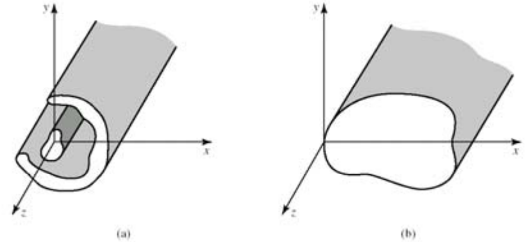
Chapter 3 Transmission Line and Waveguide

3.0 Introduction

- Transmission Lines are used for low-loss transmission of microwave power.
- Are two conductors required for the transfer of EM wave energy?
- L. Rayleigh, 1897
  TE and TM modes propagation in hollow waveguides with rectangular or circular cross sections.
- Experiments in 1936：
  (1) G. C. Southworth at AT&T: Rectangular waveguide.
  (2) M. L. Barrow at MIT : Circular waveguide.

3.1 General Solutions for TEM, TE, and TM Waves



(a) General two-conductor transmission line

(b) Closed waveguide as a transmission line

- Assume that all fields have a time–dependence of $e^{j\omega t}$ and propagation factor $e^{-j\beta z}$.
- EM fields in a waveguide or transmission line are decomposed into longitudinal and transverse components as

$$\mathbf{E}(x,y,z)=\left[\mathbf{E}_t(x,y)+\hat{\mathbf{z}}e_z(x,y)\right]e^{-j\beta z}$$

$$\mathbf{H}(x,y,z)=\left[\mathbf{H}_t(x,y)+\hat{\mathbf{z}}h_z(x,y)\right]e^{-j\beta z}$$

Field Solution

$$\left(\nabla^2+k^2\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix}=0\Rightarrow\left(\nabla_t^2-\beta^2+k^2\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix}=0\Rightarrow\left(\nabla_t^2+k_c^2\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix}=0$$

$$\nabla\cdot\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix}=0\Rightarrow\nabla_t\cdot\begin{Bmatrix}\mathbf{E}_t\\ \mathbf{H}_t\end{Bmatrix}-j\beta\begin{Bmatrix}E_z\\ H_z\end{Bmatrix}=0$$

$$\nabla\times\mathbf{E}=-j\omega\mu\mathbf{H}$$

$$\Rightarrow\left(\nabla_t+\hat{\mathbf{z}}\frac{\partial}{\partial z}\right)\times\left(\mathbf{E}_t+\hat{\mathbf{z}}E_z\right)=\left(\nabla_t-j\beta\hat{\mathbf{z}}\right)\times\left(\mathbf{E}_t+\hat{\mathbf{z}}E_z\right)=-j\omega\mu\left(\mathbf{H}_t+\hat{\mathbf{z}}H_z\right)$$

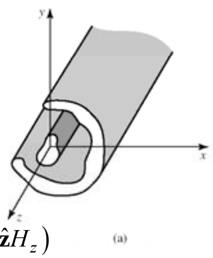
$$\Rightarrow\nabla_t\times\mathbf{E}_t=-\hat{\mathbf{z}}j\omega\mu H_z$$

$$\Rightarrow\nabla_t\times\nabla_t\times\mathbf{E}_t=\nabla_t\nabla_t\cdot\mathbf{E}_t-\nabla_t^2\mathbf{E}_t=j\beta\nabla_t E_z+k_c^2\mathbf{E}_t=-j\omega\mu\nabla_t\times\hat{\mathbf{z}}H_z$$

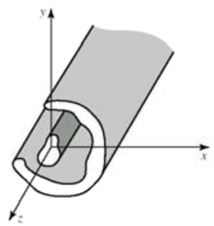
$$\Rightarrow\mathbf{E}_t=\frac{1}{k_c^2}\left(-j\beta\nabla_t E_z-j\omega\mu\nabla_t\times\hat{\mathbf{z}}H_z\right)$$

Similarly,

$$\mathbf{H}_t=\frac{1}{k_c^2}\left(-j\beta\nabla_t H_z+j\omega\varepsilon\nabla_t\times\hat{\mathbf{z}}E_z\right)$$



Field Solution (Cont'd)



In the Cartesian coordinates,

$$\nabla_t=\hat{\mathbf{x}}\frac{\partial}{\partial x}+\hat{\mathbf{y}}\frac{\partial}{\partial y}$$

$$E_x=\frac{1}{k_c^2}\left(-j\beta\frac{\partial}{\partial x}E_z-j\omega\mu\frac{\partial}{\partial y}H_z\right)$$

$$E_y=\frac{1}{k_c^2}\left(-j\beta\frac{\partial}{\partial y}E_z+j\omega\mu\frac{\partial}{\partial x}H_z\right)$$

$$H_x=\frac{1}{k_c^2}\left(j\omega\varepsilon\frac{\partial}{\partial y}E_z-j\beta\frac{\partial}{\partial x}H_z\right)$$

$$H_y=\frac{1}{k_c^2}\left(-j\omega\varepsilon\frac{\partial}{\partial x}E_z-j\beta\frac{\partial}{\partial y}H_z\right)$$

In arbitrary coordinates

$$\mathbf{E}_t=\frac{1}{k_c^2}\left(-j\beta\nabla_t E_z-j\omega\mu\nabla_t\times\hat{\mathbf{z}}H_z\right)$$

$$\mathbf{H}_t=\frac{1}{k_c^2}\left(-j\beta\nabla_t H_z+j\omega\varepsilon\nabla_t\times\hat{\mathbf{z}}E_z\right)$$

- In a cylindrical waveguide, the transversal EM field components can be expressed in terms of E_z and H_z the longitudinal fields.

(1) TEM Waves or TEM modes

$E_z = 0, H_z = 0 \;\; \Rightarrow \;\; k_c = 0$ with a nontrivial solution.

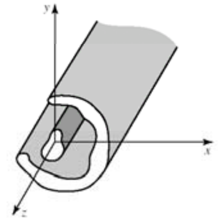
Wave Equations：
$$\left(\nabla^2 + k^2\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = 0 \Rightarrow \left(\nabla_t^{\,2} + \underbrace{k^2 - \beta^2}_{k_c^{\,2}=0}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = 0$$

$$\nabla_t^{\,2}\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = 0 \qquad \text{e.g.} \; \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{H}\end{Bmatrix} = 0$$

- The transverse fields of a TEM wave are thus the same as the static fields that can exist between the conductors.
- Electrostatic field problem: (Ref. Jackson Chap. 8)

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \;\Rightarrow\; \left(\nabla_t + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) \times \left(\mathbf{E}_t + \hat{\mathbf{z}}E_z\right)$$
$$= \left(\nabla_t - j\beta\hat{\mathbf{z}}\right) \times \left(\mathbf{E}_t + \hat{\mathbf{z}}E_z\right) = -j\omega\mu\left(\mathbf{H}_t + \hat{\mathbf{z}}H_z\right),$$
z-component $\nabla_t \times \mathbf{E}_t = 0 \;\; \Rightarrow \;\; \mathbf{E}_t(x,y) = -\nabla_t\phi(x,y)$

Wave Impedance



$$Z_{TEM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{\beta}{\omega\varepsilon} = \eta$$
$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$
$$\Rightarrow \left(\nabla_t + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) \times \left(\mathbf{E}_t + \hat{\mathbf{z}}E_z\right) = \left(\nabla_t - j\beta\hat{\mathbf{z}}\right) \times \left(\mathbf{E}_t + \hat{\mathbf{z}}E_z\right) = -j\omega\mu\left(\mathbf{H}_t + \hat{\mathbf{z}}H_z\right)$$

