

Chapter 40 Early Quantum Theory

- The theory of special relativity was the first part of the revolution in 20th century physics. Classical mechanics is not correct when particles move at high speed.
- The second part of the revolution was the formulation of the theory of quantum mechanics. Its origin lay in the study of the radiation emitted by hot bodies and in the photoelectric effect.

In this chapter, we are going to discuss the development of early quantum theory from blackbody radiation, photoelectric effect, Compton's scattering, Bohr's hydrogen model, to wave-particle duality and correspondence principle.

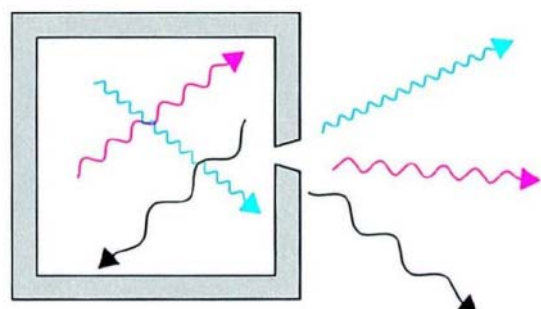
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40.1 Blackbody Radiation

Hot bodies emit radiation, which we experience as heat. As the temperature of an object is raised it first glows a dull red, then changes to orange-yellow, and finally becomes “white hot”.

Why is the distribution of thermal radiation among the various wavelengths the same for all bodies at a given temperature?

The radiation emitted by the opening is called blackbody or cavity radiation.



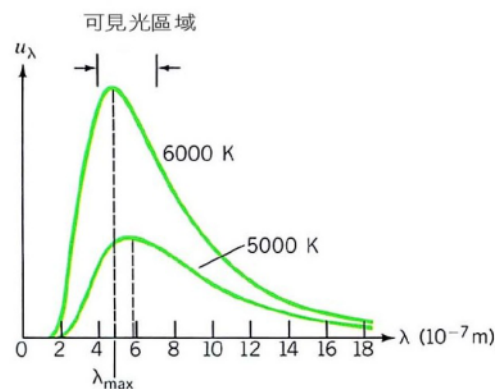
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40.1 Blackbody Radiation (II)

The spectrum of cavity radiation is independent of the material in the wall of the cavity.



A typical curves for the distribution of energy among the wavelengths. The spectral energy density, $u_\lambda(T)$, is defined so that $u_\lambda(T)d\lambda$ is the energy per unit volume of the cavity in the wavelength λ to $\lambda+d\lambda$.



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40.1 Blackbody Radiation (III)

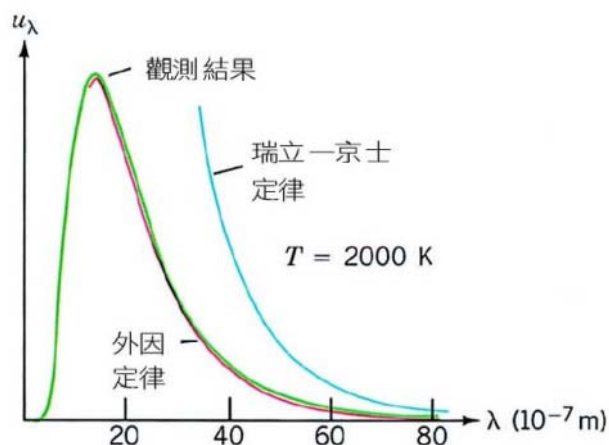
Wein's displacement law: The wavelength at which the energy density is a maximum is related to the temperature.

$$\lambda_{\max} T = 0.00298 \text{ m} \cdot \text{K}$$

Models:

Wein's radiation law: valid at short wavelengths.

Rayleigh-Jeans law: valid at long wavelengths.



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Planck's Radiation Law

Planck was impressed by the fact that the spectrum of cavity radiation was a universal property---independent of the nature of the material in the wall.

Planck then simply combined these two conditions into one and obtained a new radiation formula:

$$u_{\lambda} = \frac{A\lambda^{-5}}{e^{B/\lambda T} - 1}$$

where A and B are constant.

