Chapter 39 Special Relativity
39.1 Introduction

Ether: Since mechanical waves require a medium to propagate, it was generally accepted that light also require a medium. This medium, called the ether, was assumed to pervade all mater and space in the universe.

“Absolute” frame: The Maxwell’s equation was inferred that the speed of light should equal *c* only with respect to ether. This meant that the ether was a “preferred” or “absolute” reference frame.

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

39.1 Introduction (II)

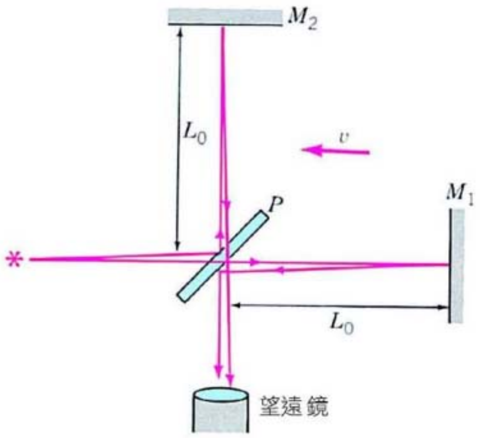
Properties of the ether: Since the light speed c is enormous, the ether had to be extremely rigid. So it did not impede the motion of light. For a substance so crucial to electro-magnetism , it was embarrassingly elusive. Despite the peculiar property just mentioned, no one could detect its ghostly presence.

Efforts to detect the ether: Michelson inspired by the Maxwell took the problem of detecting the ether as a challenge. He developed his interferometer and used it to try to detect the earth’s motion relative to the ether. The result were not conclusive.

39.2 The Michelson-Morley Experiment

Michelson and Morley wanted to detect the speed of the earth relative to the ether. If the earth were moving relative to the ether, there should be an "ether wind" blowing at the same speed relative to the earth but in the opposite direction.

Michelson-Morley interferometer: Use light speed variation to verify the existence of ether.



39.2 The Michelson-Morley Experiment (II)

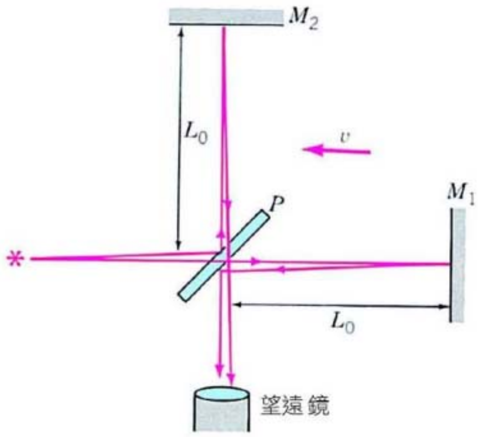
Parallel:

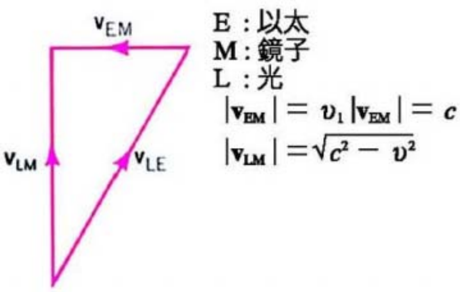
$$T_1 = \frac{L_0}{(c-v)} + \frac{L_0}{(c+v)} = \frac{(2L_0/c)}{(1-v^2/c^2)}$$

Perpendicular:

$$T_2 = \frac{2L_0}{(c^2-v^2)^{1/2}} = \frac{(2L_0/c)}{(1-v^2/c^2)^{1/2}}$$

$$\Delta T = T_1 - T_2 \cong \frac{L_0}{c}\left(\frac{v^2}{c^2}\right)$$





39.2 The Michelson-Morley Experiment (III)

Using v=30 km/s, the expected shift was about 0.4 fringe. Even though they were able to detect shifts smaller than 1/20 of a fringe, ***they found nothing***.