Similarly,

$$\Rightarrow \left(\nabla_t - j\beta\hat{\mathbf{z}}\right) \times \mathbf{E}_t = -j\omega\mu\mathbf{H}_t \quad \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \Rightarrow \left(\nabla_t - j\beta\hat{\mathbf{z}}\right) \times \mathbf{H}_t = j\omega\varepsilon\mathbf{H}_t$$
$$\Rightarrow -j\beta\hat{\mathbf{z}} \times \mathbf{E}_t = -j\omega\mu\mathbf{H}_t \qquad \Rightarrow -j\beta\hat{\mathbf{z}} \times \mathbf{H}_t = j\omega\varepsilon\mathbf{E}_t, \;\; \mathbf{H}_t \times j\beta\hat{\mathbf{z}} = j\omega\varepsilon\mathbf{E}_t$$
$$\Rightarrow \mathbf{H}_t = \hat{\mathbf{z}}\frac{\beta}{\omega\mu} \times \mathbf{E}_t = \hat{\mathbf{z}}\frac{1}{\eta} \times \mathbf{E}_t \qquad \Rightarrow \mathbf{E}_t = \mathbf{H}_t \times \hat{\mathbf{z}}\frac{\beta}{\omega\varepsilon} = \mathbf{H}_t \times \hat{\mathbf{z}}\eta$$

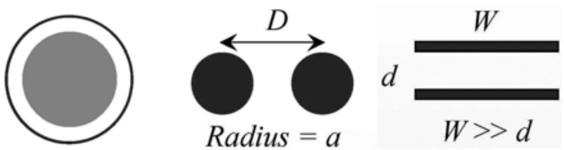
- $\mathbf{E}_t(x,y) = \mathbf{H}_t(x,y) \times \eta\hat{\mathbf{z}}$
- $\mathbf{H}_t(x,y) = \hat{\mathbf{z}}/\eta \times \mathbf{E}_t(x,y)$

(Wave impedance is in general not the characteristic impedance of the line)

Procedure for Analyzing a TEM Line

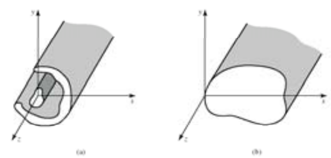
1. Solve the 2D Laplace equation, $\nabla_t^{\,2}\phi(x,y) = 0$.
2. B. C., tangential E or normal H.
3. $\mathbf{E}_t(x,y) = -\nabla_t\phi(x,y)$, $\mathbf{H}_t(x,y) = \hat{\mathbf{z}}/\eta \times \mathbf{E}_t(x,y)$
4. Calculate V_{12} and I
5. $\beta = k$, $Z_o = V_{12}/I$.

Examples:
Coaxial line, the two-wire line, and the parallel-plate waveguide in Table 2.1.



(2) General Properties of TE Waves

$$E_z = 0$$
$$\mathbf{H}_t = \frac{-j\beta}{k_c^{\,2}}\nabla_t H_z \Rightarrow H_x = \frac{-j\beta}{k_c^{\,2}}\frac{\partial}{\partial x}H_z, \; H_y = \frac{-j\beta}{k_c^{\,2}}\frac{\partial}{\partial y}H_z$$
$$\mathbf{E}_t = \mathbf{H}_t \times Z_{TE}\hat{\mathbf{z}} \Rightarrow E_x = \frac{-j\omega\mu}{k_c^{\,2}}\frac{\partial}{\partial y}H_z, \; E_y = \frac{j\omega\mu}{k_c^{\,2}}\frac{\partial}{\partial x}H_z$$



(a) Solve H_z from the Helmholtz wave equation :
$$\left(\nabla^2 + k^2\right)H_z = 0 \quad \text{or} \quad \left(\nabla_t^{\,2} + k_c^{\,2}\right)h_z = 0 \;\; \text{with } H_z = h_z e^{-j\beta z}$$

(b) Dispersive wave impedance
$$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$$

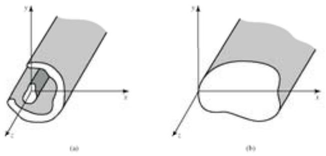
(c) Dispersive phase constant $\beta = \sqrt{k^2 - k_c^{\,2}}$ depends on line geometry.

(3) General Properties of TM Waves

$$H_z = 0$$

$$\mathbf{E}_t = \frac{-j\beta}{k_c^2}\nabla_t E_z \Rightarrow E_x = \frac{-j\beta}{k_c^2}\frac{\partial}{\partial x}E_z,\ E_y = \frac{-j\beta}{k_c^2}\frac{\partial}{\partial y}E_z$$

$$\mathbf{H}_t = \frac{\hat{\mathbf{z}}}{Z_{TM}}\times\mathbf{E}_t \Rightarrow H_x = \frac{j\omega\varepsilon}{k_c^2}\frac{\partial}{\partial y}E_z,\ H_y = \frac{-j\omega\varepsilon}{k_c^2}\frac{\partial}{\partial x}E_z$$



(a) Solve E_z from the Helmholtz wave equation :

$$\left(\nabla^2 + k^2\right)E_z = 0 \ \text{ or } \ \left(\nabla_t^2 + k_c^2\right)e_z = 0 \ \text{ with } \ E_z = e_z e^{-j\beta z}$$

(b) Dispersive wave impedance

$$Z_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\varepsilon} = \frac{\beta\eta}{k}$$

(c) Dispersive phase constant $\beta = \sqrt{k^2 - k_c^2}$ depends on line geometry.

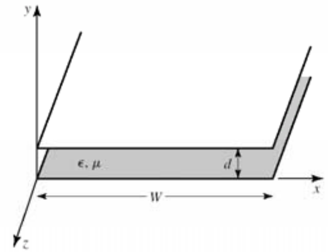
3.2 Parallel-Plate Waveguide: TEM, TE, and TM Waves

(1) TEM modes or TEM waves

$$E_z = H_z = 0$$

$$\nabla^2\phi(x,y) = 0,\quad 0 \le x \le W,\quad 0 \le y \le d$$

with $B.C.: \phi(x,0) = 0,\ \phi(x,d) = V_0$



Assume that $W \gg d$, and ϕ has no variation in x: $\frac{\partial}{\partial x} = 0$

$$\phi(x,y) = A + By \Rightarrow \phi(x,y) = V_0\, y/d$$

$$e(x,y) = -\nabla\phi(x,y) = -\hat{\mathbf{y}}\, V_0/d$$

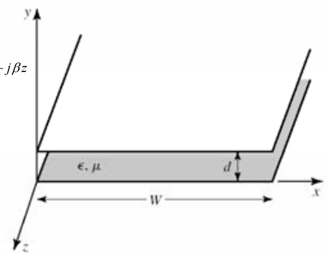
$$\mathbf{E} = \mathbf{e}(x,y)e^{-j\beta z} = -\hat{\mathbf{y}}\left(V_0/d\right)e^{-j\beta z},\ \beta = k$$

$$\mathbf{H} = \hat{\mathbf{z}}/\eta\times\mathbf{E} = \hat{\mathbf{x}}\left(V_0/\eta d\right)e^{-j\beta z}$$

Line Parameters for Parallel Plate Waveguides

- Total current on the top plate

$$I = \int_0^W \mathbf{J}_s\cdot\hat{\mathbf{z}}dx = \int_0^W -\hat{\mathbf{y}}\times\mathbf{H}\cdot\hat{\mathbf{z}}dx = \int_0^W H_x dx = \frac{WV_0}{\eta d}e^{-j\beta z}$$



- Voltage difference between top and bottom plates

$$V = -\int_0^d E_y dy = V_0 e^{-j\beta z}$$

- The characteristic impedance

$$Z_0 = \frac{V}{I} = \eta\frac{d}{W} \neq \eta \text{ (wave impedance)}$$

$$E_y = -\frac{V_o}{d}e^{-j\beta z}$$

$$H_x = \frac{V_o}{\eta d}e^{-j\beta z}$$

- Phase velocity

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \ \rightarrow \ \text{TEM mode}$$