At short wavelengths, $\exp(B/\lambda T) - 1 \cong \exp(B/\lambda T)$: $u_{\lambda} = \frac{A\lambda^{-5}}{e^{B/\lambda T}}$
(Wein's law)

At long wavelength, $\exp(B/\lambda T) - 1 \cong (B/\lambda T)$: $u_{\lambda} = C\lambda^{-4}$
(Rayleigh-Jeams law)

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Planck's Radiation Law (II)

To calculate the entropy of the system, Planck determine the number of ways a given total energy could be distributed among a fixed number of the oscillators in the cavity walls.

He divided the total energy of the oscillators into energy "elements" of size ε . He found that he can obtain the same expression provided he set $\varepsilon = hf$, where f is the frequency and h is a constant.

$$u_{\lambda} = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

This function correctly yields the complete spectrum of the cavity radiation.

At this stage, Planck did not realize the true significance of what he had done. Neither did anyone else.

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Einstein's Quantum Hypothesis

Einstein's quantum Hypothesis: Einstein proved that Planck's radiation law could be derived if the energy of each individual oscillator is quantized in steps of hf . In the n th "level", the energy is $E_n = nhf$

Einstein's hypothesis implies that an oscillator can emit or absorb radiation only in multiples of hf .

Einstein hypothesis was more or less ignored since few scientists were concerned with the problem of cavity radiation. However, by 1908 most physicists had become aware of the drastic disagreement between the prediction of classical physics and the radiation curve at short wavelength.

Although Planck had introduced the constant h , the idea that the energy of an oscillator is really quantized came from Einstein.

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Example 40.1

The peak in the radiation from the sun occurs at about 500 nm. What is the sun's surface temperature, assuming that it radiates as a blackbody.

Solution:

From Wein's law

$$T = 0.00298 / 500 \times 10^{-9} = 5800 \text{ K}$$

In comparison, the temperature of the filament of an incandescent bulb is about 2000 K.

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Example 40.2

A block of mass 0.2 kg oscillates at the end of a spring ($k=5$ N/m) with an amplitude of 10 cm. what is its “quantum number” n ?

Solution:

$$E = \frac{1}{2} kA^2 = 0.025 \text{ J}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 0.8 \text{ Hz}$$

$$n = E_n / hf = 10^{32} \text{ (extremely large)}$$

Under what condition the quantum effect becomes significant? E small, f high.

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40.2 The Photoelectric Effect

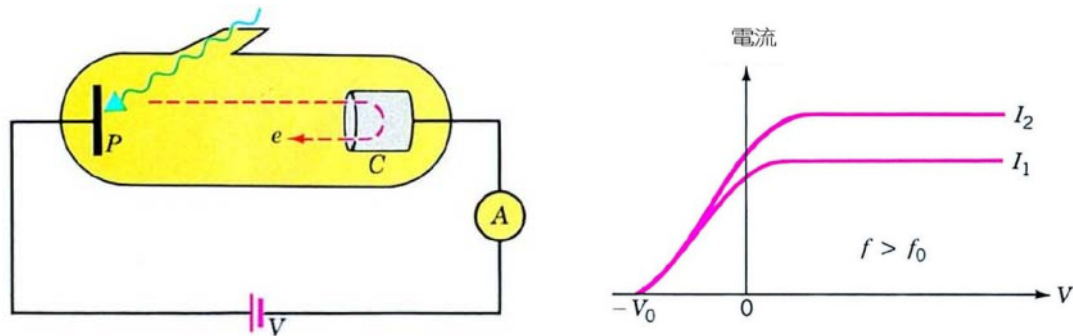
The work of Young and Fresnel had converted the scientists from the corpuscular theory to the wave theory of light. Maxwell predicted that light is an electromagnetic wave and Hertz' experiment was its crowning glory.

Ironically, the very experiment that demonstrates that light is an electromagnetic wave also produced the first evidence of its corpuscular nature.

Hallwachs found that when a zinc plate is illuminated with ultraviolet light, it becomes positively charged, and in some alkali metals, visible light can produce this emission of electrons, now called the **photoelectric effect**.

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40.2 The Photoelectric Effect (II)



Lenard conducted the experiment and found that

- Photoelectron is proportional to the light intensity, even at very low intensity. The absence of a threshold intensity is puzzling.
- Furthermore, the maximum kinetic energy of the electrons depends on the light source and the plate material, but not intensity of the source.

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The Photon

Particles: Planck had treated the total energy of the oscillators as consisting of discrete “element” but had assume the energy of the radiation is continuous.

