微波物理與應用(I) PHYS5370 (3 credits)
Microwave Physics and Applications (I)

Tsun-Hsu Chang Professor
張存續 教授

國立清華大學 物理系

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Microwave Physics and Applications (I)

Textbook ：

David M. Pozar, Microwave Engineering, 3rd Edition (歐亞書局)
or 郭仁財教授翻譯中文版

References ：

- David K. Cheng, Field and Wave Electromagnetics, 2nd Edition.
- Robert E. Collin, Foundations for Microwave Engineering, 2nd Edition.

Time： Thursdays ~~(R7R8R9: 15:30–16:50 and 17:00–18:10)~~
(R6R7R8: 14:20–15:40 and 15:50–17:00) if you agree.

Classroom：Physics Building R019

Teacher： Professor Tsun-Hsu Chang (張存續)

Office: Physics Building 417R (Ext. 42978)
Lab: Physics Building 119R (Ext. 42560)

Fields of Interest:

- Development of Terahertz Sources and Devices
- Microwave Physics and Applications
- Microwave/Materials Interaction

Lecture Notes Download:
http://www.phys.nthu.edu.tw/~thschang/MWPA.htm
or http://www.phys.nthu.edu.tw/~hf5/

Schedule (depending on the students' condition)

週次	時間	上課內容
一	02/21(四)	Introduction to MWPA Chap.1 + Chap. 2 Transmission Line Theory
二	02/28(四)	和平紀念日
三	03/07(四)	Chap. 2 Transmission Line Theory
四	03/14(四)	Chap. 3 Transmission Line and Waveguides
五	03/21(四)	Characteristics of Waveguide Modes and Their Applications
六	03/28(四)	Chap. 4 Microwave Network Analysis
七	04/04(四)	民族掃墓節、兒童節
八	04/11(四)	Chap. 4 Microwave Network Analysis
九	04/18(四)	Excitation of a Specific Waveguide Modes
十	04/25(四)	Modal Analysis for Group Delay and Millimeter-Wave Diffraction
十一	05/02(四)	Chap. 5 Impedance Matching and Tuning
十二	05/09(四)	Chap. 5 Impedance Matching and Tuning
十三	05/16(四)	Chap. 6 Microwave Resonators
十四	05/23(四)	Open Cavity: Introduction and Simulation
十五	05/30(四)	Chap. 9 Theory and Design of Ferrimagnetic Components
十六	06/06(四)	Chap. 9 Theory and Design of Ferrimagnetic Components
十七	06/13(四)	Applications of Ferrite Materials to Circulator and Isolators
十八	06/20(四)	Return the 6-page term paper

How to evaluate students' performance?

- No mid-term and final exams.
- Term-paper: Return a term-paper of six pages in IEEE-MTTs template (doc format). The topic of the term-paper is open. Any subject that related to this course is acceptable.
- Grading policy: The final score will be normalized to reflect an average consistency with other courses. It also depends on your attendance and participation.
- High attendance rate and active participation are highly encouraged. Total number of attendance: 15

  15→100%, 13→95%, 11→90%, 9→85%, 7→80%

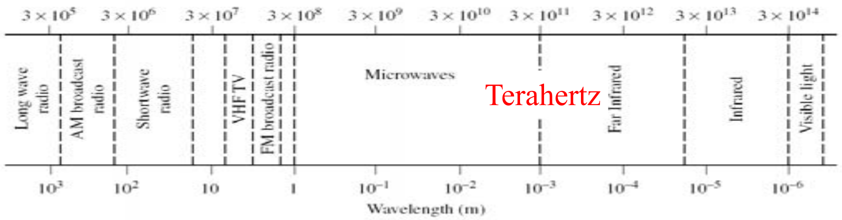
➢ (final score) = (term paper) × (attendance & participation)

Others

- The contents of this course are designed for senior and graduate level students. Only passive devices are addressed.
- This book shows that microwave circuits and devices can be explained through the use of circuit theory, Maxwell's equations, and related concepts.
- If you have any question, do not hesitate to raise your hand.
- Any comment on improving the pedagogy is more than welcome and is highly appreciated.

Chapter 1 Electromagnetic Theory: Introduction

The electromagnetic spectrum: Frequency (Hz)



Typical Frequencies	
AM broadcast band	535–1605 kHz
Short wave radio band	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
US cellular telephone	824–849 MHz
	869–894 MHz
European GSM cellular	880–915 MHz
	925–960 MHz
GPS	1575.42 MHz
	1227.60 MHz
Microwave ovens	2.45 GHz
US DBS	11.7–12.5 GHz
US ISM bands	902–928 MHz
	2.400–2.484 GHz
	5.725–5.850 GHz
US UWB radio	3.1–10.6 GHz

Approximate Band Designations	
Medium frequency	300 kHz to 3 MHz
High frequency (HF)	3 MHz to 30 MHz
Very high frequency (VHF)	30 MHz to 300 MHz
Ultra high frequency (UHF)	300 MHz to 3 GHz
L band	1–2 GHz
S band	2–4 GHz ✓
C band	4–8 GHz
X band	8–12 GHz ✓
Ku band	12–18 GHz
K band	18–26 GHz
Ka band	26–40 GHz ✓
U band	40–60 GHz
V band	50-75 GHz
E band	60-90 GHz
W band	75-110 GHz ✓
F band	90-140 GHz

General Form of **Time-Varying** Maxwell Equations

$$
\begin{aligned}
\nabla \times \bar{\mathcal{E}} &= \frac{-\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}}, \\
\nabla \times \bar{\mathcal{H}} &= \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}}, \\
\nabla \cdot \bar{\mathcal{D}} &= \rho, \\
\nabla \cdot \bar{\mathcal{B}} &= 0.
\end{aligned}
\qquad \text{(1.1a-d)}
$$

$\bar{\mathcal{E}}$ is the electric field intensity, in V/m.
$\bar{\mathcal{H}}$ is the magnetic field intensity, in A/m.
$\bar{\mathcal{D}}$ is the electric flux density, in Coul/m^2.
$\bar{\mathcal{B}}$ is the magnetic flux density, in Wb/m^2.
$\bar{\mathcal{M}}$ is the (fictitious) magnetic current density, in V/m^2.
$\bar{\mathcal{J}}$ is the electric current density, in A/m^2.
ρ is the electric charge density, in Coul/m^3.

Maxwell's Equations in **Phasor Form**

Phasors: In linear equations, harmonic quantities can be represented by complex variables as follows:

$$
\underbrace{\begin{Bmatrix} \mathbf{E}(\mathbf{x},t) \\ \mathbf{D}(\mathbf{x},t) \\ \mathbf{B}(\mathbf{x},t) \\ \mathbf{H}(\mathbf{x},t) \\ \mathbf{J}(\mathbf{x},t) \\ \rho(\mathbf{x},t) \end{Bmatrix}}_{\text{real}} = \mathrm{Re}\left[\underbrace{\begin{Bmatrix} \mathbf{E}(\mathbf{x}) \\ \mathbf{D}(\mathbf{x}) \\ \mathbf{B}(\mathbf{x}) \\ \mathbf{H}(\mathbf{x}) \\ \mathbf{J}(\mathbf{x}) \\ \rho(\mathbf{x}) \end{Bmatrix}}_{\text{Phasors}} e^{j\omega t} \right]
$$

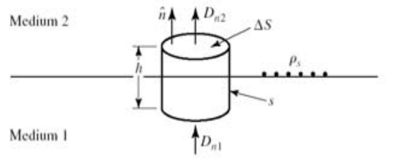
It is assumed that the LHS is given by the real part of the RHS.