Self-study

(2) TM_n Waves or TM_n Modes --- H_z=0

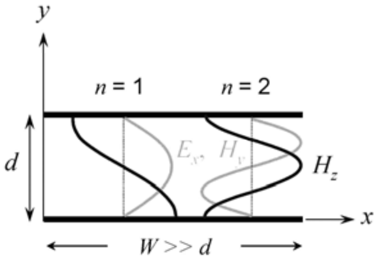
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right)e_z(x,y,z) = 0$$

$$\left(\frac{\partial^2}{\partial y^2} + k_c^2\right)e_z(y) = 0 \Leftarrow e_z(x,y,z) = e_z(x,y)e^{-j\beta z}, \frac{\partial}{\partial z} = -j\beta, \frac{\partial}{\partial x} = 0$$

$$e_z(y) = A\sin k_c y + B\cos k_c y$$

$$B.C.: e_z(y)\big|_{y=0,d} = 0, \therefore e_z(y) = A_n\sin\frac{n\pi}{d}y, k_c = \frac{n\pi}{d}, n = 0,1,2,...(n \neq 0)$$

$$TM_n \text{ mode} \begin{cases} E_z = A_n\sin\dfrac{n\pi y}{d}e^{-j\beta z} \\ H_x = \dfrac{j\omega\varepsilon}{k_c}A_n\cos\dfrac{n\pi y}{d}e^{-j\beta z} \\ E_y = \dfrac{-j\beta}{k_c}A_n\cos\dfrac{n\pi y}{d}e^{-j\beta z} \\ E_x = H_y = H_z = 0 \end{cases}$$



- TM_0 mode is actually the TEM mode.

Self-study

(3) TE_n Waves or TE_n Modes --- E_z=0

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right)h_z(x,y,z)=0$$

$$\left(\frac{\partial^2}{\partial y^2}+k_c{}^2\right)h_z(y)=0 \Leftarrow h_z(x,y,z)=h_z(x,y)e^{-j\beta z}, \frac{\partial}{\partial z}=-j\beta, \frac{\partial}{\partial x}=0$$

$$h_z(y)=A\sin k_c y+B\cos k_c y$$

$$B.C.: \frac{\partial}{\partial y}h_z(y)\bigg|_{y=0,d}=0, \therefore h_z(y)=B_n\cos\frac{n\pi}{d}y, k_c=\frac{n\pi}{d}, n=0,1,2,...(n\neq 0)$$

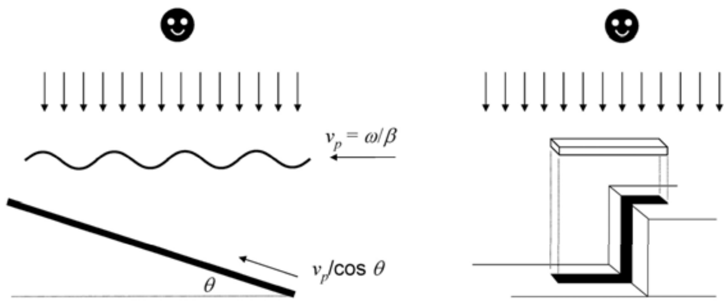
$$TE_n \text{ mode}\begin{cases} H_z=B_n\cos\dfrac{n\pi y}{d}e^{-j\beta_n z} \\ E_x=\dfrac{j\omega\mu}{k_c}B_n\sin\dfrac{n\pi y}{d}e^{-j\beta_n z} \\ H_y=\dfrac{j\beta}{k_c}B_n\sin\dfrac{n\pi y}{d}e^{-j\beta_n z} \\ E_y=E_z=H_x=0 \end{cases} \qquad \beta_n=\sqrt{k^2-(n\pi/d)^2}$$

Self-study

What does β=0 mean?

(1) Phase velocity $v_p=\omega/\beta$. $v_p=\infty$ if $\beta=0$.

(2) What does $v_p=\infty$ mean?



3.3 Rectangular Waveguides

TE modes: E_z=0 & TM modes: H_z=0

(1) TE modes: $H_z(x,y,z)=h_z(x,y)e^{-j\beta z}$, $\beta=k^2-k_c{}^2$

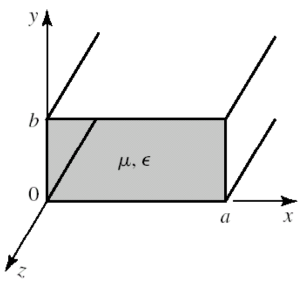
Separation of Variables: $h_z(x,y)=X(x)Y(y)$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c{}^2\right)h_z(x,y)=0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+k_c{}^2=0$$

$$\frac{\partial^2 X}{\partial x^2}=-k_x{}^2X, \frac{\partial^2 Y}{\partial y^2}=-k_y{}^2Y,$$

where $k_x{}^2+k_y{}^2=k_c{}^2$



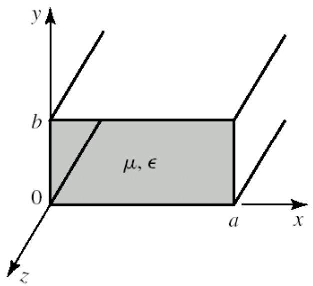
Geometry of a rectangular waveguide

Rectangular Waveguides (Cont'd)

(1) TE_{mn} mode $H_z(x,y,z)=h_z(x,y)e^{-j\beta z}$, $\beta^2=k^2-k_c{}^2$

$$h_z(x,y)=X(x)Y(y)$$
$$X=A\cos k_x x+B\sin k_x x$$
$$Y=C\cos k_y y+D\sin k_y y$$
$$h_z(x,y)=\cos\left(\frac{m\pi y}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$B.C.: e_y(x,y)=0, x=0,a$$
$$B=0, k_x=m\pi/a, m=0,1,2,...$$
$$e_x(x,y)=0, y=0,b$$
$$D=0, k_x=n\pi/b, n=0,1,2,...$$



$$H_z=A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$
$$E_x=\frac{j\omega\mu n\pi}{k_{cmn}{}^2 b}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$
$$E_y=\frac{-j\omega\mu m\pi}{k_{cmn}{}^2 a}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$
$$H_x=-\frac{E_y}{Z_{TEmn}}, H_y=\frac{E_x}{Z_{TEmn}}$$
$$Z_{TEmn}=\frac{k\eta}{\beta}=\frac{\omega\mu}{\sqrt{k^2-k_{cmn}{}^2}}$$

Rectangular Waveguides (Cont'd)

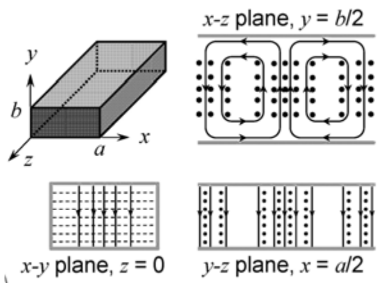
TE_{mn} modes

$$H_z = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

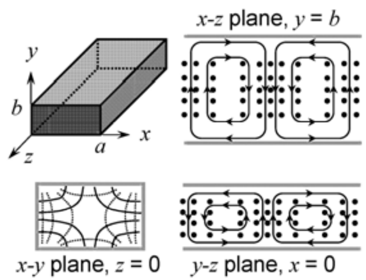
$$E_x = \frac{j\omega\mu n\pi}{k^2{}_{cmn}b}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \qquad H_x = -\frac{E_y}{Z_{TEmn}}$$

$$E_y = \frac{-j\omega\mu m\pi}{k^2{}_{cmn}a}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \qquad H_y = \frac{E_x}{Z_{TEmn}}$$

TE_{10}



TE_{11}



Self-study

Rectangular Waveguides (Cont'd)

