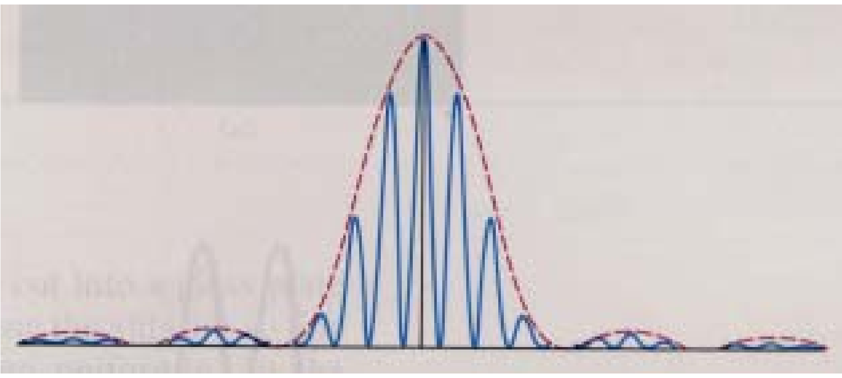
$$\text{y} = 3.6\,\text{cm}$$

Interference and Diffraction Combined

The interference pattern has a single slit diffraction envelope.

If the interference equation predicts a maximum at an angle for which the diffraction pattern has a minimum, the screen is dark.

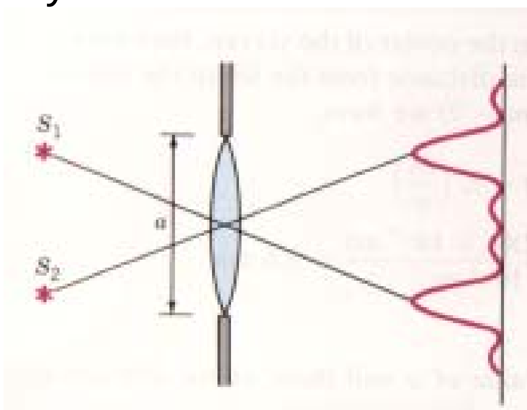
Similarly, an interference minimum will eliminate that part of a diffraction peak that it overlaps.



38.3 The Rayleigh Criterion (I)

Plane wavefronts passing through an aperture, such as a lens, undergo diffraction. Thus the image of a point source is not a point, but is instead a diffraction pattern.

The *resolution* of any optical system, its ability to produce sharp images, is limited by diffraction.

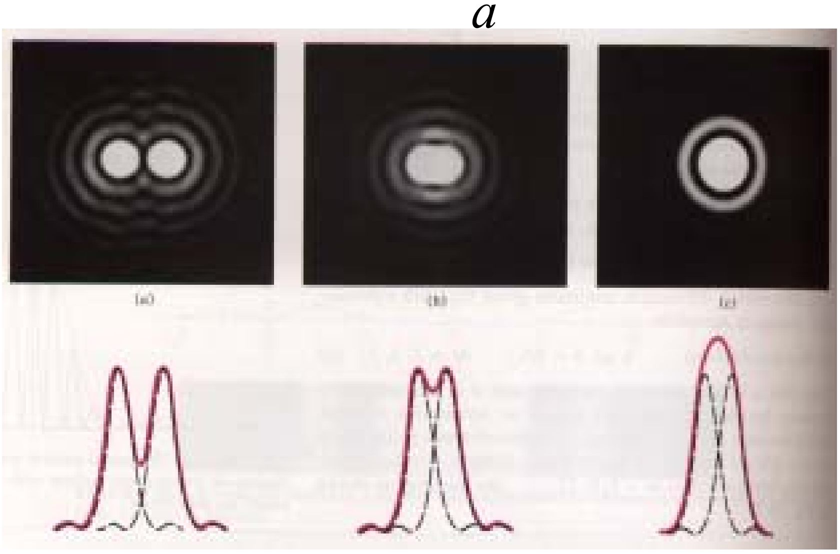


Why a laser pointer do not generate diffraction pattern when it passing through a aperture?

The Rayleigh Criterion (II)

For a circular aperture, it can be shown that the position of the first minimum in each diffraction pattern is given by

$$\sin\theta = \frac{1.22\lambda}{a}$$



Example 38.3

The optical telescope at Mount Palomar has a diameter of 200 inches (5.08 m). At a wavelength of 550 nm, what is the minimum resolvable detail on the moon? The distance to the moon is 3.84x10^8 m?