Wave: Maxwell’s theory is extremely successful in dealing with interference, diffraction, and other properties of electromagnetic radiation.

Photons: Einstein noted that optical observations refer to values over time, not to instantaneous values. Einstein proposed that radiation behaves as if it were composed of a collection of discrete energy quanta of magnitude $E=hf$, where E is the energy, h is Planck constant, and f is the frequency of the radiation.

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The Photoelectric Equation

Einstein immediately applied the idea of **light quanta** to the photoelectric effect. In the process of photoemission a *single photon gives up all its energy to a single electron*.

The maximum possible kinetic energy of the photoelectrons is determined by the energy of each photon:

$$hf = \frac{1}{2}mv_{\max}^2 + \phi$$

Where the **work function**, ϕ , is the minimum energy needed to extract an electron from the surface of the material. The minimum frequency needed to excite a photoelectron is called the **threshold frequency**.

$$hf_0 = \phi$$

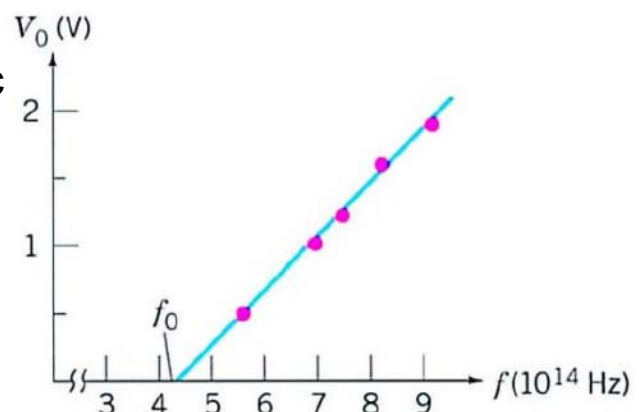
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The Photoelectric Equation (II)

Not only did Einstein explain all the known facts, but he also predicted the existence of a threshold frequency, and that a plot of V_0 versus f should produce a straight line with a slope h/e , independent of the nature of the material.

The **Einstein's photoelectric equation**:

$$eV_0 = h(f - f_0)$$



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Example 40.3

Ultraviolet light of wavelength 207 nm causes photoemission from a surface. The **stopping potential** is 2 V. Find the work function in eV; (b) the maximum speed of the photon electrons.

Solution:

$$(a) \phi = \frac{hc}{\lambda} - eV_0 = \frac{6.626 \times 10^{-34} \times 3.0 \times 10^8}{2.07 \times 10^{-9} \times 1.6 \times 10^{-19}} - 2 \\ = 6 - 2 = 4 \text{ eV}$$

$$(b) \frac{1}{2}mv_{\max}^2 = eV_0 \Rightarrow v_{\max} = \sqrt{\frac{2eV_0}{m}} = 8 \times 10^5 \text{ m/s}$$

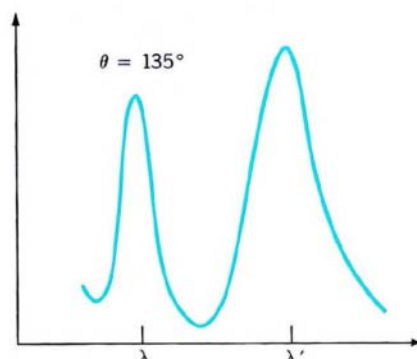
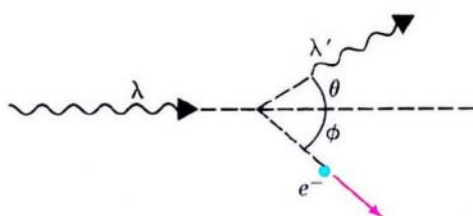
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40.3 The Compton Effect

The striking successes in blackbody radiation and the photoelectric effect were still not sufficient to convince some scientists of the validity of the quantum concept.

Compton found further evidence for the photon concept while he was studying the scattering of X rays by graphite.

He found that the scattered radiation had two components: *one* at the original wavelength and a *second* at a longer wavelength.