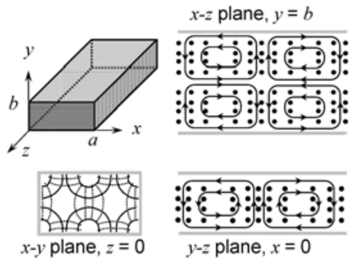
TE_{mn} modes

$$H_z = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_x = \frac{j\omega\mu n\pi}{k^2{}_{cmn}b}A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \qquad H_x = -\frac{E_y}{Z_{TEmn}}$$

$$E_y = \frac{-j\omega\mu m\pi}{k^2{}_{cmn}a}A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z} \qquad H_y = \frac{E_x}{Z_{TEmn}}$$

TE_{21}



Modal Parameters for the TE_{mn} Mode

Phase constant:

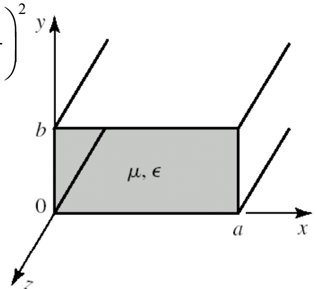
$$\beta^2 = \beta_{mn}{}^2 = \sqrt{k^2 - k^2{}_{cmn}} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Cutoff frequency:

$$f_{cmn} = \frac{k_{cmn}}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Wave impedance:

$$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{k\eta}{\beta}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$$



Modal Characteristics of the TE_{10} Mode ---Dominant Mode ($a>b$)

$$H_z = A_{10}\cos\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$E_z = \frac{-j\omega\mu\pi}{k^2{}_{c10}a}A_{10}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

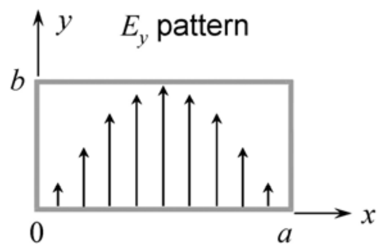
$$H_z = \frac{j\beta\pi}{k^2{}_{c10}a}A_{10}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$E_x = H_y = 0$$

$$\beta = \beta_{10} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{\pi}{a}\right)^2}$$

$$k_{c10} = \frac{\pi}{a}, f_{c10} = \frac{c}{2a\sqrt{\varepsilon_r}}$$

$$Z_{TE10} = \frac{\omega\mu}{\beta}$$



The TE_{10} mode has the lowest cutoff frequency. It is used as the dominant mode.

Self-study

Rectangular Waveguides (Cont'd) (2) TM Modes

(2) TM modes: $E_z(x,y,z) = e_z(x,y)e^{-j\beta z}, \beta^2 = k^2 - k_c^2$

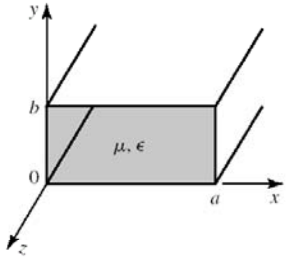
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) e_z(x,y) = 0$$

Separation of Variables: $e_z(x,y) = X(x)Y(y)$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + k_c^2 = 0$$

$$\frac{\partial^2 X}{\partial x^2} = -k_x^2 X, \frac{\partial^2 Y}{\partial y^2} = -k_y^2 Y, \quad \text{where } k_x^2 + k_y^2 = k_c^2$$

$$X = A\cos k_x x + B\sin k_x x \quad \& \quad Y = C\cos k_y y + D\sin k_y y$$



$B.C.: e_z(x,y) = 0, x = 0, a$

$A = 0, k_x = m\pi/a, m = 0,1,2,\ldots$

$e_z(x,y) = 0, y = 0, b$

$C = 0, k_y = n\pi/b, n = 0,1,2,\ldots$

$$e_z(x,y) = \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

Self-study

Rectangular Waveguides -- TM Modes

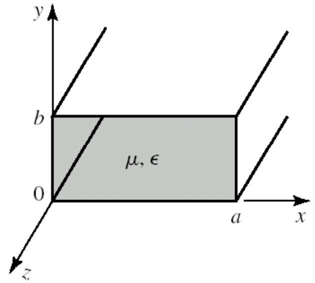
$$E_z = B_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_x = \frac{-j\beta m\pi}{k^2_{cmn}a} B_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_y = \frac{-j\beta n\pi}{k^2_{cmn}b} B_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$H_x = -\frac{E_y}{Z_{TMmn}}, H_y = \frac{E_x}{Z_{TMmn}}$$

$$Z_{TEmn} = \frac{\beta\eta}{k} = \frac{\sqrt{k^2 - k^2_{cmn}}}{\omega\varepsilon}$$



Phase constant:

$$\beta = \beta_{mn} = \sqrt{k^2 - k^2_{cmn}} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Cutoff frequency:

$$f_{cmn} = \frac{k_{cmn}}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Wave impedance:

$$Z_{TMmn} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta\eta}{k}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

Self-study

Rectangular Waveguides -- TM Modes

$$E_z(x,y) = \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

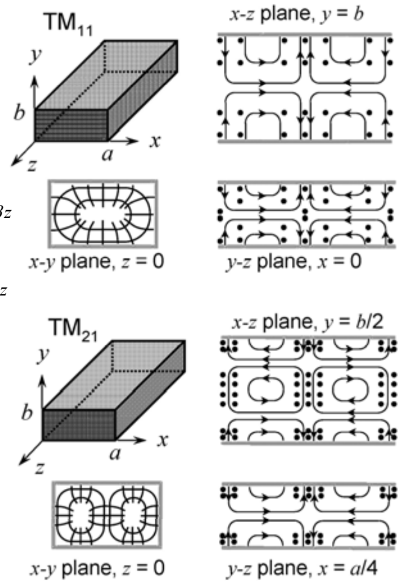
$$E_z = B_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_x = \frac{-j\beta m\pi}{k^2_{cmn}a} B_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_y = \frac{-j\beta n\pi}{k^2_{cmn}b} B_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

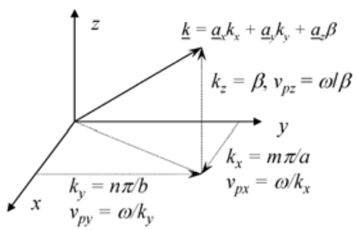
$$H_x = -\frac{E_y}{Z_{TMmn}}, H_y = \frac{E_x}{Z_{TMmn}}$$

$$Z_{TMmn} = \frac{\beta\eta}{k} = \frac{\sqrt{k^2 - k^2_{cmn}}}{\omega\varepsilon}$$



The Propagation Factor and the Wave Vector

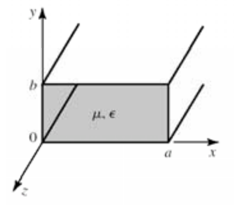
- Propagation factor $e^{-j\mathbf{k}\cdot\mathbf{r}} = e^{-jk_x x - jk_y y - j\beta z}$
- $\mathbf{k} = \hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_z$
- $k^2 = (m\pi/a)^2 + (n\pi/b)^2 + \beta^2$
- Phase velocity
  (a) in "wave direction" $v_p = \omega/k$, and
  (b) in the $u(x, y, \text{or } z)$ direction $v_{pu} = \omega/k_u$



- The two conductor planes will cause the wave propagating in the x and y directions to be totally reflected ($|\Gamma| = 1$).
- Such reflection cause a standing wave. This explains that why the EM field components are expressed in terms of cos or sin functions.

$$e^{-jk_x x} \pm e^{jk_x x} \rightarrow \cos k_x x \text{ or } \sin k_x x$$

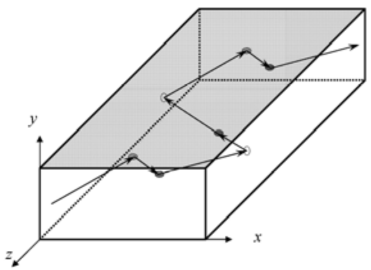
$$e^{-jk_y y} \pm e^{jk_y y} \rightarrow \cos k_y y \text{ or } \sin k_y y$$



Wave Propagation in a Rectangular Waveguide

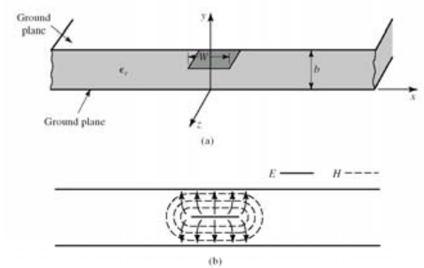
- Propagation factor
  $$e^{-j\mathbf{k}\cdot\mathbf{r}} = e^{-jk_x x - jk_y y - j\beta z}$$
- $\mathbf{k} = \hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_z$
- $k^2 = (m\pi/a)^2 + (n\pi/b)^2 + \beta^2$

- How does the wave propagate in each of x , y and z direction?