Assume an $e^{j\omega t}$ time dependence. $\mathbf{E}(x,y,x,t) = \mathrm{Re}\left[\mathbf{E}(x,y,z)e^{j\omega t}\right]$

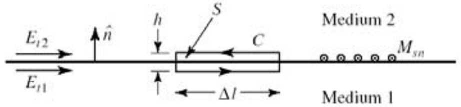
$$
\begin{cases}
\nabla \times \mathbf{E} = -j\omega\mathbf{B} - \mathbf{M} \\
\nabla \times \mathbf{H} = j\omega\mathbf{D} + \mathbf{J}_{free} \\
\nabla \cdot \mathbf{D} = \rho_{free} \\
\nabla \cdot \mathbf{B} = 0
\end{cases}
$$

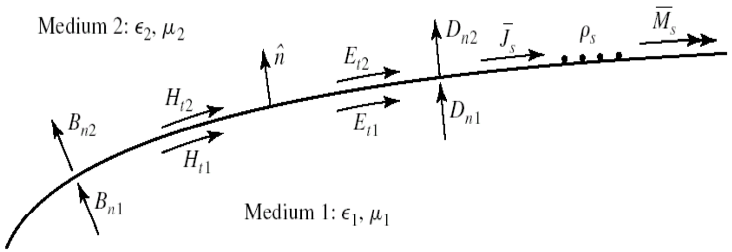
Boundary Conditions

$$
\begin{cases}
(\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n} = \sigma_{free} \\
(\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n} = 0
\end{cases}
$$



$$
\begin{cases}
(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \\
\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_{free}
\end{cases}
$$





More Boundary Conditions

Dielectric interface
$$
\begin{cases}
(\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n} = 0 \\
(\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n} = 0
\end{cases}
\qquad
\begin{cases}
(\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \\
\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0
\end{cases}
$$

Electric wall (tangential components of **D** must vanish, e.g. perfect conductor.)
$$
\begin{cases}
\mathbf{D}_2 \cdot \mathbf{n} = \sigma_{free} \\
\mathbf{E}_2 \times \mathbf{n} = 0
\end{cases}
\qquad
\begin{cases}
\mathbf{B}_2 \cdot \mathbf{n} = 0 \\
\mathbf{n} \times \mathbf{H}_2 = \mathbf{K}_{free}
\end{cases}
$$

Magnetic wall (tangential components of **H** must vanish)
$$
\begin{cases}
\mathbf{D}_2 \cdot \mathbf{n} = 0 \\
\mathbf{E}_2 \times \mathbf{n} = -\mathbf{M}_s
\end{cases}
\qquad
\begin{cases}
\mathbf{B}_2 \cdot \mathbf{n} = 0 \\
\mathbf{n} \times \mathbf{H}_2 = 0
\end{cases}
$$

Radiation condition
(Absorbing boundary condition, no reflection)

Chapter 2 Transmission Line Theory

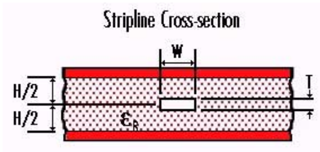
What is a transmission line?
http://www.wikipedia.org/wiki/Transmission_line

A transmission line is the material medium or structure that forms all or part of a path from one place to another for directing the transmission of energy, such as electric currents, magnetic fields, acoustic waves, or electromagnetic waves.

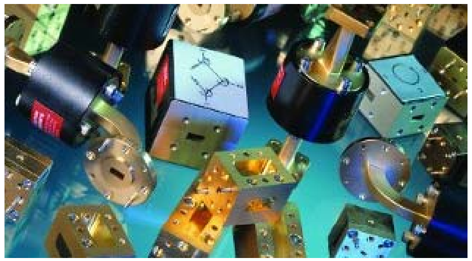
Examples of transmission lines include wires, microstrip lines, coplanar waveguides, optical fibers, coaxial cables, circular or rectangular closed waveguides, and dielectric slabs.

Transmission Line Examples

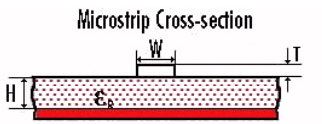
Coaxial cable



Stripline

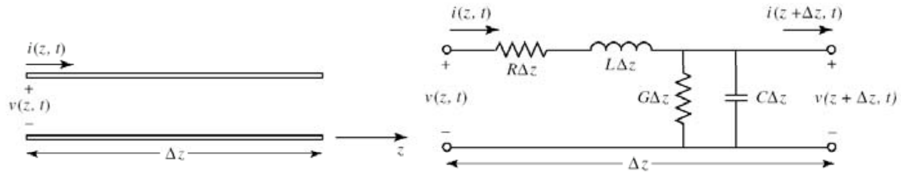


Rectangular waveguide



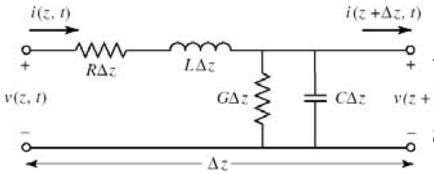
Microstrip Line

Lumped Element Circuit Model



L = Series inductance per unit length, for both conductors, H/m
C = Shunt capacitance per unit length , F/m
R = Series resistance per unit length , for both conductors, Ω/m
G = Shunt conductance per unit length , Ω^{-1}/m or S/m
(1) Δz must be electrically small, i.e. smaller than a guided wavelength.
(2) Within Δz, it is also possible to use shunt *GC* followed by the series *RL*.
(3) Also possible to use π- or *T*-networks. See problems in textbook.
(4) The parameters *RLCG* are required for describing a transmission line.

Time-Domain Transmission Line Equations



$$v(z+\Delta z,t) = v(z,t) - \left[R\Delta z i(z,t) + L\Delta z \frac{d}{dt} i(z,t)\right]$$

$$i(z+\Delta z,t) = i(z,t) - \left[G\Delta z v(z,t) + C\Delta z \frac{d}{dt} v(z,t)\right]$$

$$\frac{1}{\Delta z}[v(z+\Delta z,t) - v(z,t)] = -\left[Ri(z,t) + L\frac{d}{dt} i(z,t)\right]$$

$$\frac{1}{\Delta z}[i(z+\Delta z,t) - i(z,t)] = -\left[Gv(z,t) + C\frac{d}{dt} v(z,t)\right]$$

KVL $\Delta z \to 0$: $$\frac{\partial}{\partial z} v(z,t) = -\left[Ri(z,t) + L\frac{\partial}{\partial t} i(z,t)\right]$$

KCL $\Delta z \to 0$: $$\frac{\partial}{\partial z} i(z,t) = -\left[Gv(z,t) + C\frac{\partial}{\partial t} v(z,t)\right]$$