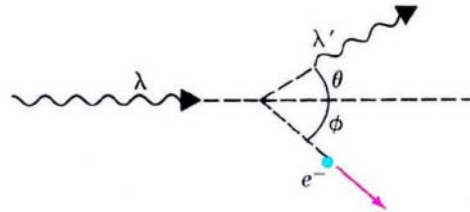
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40.3 The Compton Effect (II)

The derivation of Compton's scattering: Classically an electromagnetic wave carries momentum given by $p=E/c$.

Conservation of linear momentum:

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$



$$\begin{cases} p_x : p_{\lambda} = p_{\lambda'} \cos \theta + p \cos \phi \\ p_y : 0 = p_{\lambda'} \sin \theta - p \sin \phi \end{cases}$$

$$\Rightarrow (p_{\lambda} - p_{\lambda'} \cos \theta)^2 + (p_{\lambda'} \sin \theta)^2 = p^2$$

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40.3 The Compton Effect (III)

Conservation of energy:

$$hf = hf' + K; \quad K = (\gamma - 1)m_0c^2$$

$$\begin{cases} (cp_{\lambda} - cp_{\lambda'}) = K \\ K^2 + 2Km_0c^2 = c^2 p^2 \end{cases}$$

$$(p_{\lambda} - p_{\lambda'})^2 + 2(p_{\lambda} - p_{\lambda'})m_0c = p^2$$

For known X ray frequency and final particle momentums

We can further solve these two equations.

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40.3 The Compton Effect (III)

$$(p_{\lambda} - p_{\lambda'} \cos \theta)^2 + (p_{\lambda'} \sin \theta)^2 = p^2$$

$$(p_{\lambda} - p_{\lambda'})^2 + 2(p_{\lambda} - p_{\lambda'})m_0c = p^2$$

Further solving these two equations, we obtain

$$(p_{\lambda} - p_{\lambda'})m_0c = p_{\lambda}p_{\lambda'}(1 - \cos \theta)$$

$$\frac{1}{p_{\lambda'}} - \frac{1}{p_{\lambda}} = \frac{1}{m_0c}(1 - \cos \theta)$$

$$\Rightarrow \Delta\lambda = \frac{h}{m_0c}(1 - \cos \theta)$$

$$\frac{h}{m_0c} = 0.00243 \text{ nm is called Compton wavelength.}$$

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Example 40.4

X rays of wavelength 0.24 nm are scattered through an angle of 40° as they pass through a block of carbon. What is the wavelength of the scattered rays?

Solution:

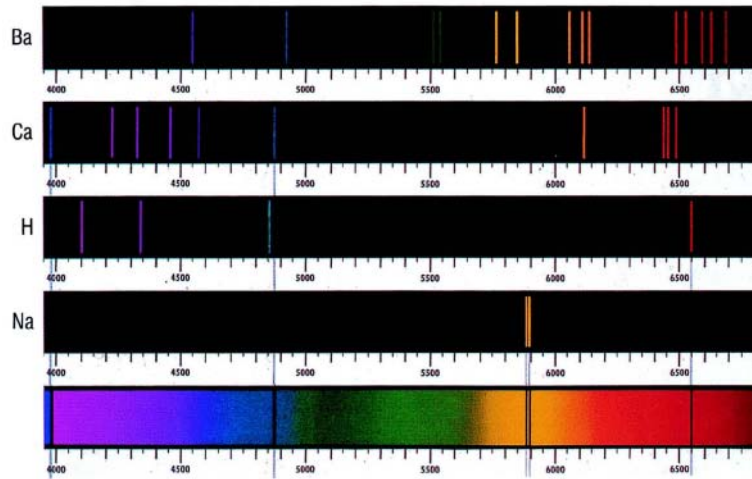
$$\theta = 40^\circ$$

$$\Delta\lambda = 0.00243(1 - \cos \theta) = 0.00057 \text{ nm}$$

$$\lambda' = 0.24 + 0.00057 = 0.24057 \text{ nm}$$

40.4 Line Spectra

The emission from a rarefied gas consists of sharp lines. Each element has a characteristic set of lines that may be used to identify it.