Possibilities:

- The ether was dragged with the Earth. ✗
- No ether. ✓
- Length contraction. ✗
- Constant light speed. ✓

39.3 Covariance

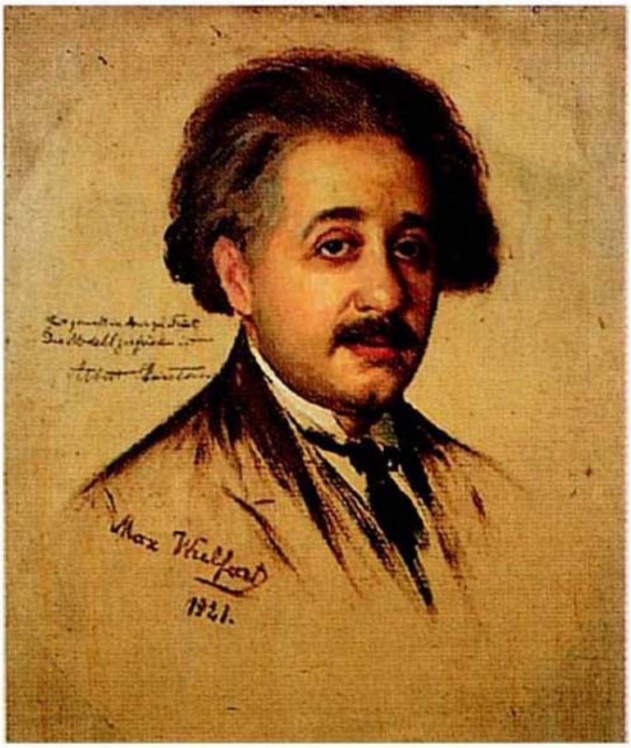
Covariance: The laws of mechanics are covariance---*they retain their form*---with respective to Galilieean transformation. Newton's second low, *F=ma*, in one frame has the same form, *F'=ma'*, in another. However, the Maxwell's equations does not satisfy this requirement when applying the Galilieean transformation.

$$x' = x - vt; \quad t' = t$$

Three Problems:

1. The force between the charge depends on the frame of reference employed.
2. Maxwell's equations are valid in only one special frame with the Galilieean transformation.
3. The applied electromagnetism law will change with reference frame.

Albert Einstein



39.4 The Two Postulates

The two postulates in the theory of special relativity are:

1. **The principle of relativity**: All physical laws have the same form in all inertia frames.
2. **The principle of the constancy of the speed of light**: The speed of light in free space is the same in all inertial frames. It does not depend on the motion of the source or the observer.

Both postulates are restricted to inertial frames. This is why the theory is special.

•The principle of relativity extends the concept of covariance from mechanics to all physical laws.

•The constancy of the speed of light is difficult to accept at first.

All the experimental consequences have confirmed its correctness.

39.5 Some Preliminaries

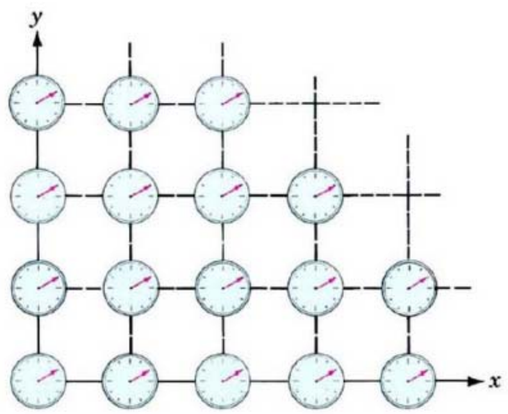
Event: In specia that occurs at a single point in space at a single instant in time.

Observer: An observer is ether a person, or an automatic device, with a clock and a meter stick. Each observers can record events only in the immediate vicinity.

Reference frame: A reference frame is a whole set of observers uniformly distributed in space. The frame in which an object is at rest is called its **rest frame**.

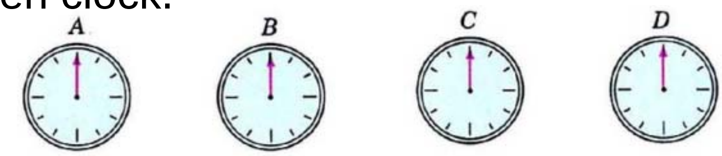
Synchronization of clocks: It is extremely important to define precisely what is meant by the time in a given reference frame. This requires a careful procedure for the synchronization of clocks.

39.5 Some Preliminaries (II)



A reference frame is assumed to consist of many observers uniformly spread through the space. Each observer has a meter stick and a clock to make measurements only in the immediate vicinity.

To synchronize four equally spaced clocks, a signal is sent out by clock A to trigger the other clocks---each of which has been set ahead by the amount of time it takes to travels from A to the given clock.