Telegrapher's Equations

Frequency-Domain Transmission Line Equations

Time-Harmonic assumption: (Sinusoidal Steady-State)

$$v(z,t) = V(z)e^{j\omega t}$$

$$i(z,t) = I(z)e^{j\omega t}$$

$$\frac{\partial}{\partial z} v(z,t) = -\left[Ri(z,t) + L\frac{\partial}{\partial t} i(z,t)\right] \Rightarrow \frac{\partial}{\partial z} V(z) = -(R + jL\omega)I(z)$$

$$\frac{\partial}{\partial z} i(z,t) = -\left[Gv(z,t) + C\frac{\partial}{\partial t} v(z,t)\right] \Rightarrow \frac{\partial}{\partial z} I(z) = -(G + j\omega C)V(z)$$

(Voltage and Current) Wave Equations:

$$\frac{\partial^2}{\partial z^2} V(z) = (R + j\omega L)(G + j\omega C)V(z) = \gamma^2 V(z)$$

$$\frac{\partial^2}{\partial z^2} I(z) = (R + j\omega L)(G + j\omega C)I(z) = \gamma^2 I(z)$$

Wave Propagation on a Transmission Line

$$\frac{d^2}{d^2 z} V(z) - \gamma^2 V(z) = 0 \qquad \gamma = \alpha + i\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\frac{d^2}{d^2 z} I(z) - \gamma^2 I(z) = 0$$

γ=(Complex) propagation constant

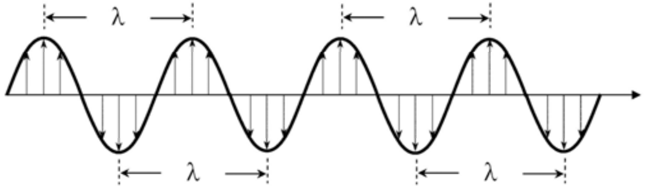
α=Attenuation constant

β=Phase constant

Solution:

$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$ $\quad V_0^+$ = forward(+z) propaating(voltge) wave

$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$ $\quad V_0^-$ = reflected(−z) propaating(voltge) wave



Characteristic Impedance Z_0

Solution of Wave Equations:

$$\frac{d^2}{d^2 z} V(z) - \gamma^2 V(z) = 0 \longrightarrow V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\frac{d^2}{d^2 z} I(z) - \gamma^2 I(z) = 0 \longrightarrow I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\frac{d}{dz} V(z) = -(R + j\omega L)\, I(z) \longrightarrow I(z) = \frac{\gamma}{R + j\omega L}[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}]$$

$$= \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} \text{ and } Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Lossless Transmission Line

$R = G = 0,\ \alpha = 0$

Proagation Constat $\gamma = \alpha + j\beta = j\omega\sqrt{LC}$

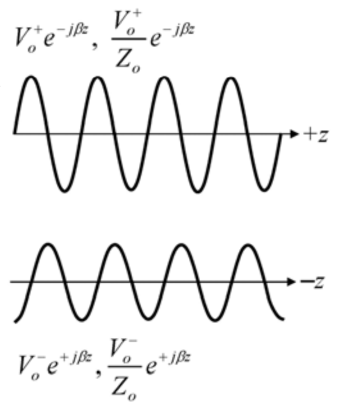
Characteristic impendance : $Z_0 = \sqrt{\dfrac{L}{C}}$

Wavelength : $\lambda = \dfrac{2\pi}{\beta}$

Phase velocity : $v_p = \dfrac{1}{\sqrt{LC}}$

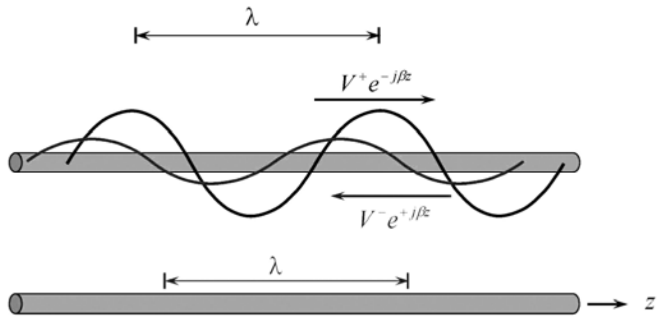
Total voltage : $V(z) = V_0^+\ e^{-j\beta z} + V_0^-\ e^{j\beta z}$

Total currnt : $I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{j\beta z}$

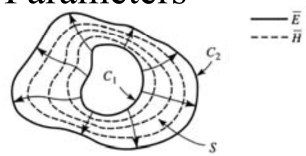


What is the difference between the above two waves? why?

Voltage Waves on Lossless Line



2.2 Transmission Line Parameters



Time-average stored magnetic energy

$$\text{Field } W_m = \frac{u}{4}\int H \cdot H^* ds, \text{ Circuit} \Rightarrow W_m = \frac{L}{4}|I_0|^2 \Rightarrow L = \frac{u}{|I_0|^2}\int H \cdot H^* ds$$

Time-average stored electric energy

$$\text{Field } W_e = \frac{\varepsilon}{4}\int E \cdot E^* ds, \text{ Circuit} \Rightarrow W_e = \frac{C}{4}|V_0|^2 \Rightarrow C = \frac{\varepsilon}{|V_0|^2}\int E \cdot E^* ds$$

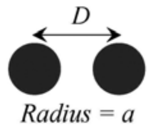
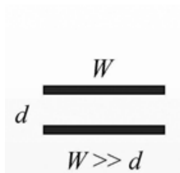
Conductor loss – Finite Conductivity, σ or R_s

$$\text{Field } P_c = \frac{R_S}{2}\int_{C1+C2} H \cdot H^* dl, \text{ Circuit} \Rightarrow P_c = \frac{R_S}{2}|I_0|^2 \Rightarrow R = \frac{R_S}{|I_0|^2}\int_{C1+C2} H \cdot H^* dl$$

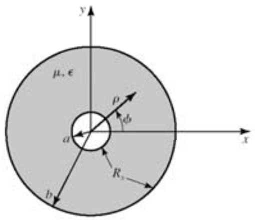
Dielectric loss – Lossy dielectric

$$\text{Field } P_d = \frac{w\varepsilon''}{2}\int E \cdot E^* ds, \text{ Circuit} \Rightarrow P_d = \frac{G}{2}|V_0|^2 \Rightarrow G = \frac{w\varepsilon^n}{|V_0|^2}\int E \cdot E^* ds$$

Some examples:

	L	C	G	R
	$\dfrac{u}{2\pi}\ln\dfrac{b}{a}$	$\dfrac{2\pi\varepsilon'}{\ln\dfrac{b}{a}}$	$\dfrac{2\pi\varepsilon''}{\ln\dfrac{b}{a}}$	$\dfrac{R_S}{2\pi}(\dfrac{1}{a}+\dfrac{1}{b})$
	$\dfrac{u}{\pi}\cosh^{-1}(\dfrac{D}{2a})$	$\dfrac{\pi\varepsilon'}{\cosh^{-1}(\dfrac{D}{2a})}$	$\dfrac{\omega\varepsilon''}{\varepsilon'}C$	$\dfrac{R_S}{\pi a}$
	$u\dfrac{W}{d}$	$\varepsilon'\dfrac{W}{d}$	$\dfrac{\omega\varepsilon''}{\varepsilon'}C$	$\dfrac{2R_s}{W}$