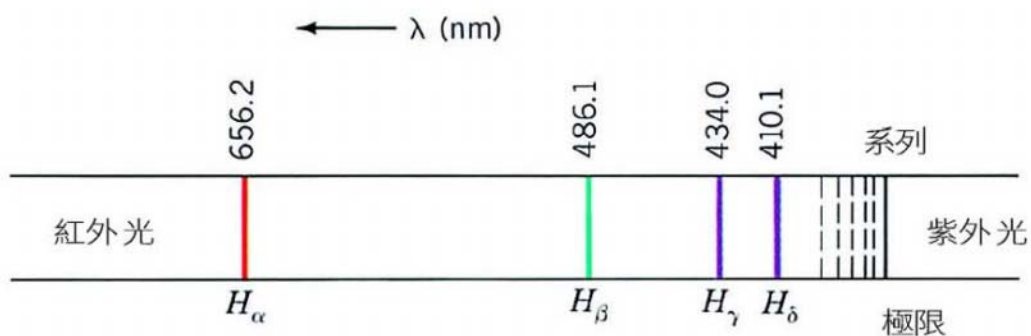
Classical physics could neither explain these line spectra nor fit them into any scheme.

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Balmer's formula

The visible spectrum of hydrogen consists of four lines. In 1884, Balmer, a Swiss mathematics teacher, found that these wavelengths (in nm) could be represented by a single formula, called Balmer formula:

$$\lambda_m = 364.56 \frac{m^2}{m^2 - 4} \quad m = 3, 4, 5, 6$$



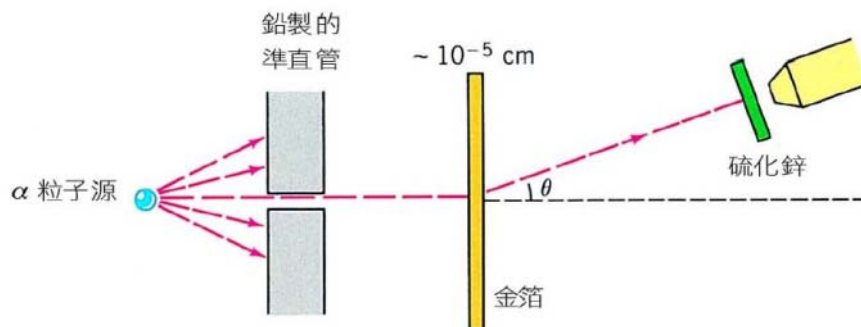
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40.5 Atomic Models

In the 19th century, there was considerable chemical and physical evidence for the existence of atoms, but nothing was known about their structure.

Thomson suggested that an atom consists of a positively charged sphere in which electrons were embedded. (The model was not satisfactory).

In 1909, Rutherford studied the scattering of alpha particles by a very thin gold foil as shown below.

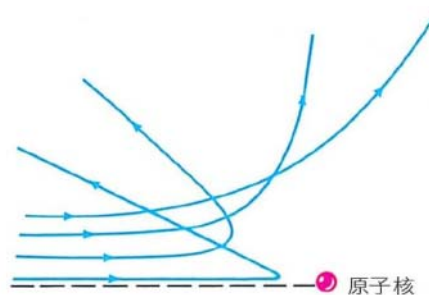


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40.5 Atomic Models (II)

Rutherford found that about one in eight thousand of the α particles were scattered through angles larger than 90° . He was stunned:

It was quite the most incredible thing that has ever happened to me in my life. It was as incredible as if you fired a 15 inch shell at a piece of paper, and it came back and hit you.



The "head-on" collision allows us to obtain an estimate of the size of the nucleus.

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The Existence of the Nuclear

Rutherford's work on α particle scattering established the existence of the nuclear.

Later, Nagaoka proposed that the electrons formed rings, but he could not explain how such a system would be stable.

A “**planetary model**” contradicts with Maxwell's theory:

1. The accelerating electron emits radiation, resulting loss of energy.
2. The radiation would cover a continuous range in frequency, in contrast to the line spectra actually observed.

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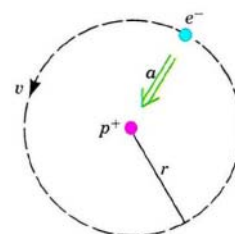
40.6 The Bohr Model

In 1913, Bohr presented a model of the hydrogen atom, which has one electron. Bohr stated two postulates:

*1. The electron moves only in certain circular orbits, called **stationary states**. This motion can be described classically.*

$$\text{Force} \quad \frac{mv^2}{r} = \frac{ke^2}{r^2}$$

$$\text{Energy} \quad E = K + U = \frac{1}{2}mv^2 - \frac{ke^2}{r} = -\frac{ke^2}{2r}$$