Solution:

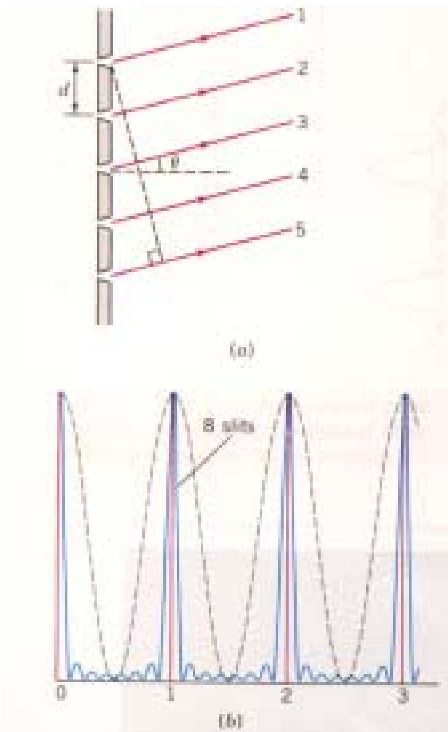
$$\theta_c = \frac{1.22\lambda}{a} = 1.32\times10^{-7}\ \text{rad}\ (= 0.03\ \text{second of arc})$$

$$\text{s} = \text{L}\theta_c = (3.84\times10^{8})\times(1.32\times10^{-7}) = 50\ \text{m}$$

In practice the resolution is limited to about 1 second of arc by atmosphere turbulence and optical aberrations in the mirror.

38.4 Gratings (interference)

A grating consists of thousands of *very fine* slits or grooves cut into a glass plate. Therefore, the single-slit diffraction pattern illuminates the screen uniformly. Only *the interference effect* will take into account.



Again, the path difference between the waves would lead to constructive or destructive interference.

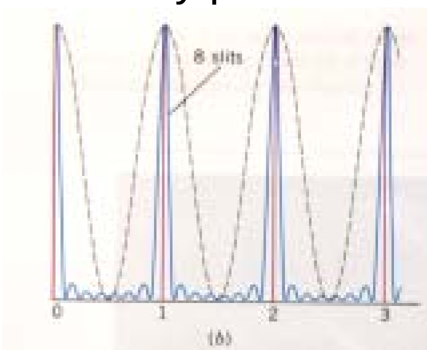
Gratings (II)

Principle maxima: The path difference between rays from adjacent slits is equal to integer times of the wavelength.

$$\text{Principal mixima: } d\sin\theta = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

The wave from all the slits are in phase.

For a grating with thousands of slits, the principal maxima are sharp lines, and the secondary peaks are not visible.



Importance of Gratings: Wavelength Analyzer

1. Gratings are extremely important in the analysis of light emitted by atoms and molecules.
2. A grating acts somewhat like a prism but has much better resolution.
3. A significant advantage offered by the use of a grating is that wavelengths can be determined.

Example 38.4: Light of wavelength 550 nm is incident normally on a grating that has 400 lines per mm. at what angles does the second-order principal maximum occur?

Sol: the spacing between the line is d=1/400 mm=2.5×10^{-6} m

Sin $\theta_2 = 2\lambda/d = 0.44$, $\theta_2 = 26.1^{\circ}$

38.5 Multiples Slits

Phasors: A more convenient way to combine three or more wavefunctions trigonometrically.

Phasor is a vector used to represent a physical quantity that varies *sinusoidally* in time. Here the physical quantity is the electric field of the light wave, $E=E_0\sin(\omega t)$.

The phasor $\mathbf{E}_0$ rotates at the angular frequency of the wave, and its magnitude is equal to the amplitude of the field.

Multiples Slits (II)

Assume the slits are so narrow and the screen is far from the sources. The *diffraction* spreads the light uniformly over the screen, i.e., each slit contributes a wave of *amplitude* E_0 at the screen.

The phase difference, ϕ, between the fields from adjacent slits is related to the path difference, $\delta = d\sin\theta$:

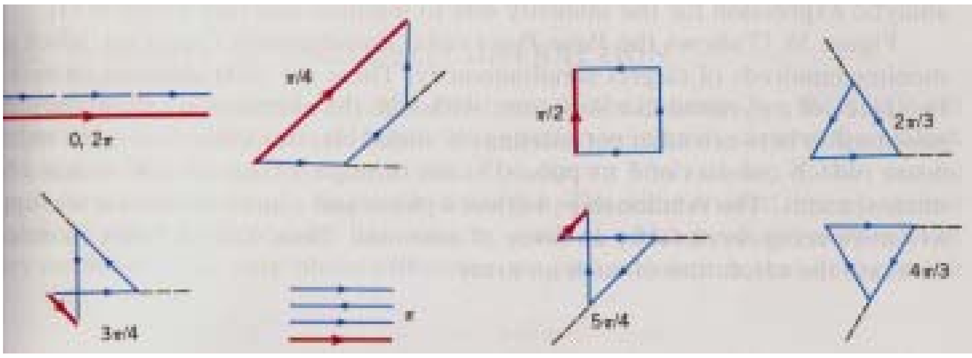
$$\phi = \frac{2\pi\delta}{\lambda} = \frac{2\pi\, d\sin\theta}{\lambda}$$

where d is the slit separation.

Three slits

$$E_1(t) = E_0 \sin(\omega t)$$
$$E_2(t) = E_0 \sin(\omega t + \phi)$$
$$E_3(t) = E_0 \sin(\omega t + 2\phi)$$

The angles between the phasors refer to the phase relationship between the fields.