Example 2.1
Transmission Line Parameters of a Coaxial Line



$$L=\frac{u}{2\pi}\int_0^{2\pi}\int_a^b \frac{1}{\rho^2}\rho\, d\rho\, d\varphi=\frac{u}{2\pi}\ln(\frac{b}{a}) \quad (H/m)$$

$$C=\frac{\varepsilon'}{[\ln(b/a)]^2}\int_0^{2\pi}\int_a^b \frac{1}{\rho^2}\rho\, d\rho\, d\varphi=\frac{2\pi\varepsilon'}{\ln(b/a)} \quad (F/m)$$

$$R=\frac{R_S}{(2\pi)^2}\left\{\int_0^{2\pi}\frac{a}{a^2}d\varphi+\int_0^{2\pi}\frac{b}{b^2}d\varphi=\frac{R_S}{2\pi}(\frac{1}{a}+\frac{1}{b})\right\}$$

$$G=\frac{\omega\varepsilon''}{\varepsilon'}C$$

If no direlectric and no condutor loss

$$\beta=\omega\sqrt{LC}=\omega\sqrt{u\varepsilon}$$

$$Z_0=\sqrt{\frac{L}{C}}=\frac{\ln(\frac{b}{a})}{2\pi}\sqrt{\frac{u}{\varepsilon}}$$

let a=1mm, b=3mm, ε_r=2.8,then

$$L=\frac{u}{2\pi}\ln(\frac{b}{a})=\frac{u}{2\pi}\ln(3)=2\ln 3\times 10^{-7}\text{ (H/m)}$$

$$C=\frac{2\pi\varepsilon'}{\ln(\frac{b}{a})}=\frac{2\pi\varepsilon'}{\ln(3)}=\frac{2.8}{18\times\ln 3}\times 10^{-9}\text{ (F/m)}$$

$$v_p=\frac{1}{\sqrt{LC}}=\frac{c}{\sqrt{2.8}}=1.793\times 10^8\text{ (m/s)}$$

$$Z_0=\sqrt{\frac{L}{C}}=\frac{\ln 3}{2\pi}\sqrt{\frac{u}{\varepsilon}}=\frac{60\ln 3}{\sqrt{2.8}}=39.39\ \Omega$$

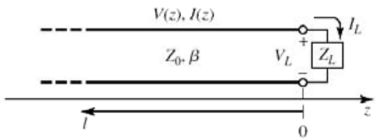
Summary and Some Concepts

Time-domain and frequency-domain transmission line equations
Fundamental transmission line parameters: R, L, C, G
Lossless lines:(a) L and C, (b) β and Z_0
β and Z_0 are determined by the cross sectional line geometry.
(What if the cross section of a line is not uniform?)

Voltage and current waves on a line share the same ordinary differential equation but have different boundary condition, which are usually specified at the source and load ends
The phase velocity and wavelength are determined solely by β

Is a rectangular waveguide a tranmission line? if yes, what are its R, L, G,and C?
All transmission lines must have two conductors?
Only the TEM mode can propagate on a transmission line?

2.3 Terminated Lossless Transmission Lines



$$V(z)=V_0^+e^{-j\beta z}+V_0^-e^{j\beta z}$$

$$I(z)=\frac{V_0^+}{Z_0}e^{-j\beta z}-\frac{V_0^-}{Z_0}e^{j\beta z}$$

$$Z_L=\frac{V(0)}{I(0)}=Z_0\frac{V_0^++V_0^-}{V_0^+-V_0^-}\longrightarrow V_0^-=\frac{Z_L-Z_0}{Z_L+Z_0}V_0^+$$

$$\Gamma\equiv\frac{V_0^+}{V_0^-}=\frac{Z_L-Z_0}{Z_L+Z_0}$$

Total Voltage Wave : $V(z)=V_0^+(e^{-j\beta z}+\Gamma e^{j\beta z})$

Total Current Wave : $I(z)=\dfrac{V_0^+}{Z_0}(e^{-j\beta z}-\Gamma e^{j\beta z})$

Time-average power flow $P_{av}=\frac{1}{2}\mathrm{Re}[V(z)I^*(z)]=\frac{1}{2}|V(z)|^2\dfrac{(1-|\Gamma|^2)}{Z_0}$

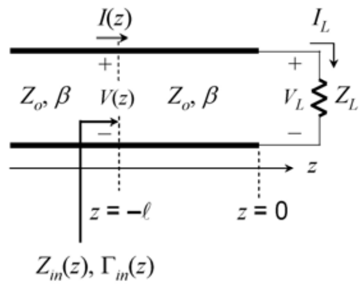
Return Loss: $RL=-20\ \log|\Gamma|$ (dB)

If $|\Gamma|$=0, $RL=-\infty$ dB, called matched load.

http://education.tm.agilent.com/index.cgi?CONTENT_ID=22

Reflection Coefficient at Arbitrary Position

$$\Gamma_{in}(z)\equiv\Gamma(z)=\frac{V^-(z)}{V^+(z)}=\frac{V_0^-e^{j\beta z}}{V_0^+e^{-j\beta z}}$$



At $z=0$, $V^-(z)=V_0^-$, $V^+(z)=V_0^+$

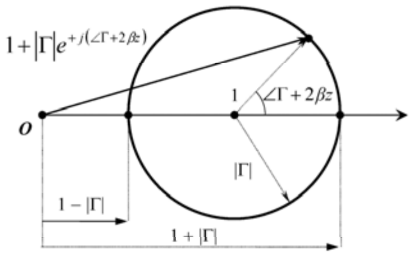
At $z=l$, $V^-(z)=V_0^-e^{-j\beta l}$, $V^+(z)=V_0^+e^{j\beta l}$

$$\Gamma_{in}(-l)=\frac{V^-(-l)}{V^+(-l)}=\frac{V_0^-e^{-j\beta l}}{V_0^+e^{j\beta l}}=\Gamma(0)e^{-j2\beta l}$$

The reflection coefficient along the line is a periodic complex function with a constant magnitude

Standing Wave Ratio, *SWR* or *VSWR*

$$SWR \equiv \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$



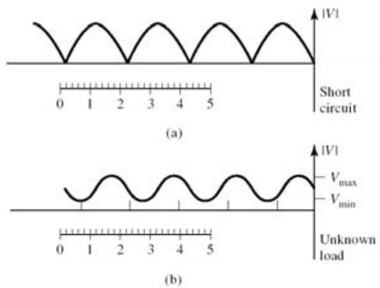
What are V_{max} and V_{min} along line?