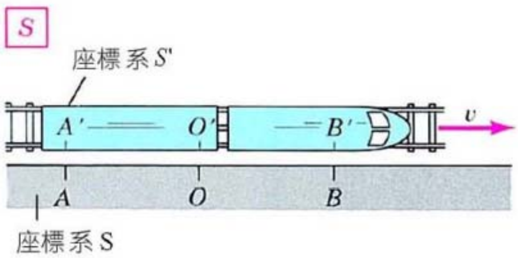


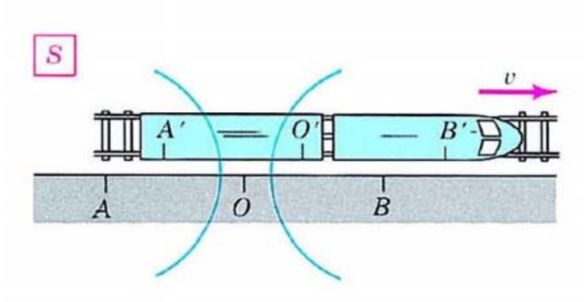
39.6 Relativity of Simultaneity

How can we determine whether two events at different locations are simultaneous?

Two events at different locations are simultaneous if an observer midway between them receives the flashes at the same instant.

Relativity of Simultaneity: Spatially separated events that are simultaneous in one frame are not simultaneous in another, moving relative to the first.

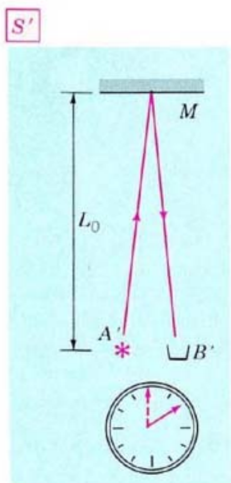




39.7 Time Dilation

How does the relative motion of two frames affects the measured time interval between two events?

$$T_0 = \Delta t' = \frac{2L_0}{c}$$



A **proper time**, T_0, is the time interval between two events as measured in the rest frame of a clock. In this frame both events occur at the same position.

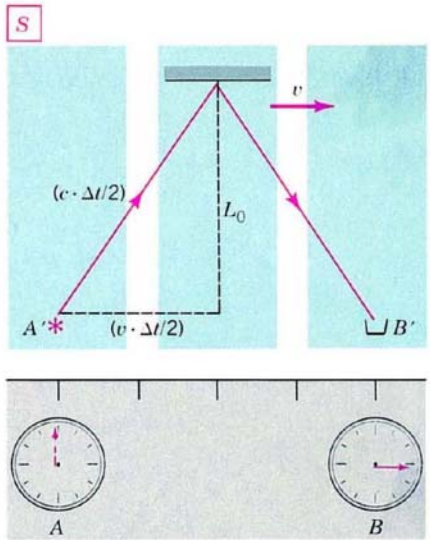
39.7 Time Dilation (II)

Now let us find the time interval recorded in the frame *S*, in which the clock has velocity *v*. The time interval Δt in frame *S* measured by two observers A and B at different positions.

$$(c \cdot \frac{\Delta t}{2})^2 = L_0^2 + (v \cdot \frac{\Delta t}{2})^2$$

$$T = \Delta t = \frac{2L_0}{c} \cdot (\frac{1}{\sqrt{1 - v^2/c^2}})$$

$$T = \gamma T_0 \quad \text{where } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



Note that we have used *c* as the speed of light in both frames---in accord with the second postulate.

39.7 Time Dilation (III)

Since γ>1, the time interval T measured in frame S (by two clocks) is greater than the proper time, T0, registed by the clock in its rest frame S'. The effect is called time dilation.

Two spatially separated clocks, A and B, record a greater time interval between two events than the proper time recorded by a single clock that moves from A to B and is present at both events.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

表 39.1*

v/c	γ
0.6	5/4
0.8	5/3
0.98	5
0.995	10
0.9965	12
0.9992	25

*有些 γ 的數值已經加以化約。

39.7 Time Dilation (IV)

Experimental evidence (muon decay):

The reality of time dilation was verified in an experiment performed in 1941.