*2. Radiation occurs **only when** an electron goes from one allowed orbit to another of lower energy. The radiated frequency is $hf = E_m - E_n$, where E_m and E_n are energies of the two states.*

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40.6 The Bohr Model (II)

What is the stationary states? We need a quantum condition that restricts the allowed values of the orbital radius. It was realized only later that this is a fundamental aspect of quantum theory, and so it serves as our “third” postulate:

3. The angular momentum of the electron is restricted to integer multiples of $h/2\pi$.

$$\text{Angular momentum } mvr = n\hbar \Rightarrow v = \frac{n\hbar}{mr}$$

$$\text{Energy } \frac{1}{2}mv^2 = \frac{ke^2}{2r} \Rightarrow r_n = \frac{n^2\hbar^2}{mke}$$

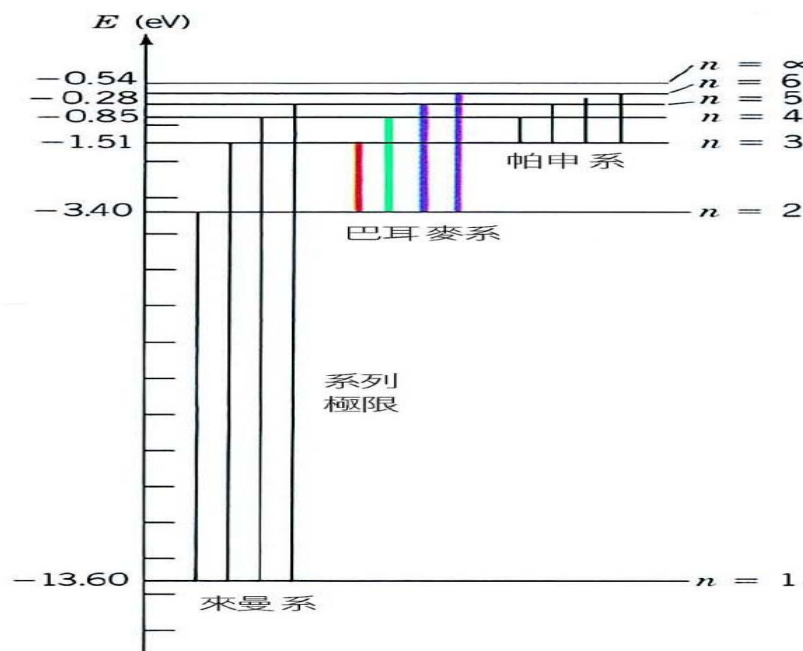
The total energy of the n th orbit is

$$E_n = -\frac{mk^2e^4}{2\hbar^2} \left(\frac{1}{n^2}\right) = -\frac{13.6}{n^2} \text{ eV}$$

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The Energy Level Diagram for Hydrogen ($Z=1$)

The photon energy must correspond exactly to the energy between two states involved in the transition.



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The Bohr Model (III)

Bohr's theory correctly predicted the frequencies of the lines, but it provided no information regarding the relative intensities of the lines or the spectra of multi-electron atoms.

In the next chapter, Bohr's theory has been replaced by quantum mechanics. The second and third postulates remain *valid*, but the picture of an electron in well-defined orbit is not correct.

Nonetheless, Bohr's approach is a good example of how a scientist pulls together different strands in developing a remarkably successful theory.

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40.7 Wave-Particle Duality of Light

Wave nature: Young's double-slit interference and single slit diffraction.

Particle nature: Photoelectric effect and Compton scattering.

What is the true nature of light?

Light exhibits a **wave-particle** duality. Depending on the experiment performed, it will behave either as a particle, or as a wave.

Generally speaking, at lower frequency it behaves like a wave, while at higher frequency it acts like a particles.

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40.8 Bohr's Corresponding Principle

Classical physics is very successful in dealing with a large number of phenomena.

Bohr felt that when a newer, more general, theory is proposed, its predictions should **reduce** to the classical result when the appropriate limit is taken.

This requirement that the results of a new theory correspond, in the limit, to classical physics is called the **correspondence principle**.

For example, at large quantum number the physics becomes to classical.

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Exercises and Problems

Ch.40:

Ex. 4, 23, 33, 46

Prob. 1, 5, 8, 10

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