$$V(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})$$

$$|V(z)| = |V_0^+|\,|1+\Gamma e^{j2\beta z}|$$

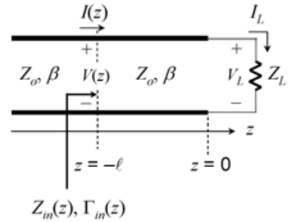
$$\longrightarrow V_{max} = |V_0^+|(1+|\Gamma|)$$

$$\longrightarrow V_{min} = |V_0^+|(1-|\Gamma|)$$



Input Impedance at Any *z* of a Line

$$Z_{in}(z) = \frac{V(z)}{I(z)} = Z_0 \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}$$



$$Z_{in}(-l) = \frac{V(z)}{I(z)}\Big|_{z=-l} = Z_0 \frac{V_0^+ e^{+j\beta l} + V_0^- e^{-j\beta l}}{V_0^+ e^{+j\beta l} - V_0^- e^{-j\beta l}} = Z_0 \frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}} = Z_0 \frac{1+\dfrac{Z_L-Z_0}{Z_L+Z_0}e^{-j2\beta l}}{1-\dfrac{Z_L-Z_0}{Z_L+Z_0}e^{-j2\beta l}}$$

$$= Z_0 \frac{(Z_L+Z_0)e^{j\beta l} + (Z_L-Z_0)e^{j\beta l}}{(Z_L+Z_0)e^{j\beta l} - (Z_L-Z_0)e^{j\beta l}} = Z_0 \frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_L\sin\beta l} = Z_0 \frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}$$

One can verify that $\Gamma(-l) = \dfrac{Z_{in}(-l) - Z_0}{Z_{in}(-l) + Z_0}$

Special Terminations

(1) Short Circuit: $Z_L = 0$

$\Gamma(0) = -1;\ Z_{in} = j\,Z_0\tan\beta l\ ;\ V_0^- = -V_0^+;$

$V^-(z) = -V_0^+ e^{j\beta z},\ V^+(z) = V_0^+ e^{-j\beta z}$

(2) Open Circuit: $Z_L = \infty$

$\Gamma(0) = 1;\ Z_{in} = -j\,Z_0\cos\beta l\ ;\ V_0^- = V_0^+;$

$V^-(z) = V_0^+ e^{j\beta z},\ V^+(z) = V_0^+ e^{-j\beta z}$

(3) Matched Load: $Z_L = Z_0$

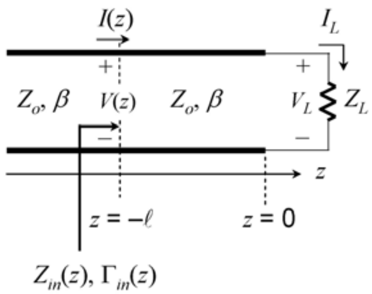
$\Gamma(0) = 0;\ Z_{in} = Z_0\ ;\ V_0^- = 0;$

$V^-(z) = 0,\ V(z) = V_0^+ e^{-j\beta z}$, a traveling wave

(4) Transmission Lines of Special Length

$(a)\ l = \dfrac{\lambda}{2},\ Z_{in} = Z_L\ \ (b) l = \dfrac{\lambda}{4},\ Z_{in} = \dfrac{Z^2{}_0}{Z_L}$

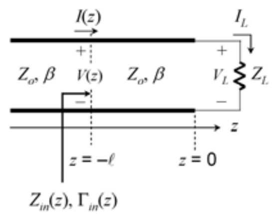
A section of $\dfrac{\lambda}{4}$ line will invert the load impedance



$$Z_{in}(-l) = Z_0 \frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}$$

$$\Gamma(-l) = \frac{V^-(-l)}{V^+(-l)} = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{+j\beta l}} = \Gamma(0)e^{-2j\beta l}$$

Short-Circuited Transmission Line



Total voltage and current at arbitrary *z*:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0}e^{-j\beta z} - \frac{V_0^-}{Z_0}e^{j\beta z}$$

Reflection coefficient at $z = 0$ (Load):

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

When $Z_L = 0$.

(1) $Z_{in}(-l) = \dfrac{V(-l)}{I(-l)} = Z_0 \dfrac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} = jZ_0\tan\beta l$

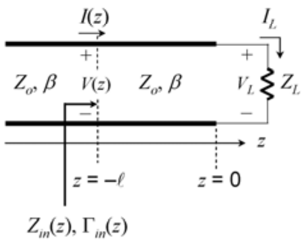
(2) $V(-l) = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l} = 2jV_0^+\sin\beta l,\ I(-l) = \dfrac{V_0^+}{Z_0}e^{j\beta l} - \dfrac{V_0^-}{Z_0}e^{-j\beta l} = 2\dfrac{V_0^+}{Z_0}\cos\beta l$

(3) $\Gamma(0) = \dfrac{Z_L - Z_0}{Z_L + Z_0} = -1,\ V_0^- = -V_0^+$, or $V_0^+ + V_0^- = 0$

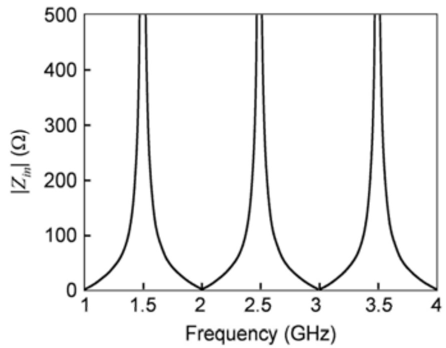
(4) $\Gamma(-l) = \Gamma_0 e^{-2j\beta l} = -e^{-2j\beta l},\ \ |\Gamma(-l)| = |\Gamma_0| = 1$ for all l

Example: Input Impedance

For a short-circuited line of 10 cm, compute the magnitude of the input impedance for $1 \le f \le 4$ GHz. Let L=209.4nH/m and C=119.5pF/m



$$Z_{in}(-l) = jZ_0 \tan\beta l = jZ_0 \tan\pi f$$



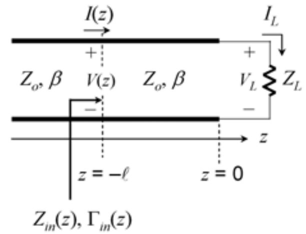
$$Z_0=\sqrt{\frac{L}{C}}=41.86\ \Omega$$

$$v_p=\frac{1}{\sqrt{LC}}=2\times10^8\ \text{m/s}$$

$$\varepsilon_{reff}=(\frac{c}{v_p})^2=2.25$$

$$\beta l=\omega\sqrt{LC}\times l=2\pi f\times\frac{10^9}{2\times10^8}\times0.1=\pi f$$

Open-Circuit Transmission Line



Total voltage and current at arbitrary z:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

Reflection coefficient at $z = 0$ (Load):

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

When $Z_L = \infty$.

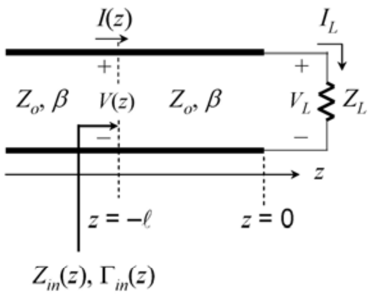
(1) $Z_{in}(-l) = \dfrac{V(-l)}{I(-l)} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = -jZ_0 \cot\beta l$

(2) $V(-l) = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l} = 2V_0^+ \cos\beta l,\ I(-l) = \dfrac{V_0^+}{Z_0} e^{j\beta l} - \dfrac{V_0^-}{Z_0} e^{-j\beta l} = 2j\dfrac{V_0^+}{Z_0}\sin\beta l$

(3) $\Gamma(0) = \dfrac{Z_L - Z_0}{Z_L + Z_0} = -1,\ V_0^- = V_0^+$

(4) $\Gamma(-l) = \Gamma_0 e^{-2j\beta l} = -e^{-2j\beta l},\quad |\Gamma(-l)| = |\Gamma_0| = 1$ for all l.