Three slits (II)

The **principal maxima** occur when all the waves at the screen are in phase, that is, when ϕ=0, 2π, 4π, and so on. The amplitude of the resultant phasor in these cases is $E_{0T}=3E_0$, so the intensity is $I_T=9I_0$.

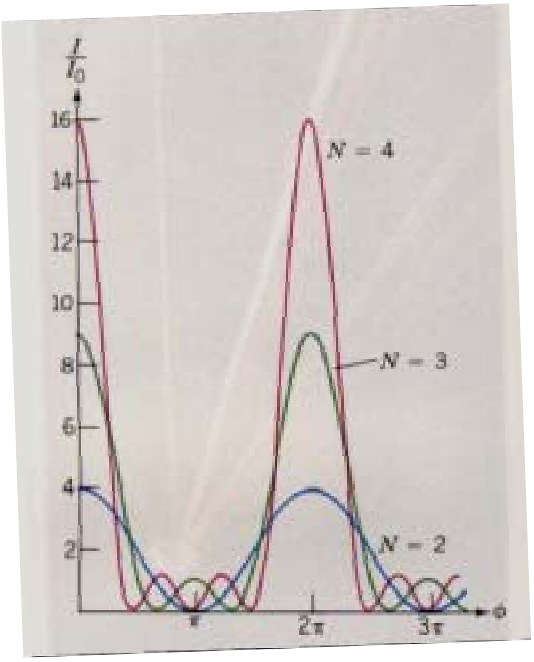
When ϕ=2π/3, 4π/3, 8π/3, and so on, the phasor diagram closes on itself, so the intensity is zero, $I_T=0$. To sum up,

$$\text{Principal mixima}: \phi = 2m\pi \qquad m = 0, 1, 2, 3, \ldots$$

$$\text{Minima}: \phi = \frac{2p\pi}{3} \qquad p = 1, 2, 4, 5, (p \neq 3, 6, 9, ...)$$

Multiples Slits (III)

The principal maxima become sharper and higher as the number of slits increases.



37.6 Intensity of Single-Slit Diffraction

The phasor analysis for N slits can be carried over to the case of a *single slit* of width *a* < λ.

The slit is divided into a large but *indefinite number* of coherent line source, each of which produces *a wavelet of tiny amplitude*.

The only phase difference we can calculate is that between the waves from the *top* and *bottom edges* of the slit.

When the screen is far away, we may treat the outgoing rays as parallel, so the path difference for the two extreme rays is δ=a sin θ , and the phase difference is

$$\alpha = \frac{2\pi a \sin\theta}{\lambda}$$

Intensity of Single-Slit Diffraction (II)

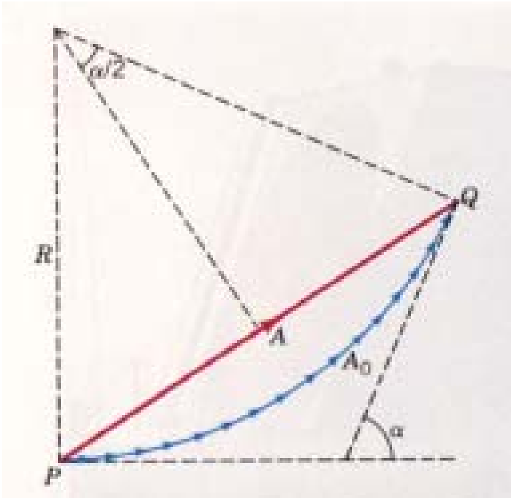
In the forward direction (θ =0, α=0) all the phasors are aligned.

At some arbitrary angle θ, the discrete set of lines of Fig.38.16 is replaced in Fig. 38.19 by continuous arc of length A_0.

$$A = 2R\sin(\frac{\alpha}{2})$$

$$A_0 = R\alpha$$

$$\therefore A = \frac{A_0 \sin(\frac{\alpha}{2})}{\frac{\alpha}{2}} \Rightarrow I = I_0 (\frac{\sin(\frac{\alpha}{2})}{\frac{\alpha}{2}})^2$$



Intensity of Single-Slit Diffraction (III)

The positions of the secondary maxima are (see problem 9)

(secondary maxima) α=2.86π, 4.92π, 6.94π, …

Notice that these are close, but not quite equal to 3π, 5π, 7π, and so on.

The intensities for these values of α are found

I=0.047I_0, 0.017I_0, 0.008I_0, …

The first secondary peak has an intensity of only 4.7% relative to the central peak.

Example 38.6

Light of wavelength 600 nm is incident normally on a slit of width 0.1 mm. what is the intensity at θ =0.2º?

Solution:

$$\alpha = \frac{2\pi a \sin\theta}{\lambda} = \frac{(2\pi \times 10^{-4})(3.5\times 10^{-3})}{6\times 10^{-7}} = 3.67\,\text{rad}$$

$$\Rightarrow I = I_0 (\frac{\sin 1.84}{1.84})^2 = 0.27 I_0$$

Exercises and Problems

Ch.38:

Ex. 11, 17, 23, 26, 49

Prob. 1, 3, 4, 9, 11