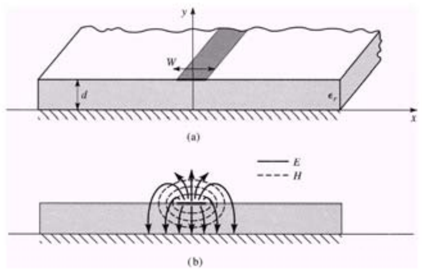
3.7 & 3.8 Stripline and Microstrip

Stripline: 1950's



TEM

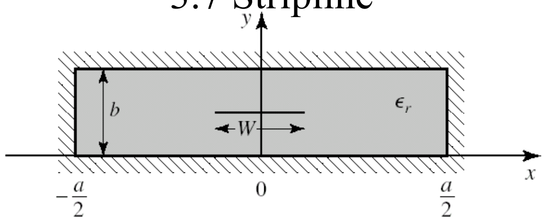
Microstrip: 1960's



Quasi-TEM

$$\begin{cases} v_p = c & \text{air} \\ v_p = \dfrac{c}{\sqrt{\varepsilon_r}} & \text{dielectric} \end{cases}$$

3.7 Stripline



- Fundamental mode: TEM mode
- A stripline also supports higher-order TM & TE modes, which are avoided in practice.
- Approximation formulas for analysis:

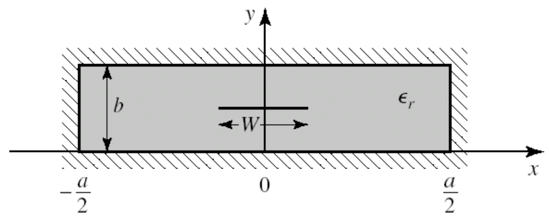
$$Z_o = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{b}{W_o + 0.441b}$$

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \dfrac{W}{d} > 0.35 \\ \left(0.35 - \dfrac{W}{b}\right)^2 & \text{for } \dfrac{W}{d} < 0.35 \end{cases}$$

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\varepsilon_r}Z_o < 120\ \Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\varepsilon_r}Z_o > 120\ \Omega \end{cases}$$

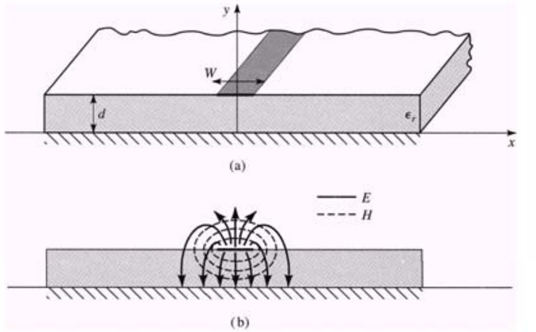
$$x = \frac{30\pi}{\sqrt{\varepsilon_r}Z_o} = -0.441$$

Does stripline have higher-order modes?



- Of course, yes. In addition to the TEM mode, it has higher-order modes.
- The structure of stripline is actually a perturbation of parallel-plate waveguide , which has TE_n and TM_n higher-order modes.
- The cutoff frequencies of TE_n and TM_n are $n\pi/b$,which can be very high if b is sufficiently small.

3.8 Microstrip



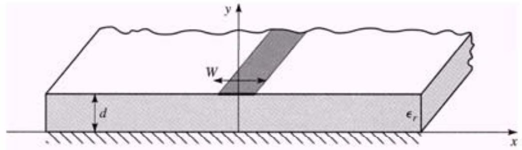
(1) The most popular planar transmission line.

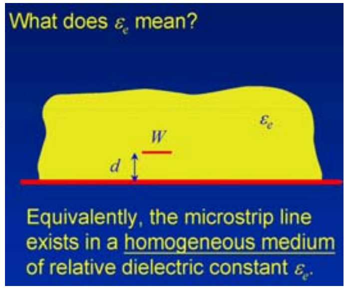
(2) Easily integrated with microwave active and passive devices.

(3) Easy fabrication with low cost.

(4) Dominant mode is quasi-TEM mode, a hybrid mode; not a pure TEM mode.

Microstrip Analysis (Approximation)



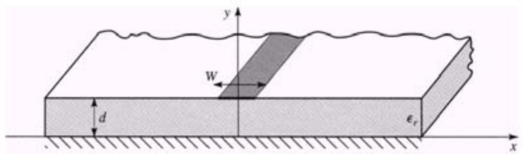


$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12\, d/W}}$$

$$Z_o = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{4W}{d}\right), & \text{for } \dfrac{W}{d} \le 1 \\[2ex] \dfrac{120\pi}{\sqrt{\varepsilon_e}\left[\dfrac{W}{d} + 1.393 + 0.667 \ln\left(\dfrac{W}{d} + 1.444\right)\right]}, & \text{for } \dfrac{W}{d} > 1 \end{cases}$$

http://chemandy.com/calculators/microstrip_transmission_line_calculator.htm

Microstrip Synthesis (Given Z_0 and ε_r)

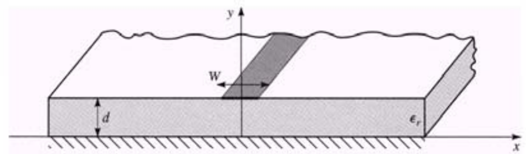


$$\frac{W}{d} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} \text{ or } \dfrac{4}{\sinh A}, & \text{for } \dfrac{W}{d} \le 2 \\[2ex] \dfrac{2}{\pi}\left[B - 1 - \ln(2B-1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left\{\ln(B-1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right\}\right], & \text{for } \dfrac{W}{d} > 2 \end{cases}$$

$$A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{377\pi}{2Z_o\sqrt{\varepsilon_r}}$$

Microstrip Synthesis (Given Z_0 and ε_0)



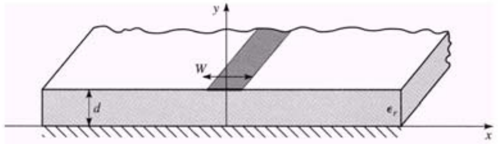
Example 3.7 Microstrip Synthesis
Calculate W and the length for a microstrip line with 50 Ω characteristic impedance and a phase shift of 90° at 2.5GHz. $d = 1.27$mm, and $\varepsilon_r = 2.2$.

Sol: $W/d = 3.081 \Rightarrow W = 3.081 \times 1.27 = 3.91mm$

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12\, d/W}} = 1.87$$

$$90^o = \beta l = \sqrt{\varepsilon_e} k_o l \Rightarrow l = \frac{c\pi/4}{2\pi f\sqrt{\varepsilon_e}} = 21.9mm$$

Does microstrip have higher-order modes?