Input Impedance of An Open-Circuit Transmission Line (l =10 cm)



$$Z_0=\sqrt{\frac{L}{C}}=41.86\ \Omega$$

$$v_p=\frac{1}{\sqrt{LC}}=2\times10^8\ \text{m/s}$$

$$\varepsilon_{reff}=(\frac{c}{v_p})^2=2.25$$

$$\lambda = \frac{2\times10^8}{f} = \frac{20}{f}\frac{\text{cm}}{\text{GHz}}$$

$$\beta l=\omega\sqrt{LC}\times l=2\pi f\times\frac{10^9}{2\times10^8}\times0.1=\pi f$$

$$Z_{in}(-l) = -jZ_0 \cot\beta l = -jZ_0 \cot\pi f$$

2.4 The Smith Chart

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} \Leftarrow z_L = \frac{Z_L}{Z_0} = r_L + jx_L$$

$$= |\Gamma| e^{j0} = \Gamma_r + j\Gamma_i$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} \Rightarrow r_L + jx_L = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

Rearranging the above equation, one can obtain

$$(\Gamma_r - \frac{r_L}{1+r_L})^2 + \Gamma_i^2 = (\frac{1}{1+r_L})^2 \quad \text{called the constant -}r\text{ circles}$$

$$(\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x_L})^2 = (\frac{1}{x_L})^2 \quad \text{called the constant -}x\text{ circles}$$

Derivation (optional)

$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} \Rightarrow r_L + jx_L = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

$$r_L + jx_L = \frac{(1+\Gamma_r + j\Gamma_i)(1-\Gamma_r + j\Gamma_i)}{(1-\Gamma_r - j\Gamma_i)(1-\Gamma_r + j\Gamma_i)} = \frac{(1+\Gamma_r + j\Gamma_i)(1-\Gamma_r + j\Gamma_i)}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r_L = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2},\; x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r_L = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-2\Gamma_r + \Gamma_r^2) + \Gamma_i^2} \Rightarrow (1-2\Gamma_r + \Gamma_r^2 + \Gamma_i^2) r_L + \Gamma_r^2 + \Gamma_i^2 + 1 = 0$$

$$(1+r_L)\Gamma_r^2 - 2r_l\Gamma_r + (1+r_L)\Gamma_i^2 = 1 - r_L$$

$$\Gamma_r^2 - 2\frac{r_L}{1+r_L}\Gamma_r + \Gamma_i^2 = \frac{1+r_L}{1-r_L}$$

$$\begin{cases} (\Gamma_r - \frac{r_L}{1+r_L})^2 + \Gamma_i^2 = \frac{1-r_L}{1+r_L} + (\frac{r_L}{1+r_L})^2 = (\frac{1}{1+r_L})^2 & \rightarrow (\Gamma_r - \frac{r_L}{1+r_L})^2 + \Gamma_i^2 = (\frac{1}{1+r_L})^2 \\ x_L = \frac{2\Gamma_i}{1-2\Gamma_r^2 + \Gamma_r^2 + \Gamma_i^2} & \rightarrow (\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x_L})^2 = (\frac{1}{x_L})^2 \end{cases}$$

Reflection Coefficient and Load Impedance

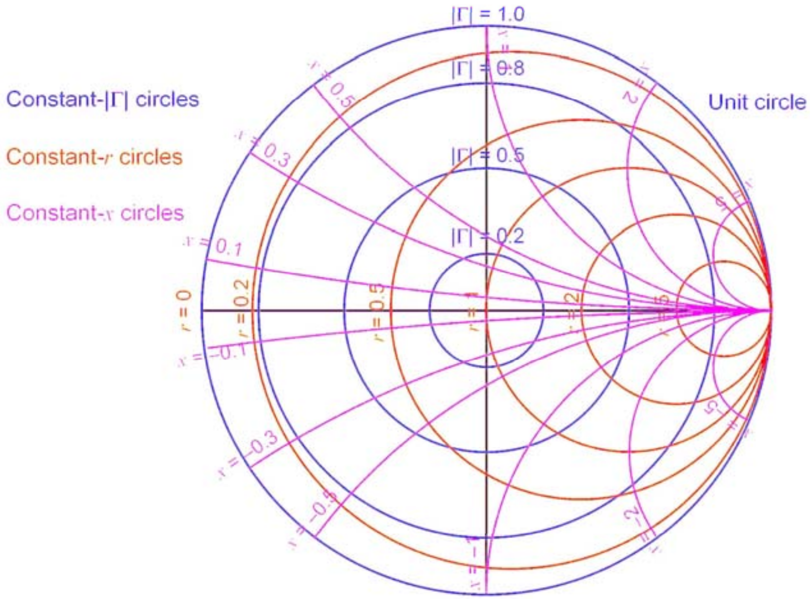
Along a transmission line, the magnitude of the reflection coefficient is not changed. It is called the constant-$|\Gamma|$ circle

$$\Gamma_{in}(-l) = \frac{V^-(-l)}{V^+(-l)} = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma(0) e^{-2j\beta l} \qquad \text{(See section 2.3)}$$

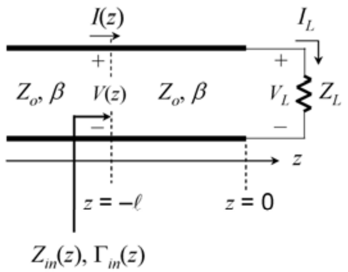
The constant-r, constant-x, constant-$|\Gamma|$ circle are defined on a plane called Γ-plane, of which the horizontal and vertical axes are Γ_r and Γ_i, respectively.

http://education.tm.agilent.com/index.cgi?CONTENT_ID=22

Review of Smith Chart



Input Impedance ($z_{in}=r+jx$) and Reflection Coefficient ($\Gamma = \Gamma_r + j\Gamma_i$)



$$Z_{in}(-l) = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\Gamma_{in}(-l) = \frac{V^-(-l)}{V^+(-l)} = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma(0) e^{-2j\beta l}$$

(a) Constant-Γ circle:

$$\Gamma_{in}(-l) = \frac{V^-(-l)}{V^+(-l)} = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma(0) e^{-2j\beta l},\; |\Gamma_{in}(z)| = |\Gamma(0)| \le 1 \text{ for any } z.$$

Load Impedance ($z_{in}=r+jx$) and Reflection Coefficient ($\Gamma= \Gamma_r+j\Gamma_i$)

Any point inside the unit circle in Γ-plane corresponds to a normalized impedance value:

$$z_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma(0)}{1-\Gamma(0)} = \frac{1+|\Gamma|e^{j\angle\Gamma}}{1-|\Gamma|e^{j\angle\Gamma}}$$

$$z_{in} = \frac{Z_{in}(-l)}{Z_0} = \frac{1+\Gamma(-l)}{1-\Gamma(-l)} = \frac{1+\Gamma(0)e^{-j2\beta l}}{1-\Gamma(0)e^{-j2\beta l}} = \frac{1+\Gamma(0)e^{j(\angle\Gamma-2\beta l)}}{1-\Gamma(0)e^{j(\angle\Gamma-2\beta l)}}$$

(1) Γ_{in} and z_{in} are periodic functions on the chart with a period $=\frac{\lambda}{2}$

(2) There are two scales, angle in degree and "wavelength" along the peripheral of the Smith chart

(3) To locate $z_{in}(-l)$ and/or $\Gamma_{in}(-l)$, first find z_L, then go along the constant $-|\Gamma|$ circle clockwise with $2\beta l$ degree.