Rest frame at ground: An elementary particle, the muon (μ), decays into other particle. The particle decay rate is

$$N = N_0 e^{-t/\tau}$$

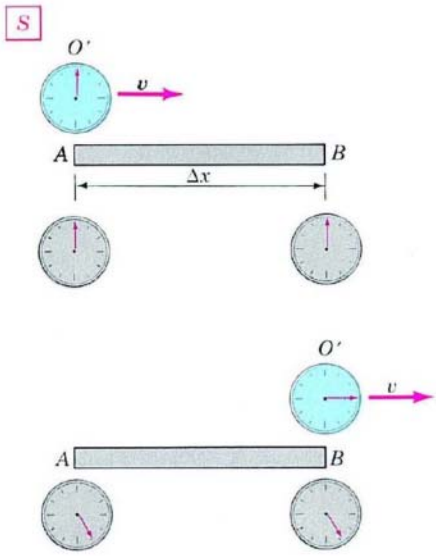
$$\text{where } \tau = 2.2\ \mu s \text{ is called the mean lifetime.}$$

Moving frame at the upper atmosphere: Another source of generating muon is bombarded with cosmic ray protons. The muon generated with this method has the speed of v=0.995c. The mean lifetime is 10 times longer than their cousins that decay at rest in the laboratory.

39.8 Length Contraction

Consider a rod *AB* at rest in frame *S*, as shown below. The distance between its ends is its proper length L_0:

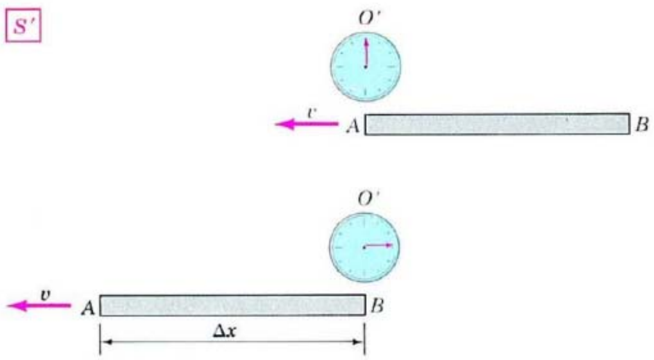
The proper length, L_0, of an object is the space interval between its ends measured in the rest frame of the object.



39.8 Length Contraction (II)

An observer O' in Frame S', which moves at velocity v relative to frame S, can measure the rod's length. By recording the interval b times at which O' passes A and B. The measurements in the two frames are

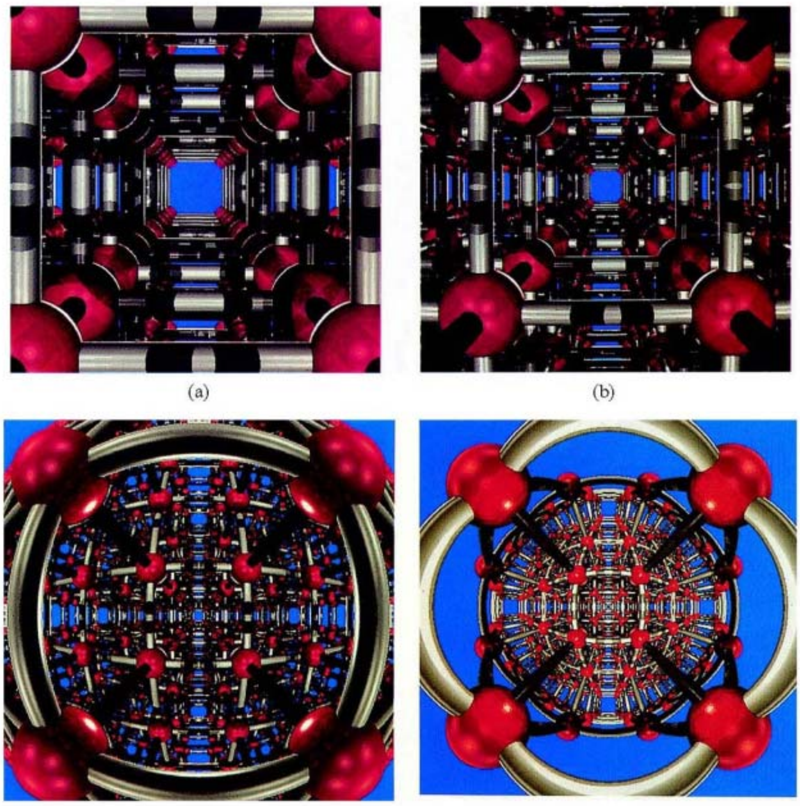
$$\left.\begin{array}{l}\text{Frame } S:\ L_0 = \Delta x = v\Delta t \\ \text{Frame } S':\ L = \Delta x' = v\Delta t'\end{array}\right\} \Rightarrow L = \frac{1}{\gamma} L_0$$