In a closed microstrip, there exists a dominant mode, called quasi-TEM modes, and many higher-order modes , including evanescent modes and complex modes.

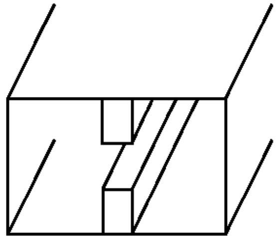
Dispersion Characteristics of Microstrip

$$\beta = k_o\sqrt{\varepsilon_e}, \sqrt{\varepsilon_e} = \frac{\beta}{k_o} \quad \Rightarrow \quad v_p = \frac{1}{\sqrt{\mu\varepsilon_o\varepsilon_e}} = \frac{v_c}{\sqrt{\varepsilon_e}}$$

ε_e (effective dielectric constant): function of frequency $\leftrightarrow$ dispersive

3.11 Other Transmission Lines and Waveguides

1. Ridge Waveguide

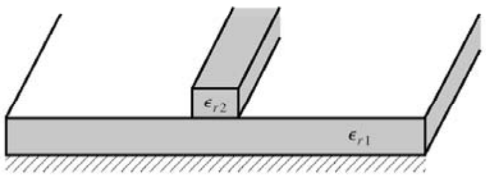


(a) TE and TM modes

(b) Structure lowers the cutoff frequency of the dominant mode.

(c) Structure increases bandwidth and has better impedance characteristics for matching purpose.

Dielectric Waveguides



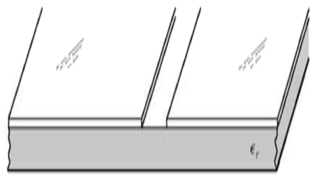
(a) Both TE and TM waves exist.

(b) $\varepsilon_{r2} > \varepsilon_{r1}$, so that most fields are confined to the ridge (ε_{r2}) region.

(c) Easily integrated with active devices.

(d) Small size, suitable for millimeter-wave to optical wave.

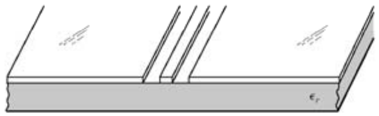
Slotline and Coplanar Waveguides



Slotline

(a) Quasi-TEM mode is available.

(b) Ranks just behind microstrip.



CPW

(a) Even and odd quasi-TEM modes exist.

(b) Particularly useful for fabricating with active circuitry.

Table 3.6
Comparison of Common Transmission Lines and Waveguides

Characteristic	Coax	Waveguide	Stripline	Microstrip
Modes: Preferred Other	TEM TM,TE	TE_{10} TM,TE	TEM TM,TE	Quasi-TEM Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	High
Loss	Medium	Low	High	High
Power capacity	Medium	High	Low	Low
Physical size	Large	Large	Medium	Small
Ease of fabrication	Medium	Medium	Easy	Easy
Integration	Hard	Hard	Fair	Easy

3.3 Circulator Waveguides

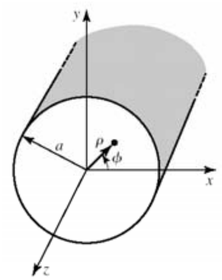
Determining the longitudinal components E_z and B_z, we could quickly calculate all the others.

$$E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$



$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{v^2} - k^2\right]E_z = 0$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{v^2} - k^2\right]B_z = 0$$

If $E_z = 0 \Rightarrow$ TE (transverse electric) waves;
If $B_z = 0 \Rightarrow$ TM (transverse magnetic) waves;
If $E_z = 0$ and $B_z = 0 \Rightarrow$ TEM waves.

TE and TM modes

TM Mode of a Waveguide ($B_z = 0$):

$$\begin{cases} (\nabla_t^2 + \gamma^2)E_z = 0 \text{ with boundary condition } E_z\big|_s = 0 \\ \mathbf{E}_t = \pm\frac{ik_z}{\gamma^2}\nabla_t E_z \\ \mathbf{H}_t = \pm\frac{\varepsilon\omega}{k_z}\mathbf{e}_z \times \mathbf{E}_t = \pm\frac{1}{Z_e}\mathbf{e}_z \times \mathbf{E}_t \\ \gamma^2 = \mu\varepsilon\omega^2 - k_z^2 \end{cases}$$

Assume perfectly conducting wall.

$Z_e \equiv k_z/\varepsilon\omega$, wave impedance of TM modes

TE Mode of a Waveguide ($E_z = 0$):

$$\begin{cases} (\nabla_t^2 + \gamma^2)H_z = 0 \text{ with boundary condition } \frac{\partial}{\partial n}H_z\big|_s = 0 \\ \mathbf{H}_t = \pm\frac{ik_z}{\gamma^2}\nabla_t H_z \\ \mathbf{E}_t = \mp\frac{\mu\omega}{k_z}\mathbf{e}_z \times \mathbf{H}_t = \mp Z_h \mathbf{e}_z \times \mathbf{H}_t \\ \gamma^2 = \mu\varepsilon\omega^2 - k_z^2 \end{cases}$$

$Z_h \equiv \mu\omega/k_z$, wave impedance of TE modes

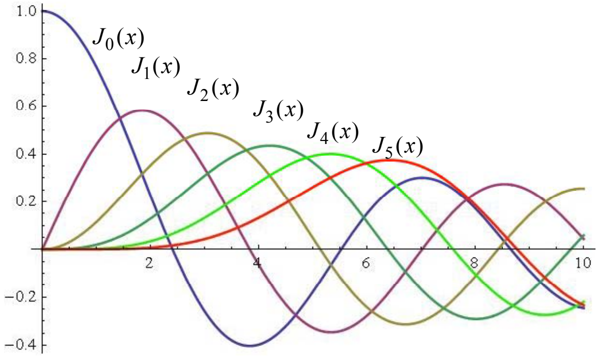
b.c. $\mathbf{n}\cdot\mathbf{H}\big|_s = 0$
$\mathbf{n} \perp \mathbf{e}_z \Rightarrow \mathbf{n}\cdot\mathbf{H}_t\big|_s = 0$
$\Rightarrow \mathbf{n}\cdot\nabla_t H_z\big|_s = 0$
$\Rightarrow \frac{\partial}{\partial n}H_z\big|_s = 0$

Table 3.5 Circular Waveguide

Quantity	TE_{nm} Mode	TM$_{nm}$ Mode
k	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}$
k_c	$\frac{p'_{nm}}{a}$ x'_{nm}	x_{nm} $\frac{p_{nm}}{a}$
β	$\sqrt{k^2 - k_c^2}$	$\sqrt{k^2 - k_c^2}$
λ_c	$\frac{2\pi}{k_c}$	$\frac{2\pi}{k_c}$
λ_g	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$
v_p	$\frac{\omega}{\beta}$	$\frac{\omega}{\beta}$
α_d	$\frac{k^2\tan\delta}{2\beta}$	$\frac{k^2\tan\delta}{2\beta}$
E_z	0	$(A\sin n\phi + B\cos n\phi)J_n(k_c\rho)e^{-j\beta z}$
H_z	$(A\sin n\phi + B\cos n\phi)J_n(k_c\rho)e^{-j\beta z}$	0
E_ρ	$\frac{-j\omega\mu n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$	$\frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J'_n(k_c\rho)e^{-j\beta z}$
E_ϕ	$\frac{j\omega\mu}{k_c}(A\sin n\phi + B\cos n\phi)J'_n(k_c\rho)e^{-j\beta z}$	$\frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$
H_ρ	$\frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J'_n(k_c\rho)e^{-j\beta z}$	$\frac{j\omega\epsilon n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$
H_ϕ	$\frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}$	$\frac{-j\omega\epsilon}{k_c}(A\sin n\phi + B\cos n\phi)J'_n(k_c\rho)e^{-j\beta z}$
Z	$Z_{TE} = \frac{k\eta}{\beta}$	$Z_{TM} = \frac{\beta\eta}{k}$