(4) Counter-clockwise toward load, and clockwise toward generator.

Admittance Smith Chart (optional)

z_L and Γ have one-on-one correspondence, i.e. a value of z_L will *map* to a unique Γ value, and vice versa, on the Γ-plane

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1},\ z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$\Gamma = \frac{y^{-1}{}_L - 1}{y^{-1}{}_L + 1} = \frac{1-y_L}{1+y_L} = -\frac{y_L - 1}{y_L + 1},\ y_L = \frac{1-\Gamma}{1+\Gamma} = \frac{1+\Gamma e^{j\pi}}{1-\Gamma e^{j\pi}}$$

The admittance chart or *y*-chart is the *z*-chart rotated by 180°

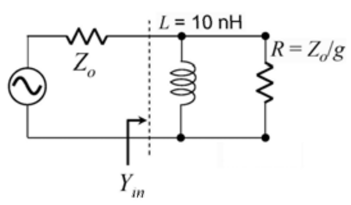
The admittance of $y_{in} = g + jb = z_{in}^{-1} = (r + jx)^{-1}$ *is* z_{in} rotated by 180° in a y-chart, the value of r and x are actually those of g and b

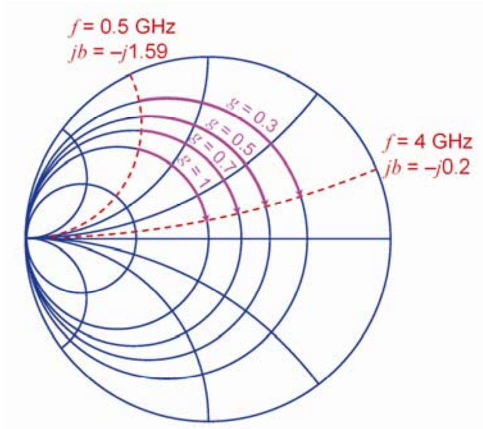
y_{in} and z_{in} are symmetric about the origin, $\Gamma=\Gamma_r+j\Gamma_i=0$, and have the same constant $-|\Gamma|$ circle

One may use a combined *yz*-chart to avoid successive rotations

Shunt Connection of *R* and *L*

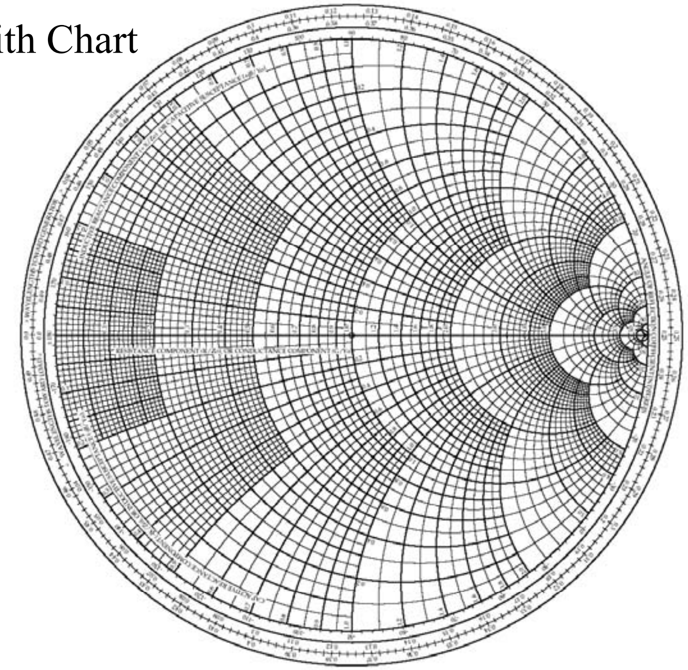
Find the input admittance of a resistor with $g = \frac{Z_0}{R} = 0.3, 0.5, 0.7$, and 1 in parallel with an inductor $L = 10nH$ for frequencies ranging from 500MHz to 4 GHz Characteristic impedance of the line $Z_0 = 50\Omega$





Sometimes admittance is more convenient than impedance.

The Smith Chart



Summary of Impedance and Reflection

$$Z_{in}(-l)=\frac{V(-l)}{I(-l)}=Z_0\frac{V_0^+e^{+j\beta l}+V_0^-e^{-j\beta l}}{V_0^+e^{+j\beta l}-V_0^-e^{-j\beta l}}=Z_0\frac{1+\Gamma(0)e^{-j2\beta l}}{1-\Gamma(0)e^{-j2\beta l}}$$

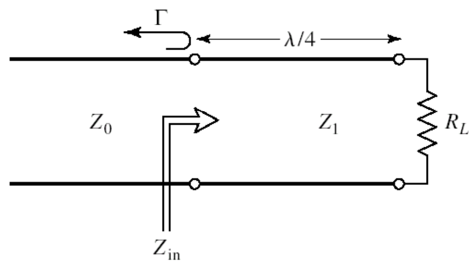
$$=Z_0\frac{1+\frac{Z_L-Z_0}{Z_L+Z_0}e^{-j2\beta l}}{1-\frac{Z_L-Z_0}{Z_L+Z_0}e^{-j2\beta l}}=Z_0\frac{Z_L+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l}$$

$$\Gamma_{in}(-l)=\frac{V^-(-l)}{V^+(-l)}=\frac{V_0^-e^{-j\beta l}}{V_0^+e^{j\beta l}}=\Gamma(0)e^{-2j\beta l}$$

$$\Gamma(-l)=\frac{Z_{in}(-l)-Z_0}{Z_{in}(-l)+Z_0}\qquad \Gamma(0)=\frac{Z_L-Z_0}{Z_L+Z_0}$$

2.5 The Quarter-Wave Transformer

A quarter-wave transformer is an impedance matching circuit



$$Z_{in}=Z_1\frac{R_L+jZ_1\tan\beta l}{Z_1+jR_L\tan\beta l}\bigg|_{\beta l=\frac{\pi}{2}}=\frac{Z_1^{\,2}}{R_L}$$

If $\Gamma_{in}=0$ is required, $Z_{in}=Z_0$, then $Z_1=\sqrt{Z_0R_L}$

How good is the performance of the impedance transformer?

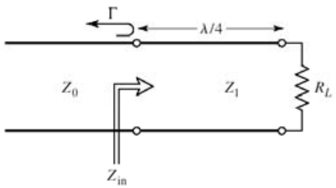
Example 2.5

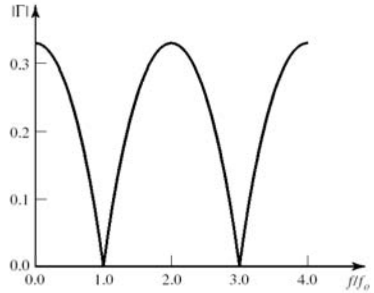
Examine the frequency response of a quarter-wave transformer.