39.8 Length Contraction (III)

$v = 0.0\,c$

$v = 0.5\,c$

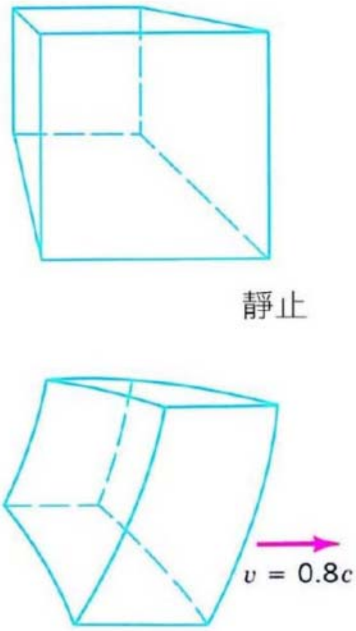


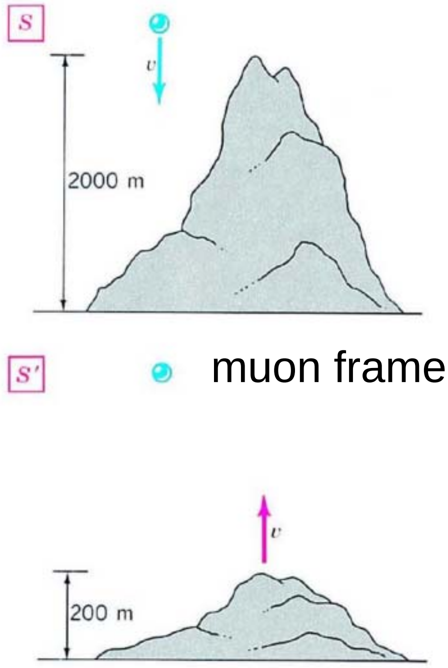
(a) (b) (c) (d)

$v = 0.95\,c$

$v = 0.99\,c$

39.8 Length Contraction (IV)





39.9 The Relativistic Doppler Effect

In the classical Doppler effect for sound waves, the observed frequency depends differently on the velocities of the source and the observer. The underlying reason is that for sound there is a medium (the air) that serves as an "**absolute" reference frame**.

(a) Source at rest, observer moves (velocity modulation)

$$f' = \frac{v'}{\lambda_o} = \frac{v \pm v_o}{v} f_o$$

(b) Source moves, observer at rest (wavelength modulation)

$$f' = \frac{v}{\lambda'} = \frac{v}{v \pm v_s} f_o$$

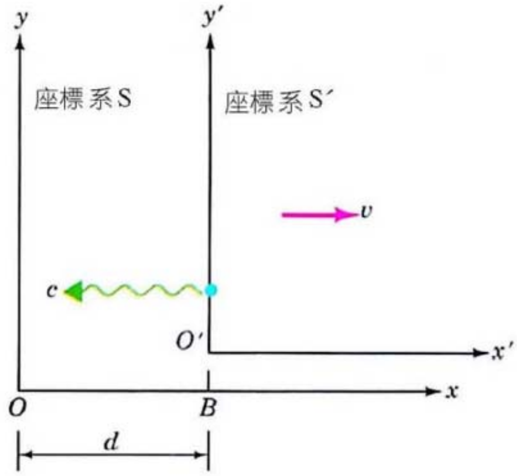
In contrast, for light there is **no absolute frame**: The relativistic Doppler effect for light depends only on the relative velocity between the source and the observer.

39.9 The Relativistic Doppler Effect (II)

In order to obtain the Doppler effect, we have to calculate the time interval measured in two frames. Note that the time dilation is assumed in the following calculation.

$$T = \Delta t + \frac{d}{c} = \gamma(1+\frac{v}{c})T_0$$

$$T = \sqrt{\frac{c+v}{c-v}}T_0$$



Longitudinal

$$f = \sqrt{\frac{c-v}{c+v}} f_o$$

Transverse

$$f = \frac{1}{\gamma} f_o$$

39.10 The Twin Paradox

Nothing in the theory of relativity catches the imagination more than the so-called twin paradox.