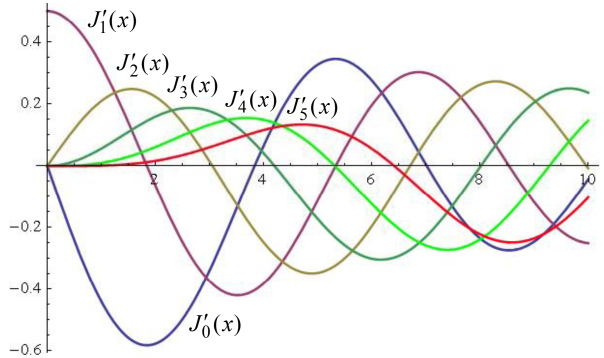
The Roots of Bessel Function (TM$_{nm}$ modes)

x_{nm}	$J_0(x_{0m})$	$J_1(x_{1m})$	$J_2(x_{2m})$	$J_3(x_{3m})$	$J_4(x_{4m})$	$J_5(x_{5m})$
m=1	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715
2	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386
3	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002
4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801
5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178

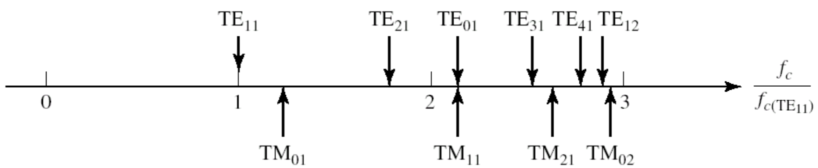


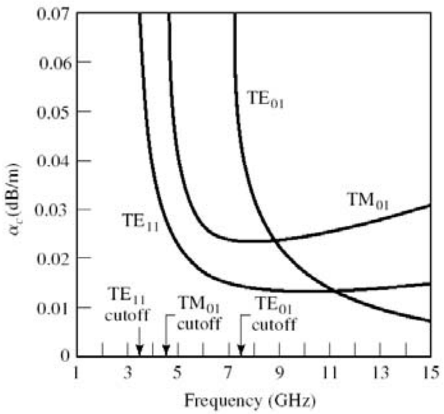
The Roots of Bessel Derivative (TEnm modes)

x'_{nm}	$J'_0(x'_{0m})$	$J'_1(x'_{1m})$	$J'_2(x'_{2m})$	$J'_3(x'_{3m})$	$J'_4(x'_{4m})$	$J'_5(x'_{5m})$
m=1	3.8317	1.8412	3.0542	4.2012	5.3175	6.4156
2	7.0156	5.3314	6.7061	8.0152	9.2824	10.5199
3	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872
4	13.3237	11.7060	13.1704	14.5858	15.9641	17.3128
5	16.4706	14.8636	16.3475	17.7887	19.1960	20.5755

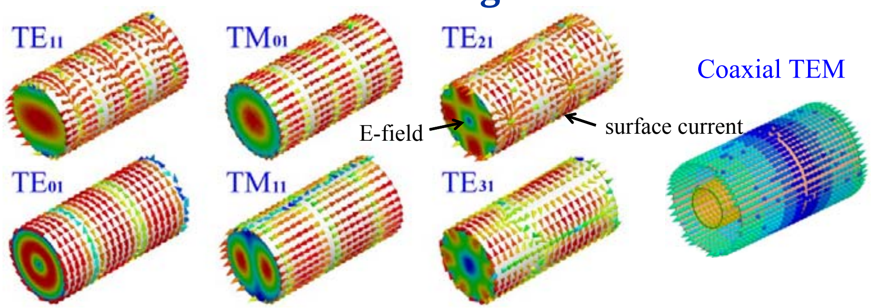


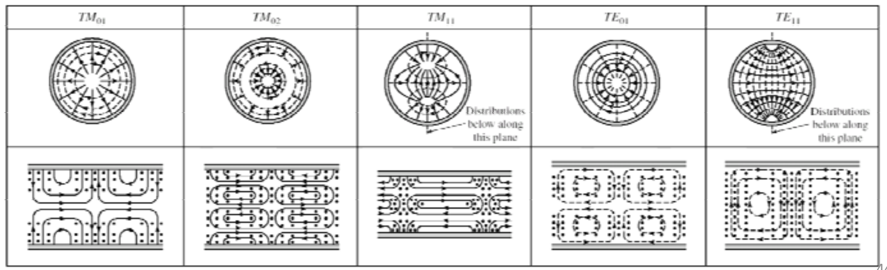
Normalized Cutoff Frequency and Attenuation





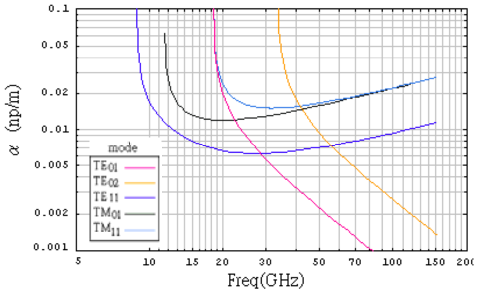
Field Patterns and Surface Current of Circular Waveguide Modes



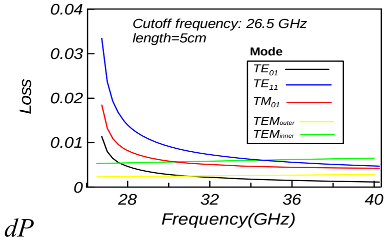


Attenuation of Circular Waveguide Modes

(a) Fix waveguide radius @ R=1mm   (b) Fix cutoff frequency @26.5GHz



mode	TE_{01}	TE_{11}	TM_{01}	TEM_{inner}	TEM_{outer}
radius	0.6904cm	0.3317cm	0.4333cm	0.1269cm	0.292cm



$$Loss = \frac{P_{in}-P_{out}}{P_{in}} = 1-e^{-2\alpha z}, \quad \text{where } \alpha \equiv -\frac{1}{2P}\frac{dP}{dz}$$

TE

$$\alpha = \frac{R_s}{R\eta}\left(1-\left(\frac{f_c}{f}\right)\right)^{-\frac{1}{2}}\left(\left(\frac{f_c}{f}\right)^2+\frac{m^2}{X_{mn}^2-m^2}\right)$$

$$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

TM

$$\alpha = \frac{R_s}{R\eta}\left(1-\left(\frac{f_c}{f}\right)^2\right)^{-\frac{1}{2}}$$

TEM

$$\alpha = \frac{R_m}{2\eta\cdot\ln\left(b/a\right)}\frac{b+a}{ab}$$

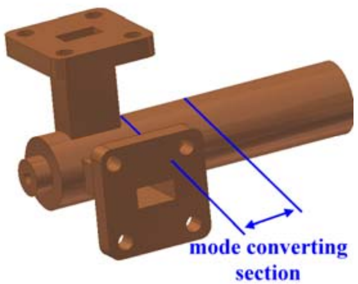
$$R_m = \frac{1}{\sigma\delta}$$

Applications of the Circular Waveguide Modes

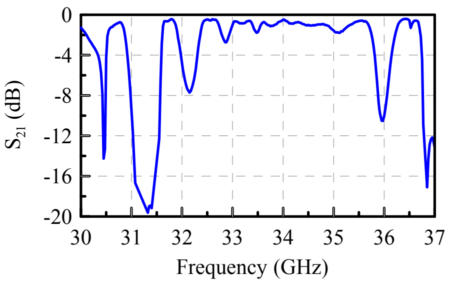
- Gyrotrons (generating millimeter and terahertz radiation sources)
  TE_{11}, TE_{21}, TE_{01}, TE_{02}, TE_{06}, …
- Millimeter-wave devices
  Rotary Joints---TE_{01}, TM_{01}, coaxial TE_{01}, coaxial TEM
  Circulator and Isolator---TM_{11}, TE_{11}
- Plasma applicator
  TE_{01}, TM_{01}, coaxial TE_{01}, coaxial TEM
- Microwave /material applicator
  TE_{11}, TM_{01}, TM_{11}, TE_{01}, …