R_L=100Ω, Z_0=50Ω. Find Z_1.

$$Z_1=\sqrt{50\times100}=70.7\Omega$$

$$l=\frac{\lambda}{4}@\ f_0\ \ or\ \ l=\frac{v_p}{4f_0}\cdot\beta l=\frac{2\pi f}{v_p}\times\frac{v_p}{4f_0}=\frac{\pi}{2}\frac{f}{f_0}$$



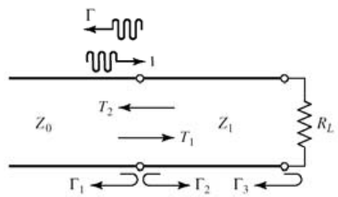


$$Z_{in}=Z_1\frac{R_L+jZ_1\tan\beta l}{Z_1+jR_L\tan\beta l}$$

$$|\Gamma|=\frac{Z_{in}-Z_0}{Z_{in}+Z_0}$$

$$Z_{in}(\beta l=\frac{\pi}{2})=\frac{Z_1^{\,2}}{R_L}=50\Omega$$

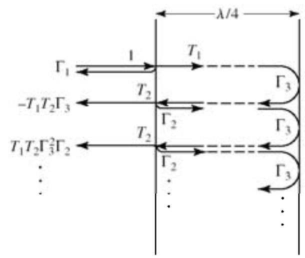
The Multiple Reflection Viewpoint



$$\Gamma_1=\frac{Z_1-Z_0}{Z_1+Z_0},T_1=\frac{2Z_1}{Z_1+Z_0}$$

$$\Gamma_2=\frac{Z_0-Z_1}{Z_0+Z_1}=-\Gamma_1,T_2=\frac{2Z_0}{Z_1+Z_0}$$

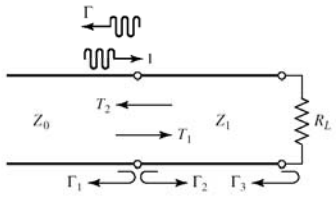
$$\Gamma_3=\frac{R_L-Z_1}{R_L+Z_1}$$



The total reflection coefficient in the frequency domain is the sum of time series of the reflection coefficient

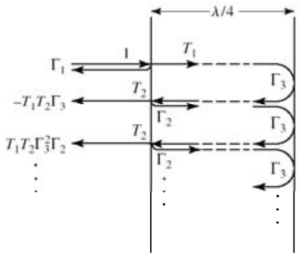
$$\Gamma=\Gamma_1-T_1T_2\Gamma_3+T_1T_2\Gamma_2\Gamma^2{}_3-T_1T_2\Gamma^2{}_2\Gamma^2{}_3+-........$$

The Multiple Reflection Viewpoint (Cont'd)



$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n$$

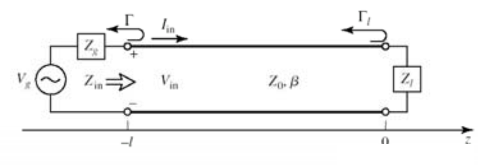
$$= \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3} = \frac{\Gamma_1 + \Gamma_1\Gamma_2\Gamma_3 - T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3}$$



Numerator$= \ldots = \dfrac{2(Z^2{}_1 - Z_0 R_L)}{(Z_1+Z_0)(Z_1+R_L)}$

If $Z_1 = \sqrt{Z_0 R_L} \quad \Rightarrow \Gamma = 0$

2.6 Generator and Load Mismatches



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$= Z_0 \frac{1+\Gamma_L e^{-2j\beta l}}{1-\Gamma_L e^{-2j\beta l}} = \frac{1+\Gamma}{1-\Gamma}$$

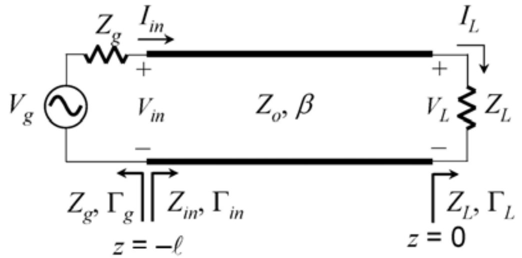
Forward and backward propagation voltage waves:

$$V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z})$$

$$V(-l) = \frac{Z_{in}}{Z_{in}+Z_g} V_g = V_0^+ (e^{j\beta l} + \Gamma e^{-j\beta l})$$

$$V_0^+ = \frac{Z_{in}}{Z_{in}+Z_g} \frac{V_g}{e^{j\beta l} + \Gamma e^{-j\beta l}} \Big|_{\substack{V_L = \ldots \\ Z_{in} = \ldots}} = V_g \frac{Z_0}{Z_0+Z_g} \frac{e^{-j\beta l}}{1-\Gamma_g \Gamma_L e^{-2j\beta l}}$$

Power Delivered to the Load



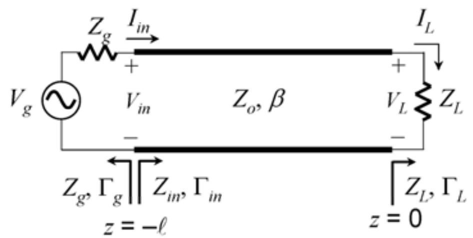
$$Z_{in} = R_{in} + jX_{in}$$
$$Z_g = R_g + jX_g$$

$$P_L = \frac{1}{2}\mathrm{Re}\left[V_{in} I^*{}_{in}\right] = \frac{1}{2}|V_{in}|^2 \mathrm{Re}\left[Z^{-1}{}_{in}\right] = \frac{1}{2}|V_g|^2 \left|\frac{Z_{in}}{Z_{in}+Z_g}\right|^2 \mathrm{Re}\left[Z^{-1}{}_{in}\right]$$

$$= \frac{1}{2}|V_g|^2 \frac{R^2{}_{in} + X^2{}_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2} \frac{R_{in}}{R^2{}_{in} + X^2{}_{in}}$$

$$= \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}$$

Power Delivered to the Load (cont'd)



$$P_L = \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}$$

(1) Load matched to line: $Z_L = Z_0,\ \Gamma_L = 0 \rightarrow P_L = \frac{1}{2}|V_g|^2 \frac{Z_0}{(Z_0+R_g)^2 + X_g{}^2}$

(2) Generator matched to loaded line: $Z_{in} = Z_g,\ R_{in} = R_g,\ X_{in} = X_g \rightarrow P_L = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g{}^2 + X_g{}^2)}$

(3) Conjungate matching: $Z_{in} = Z_g^*,\ R_{in} = R_g,\ X_{in} = -X_g,\ \frac{\partial P}{\partial R_{in}}\Big|_{2.75} = \frac{\partial P}{\partial X_{in}}\Big|_{2.75} = 0 \rightarrow P_L = \frac{1}{2}\frac{|V_g|^2}{4R_g}$

Γ_L, Γ_g, and Γ may be non-zero.

If $Z_L = Z_g = Z_0$, $\Gamma_L = \Gamma_g = 0$, only half the power produced by the generator is delivered to the load. The other half is lost in Z_g.

The End of Chap. 2