Twin A stays on earth while twin B travels at high speed to a nearby start. When B returns, they both find that A has aged more than B.

The paradox arises because of the apparent symmetry of the situation: In B's frame, it is A that leaves and returns, so one should also find that B has aged more than A.

$$\left.\begin{matrix} A > B \\ B > A \end{matrix}\right\} ?$$

What's going on?

39.11 The Lorentz Transformation

The laws of electromagnetism are not covariant with respect to the Galiliean transformation. However with Lorentz transformation they are covariant. The space and time are related shown as follows:

Rest frame

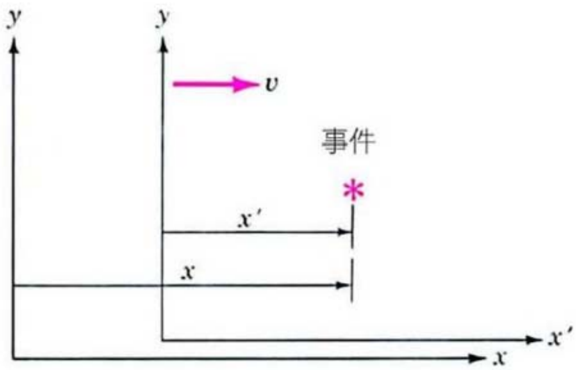
$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

Moving frame

$$x = \gamma(x' + vt')$$

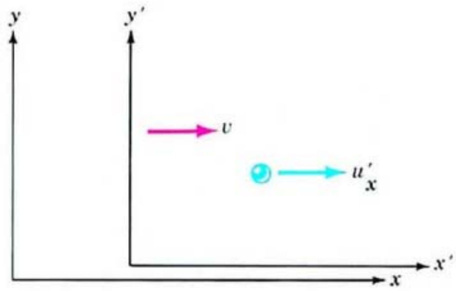
$$t = \gamma(t' + \frac{vx'}{c^2})$$



39.12 The Addition of Velocity

$$dx = \gamma(dx' + vt') = \gamma dt'(u'_x + v)$$

$$dt = \gamma(dt' + \frac{vdx'}{c^2}) = \gamma dt'(1 + \frac{u'_x v}{c^2})$$



Taking the ratio of these equations we find

$$u_x = \frac{u'_x + v}{1 + u'_x v / c^2}$$

A extreme case

when $u'_x = c$, we have $u_x = \frac{c + v}{1 + cv/c^2} = c$

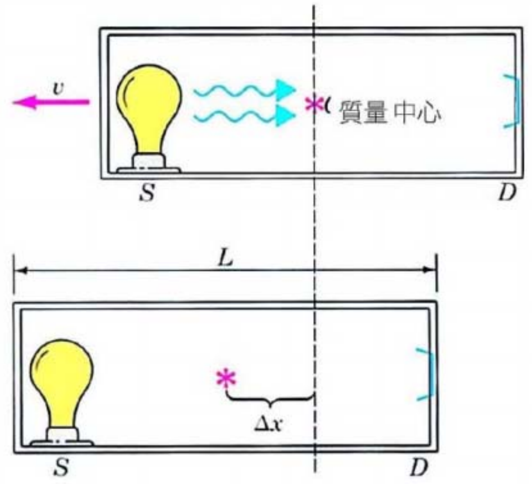
39.13 Momentum and Energy

Mass-energy equivalence: Since no internal process can even move the center of mass of a system, we can derive mass-energy equivalence.

$$\frac{E}{c} = mv, \quad \Delta x = v\Delta t = \frac{EL}{Mc^2}$$

$$-M\Delta x + mL = 0$$

$$E = mc^2$$



39.13 Momentum and Energy (II)

The relativistic definition of the **linear momentum** is

$$p = mv = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

The **relativistic kinetic energy** is the **total energy** minus the **rest energy**:

$$K = E - m_0c^2 = m_0c^2(\gamma - 1)$$

$$\cong \frac{1}{2}mv^2, \quad \text{when } v << c$$

The relativistic momentum and the relativistic energy can be related through:

$$E^2 = p^2c^2 + m_0^2c^4$$