Applications: Gyrotrons (I)

- Fundamental cyclotron harmonic: TE_{11},
  TE_{11}: NTHU gyrotron experiment before 2003



Reflection method

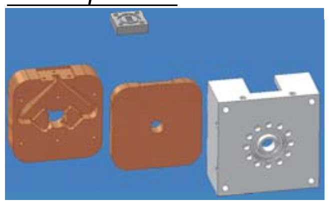


- High conversion efficiency
- Low reflection
- Mode purity
- Broad bandwidth

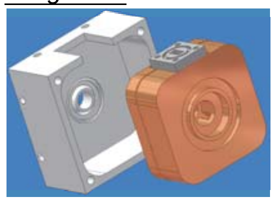
- T. H. Chang, L. R. Barnett, K.R. Chu, F. Tai and C.L. Hsu, Rev. Sci. Instruments, 70(2), 1530 (Feb. 1999).
- T. H. Chang, S. H. Chen, L. R. Barnett, and K. R. Chu, "Characterization of Stationary and Nonstationary Behavior of Gyrotron Oscillators", Phys. Rev. Lett. 87, 064802, (2001).

TE_{01} Mode Converter From W-band to Terahertz

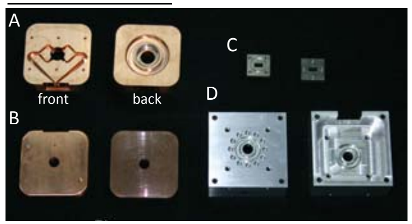
Decomposition

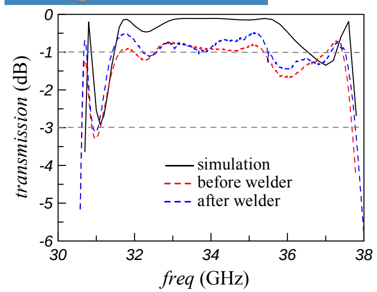


Integration



Finish Product



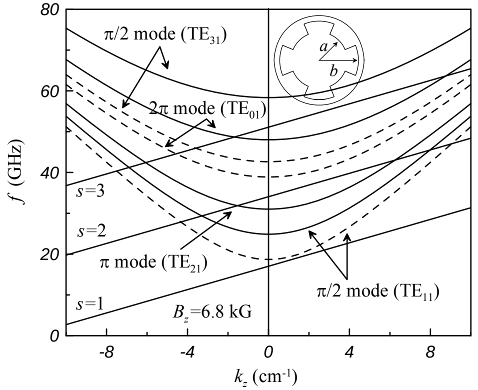


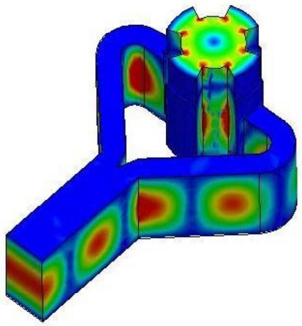
N. C. Chen, C. F. Yu, C. P. Yuan, and T. H. Chang, "A mode-selective circuit for TE_{01} Gyrotron Backward-wave Oscillator with wide-tuning range", Appl. Phys. Lett. 94, 101501 (2009).

Applications: Gyrotrons (II)

- Second cyclotron harmonic:
  Slotted TE_{21}: Reduce the magnetic field requirement

borken line : smooth-bore waveguide
solid line : slotted-bore waveguide (b/a=1.5)

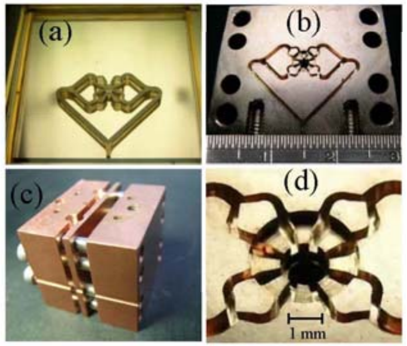


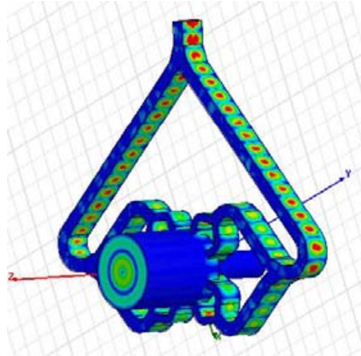


N. C. Chen, C. F. Yu, and T. H. Chang*, "A TE_{21} second harmonic gyrotron backward-wave oscillator with slotted structure", Phys. Plasmas, 14, 123105 (2007).

Applications: Gyrotrons (III)

- Terahertz higher-order mode:
  TE_{02}: 203GHz using micro-fabrication technique (LIGA)
  TE_{06}: Mode converter free (Why?)

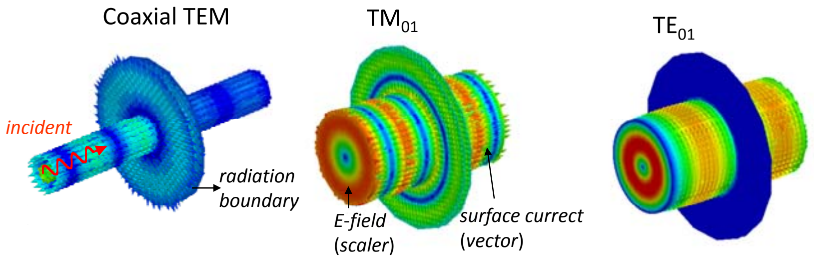




- T. H. Chang*, B. Y. Shew, C. Y. Wu, and N. C. Chen, "X-ray microfabrication and measurement of a terahertz mode converter", Rev. Sci. Instrum. 81, 054701 (2010).
- N. C. Chen, T. H. Chang*, C. P. Yuan, T. Idehara and I. Ogawa, "Theoretical investigation of a high efficiency and broadband sub-terahertz gyrotron", Appl. Phys. Lett. 96, 161501 (2010).

Applications: Rotary joint

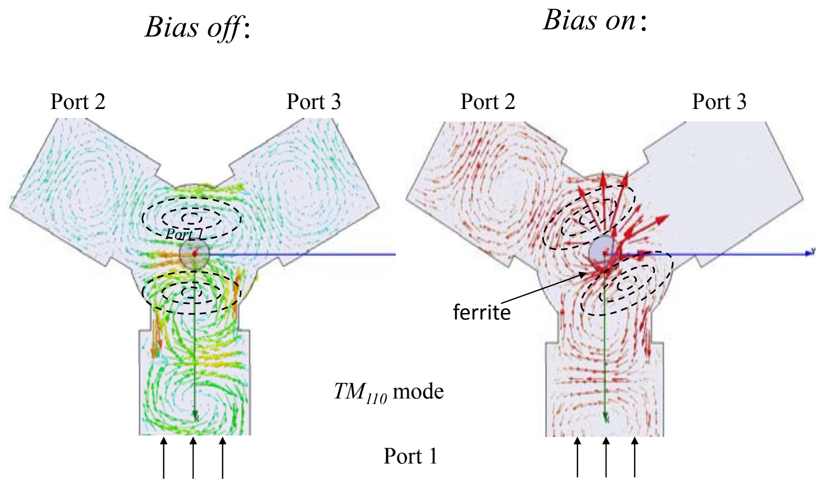
Optimal choice: TE_{01} mode



T. H. Chang and B. R. Yu, "High-Power Millimeter-Wave Rotary Joint", Rev. Sci. Instrum. 80, 034701 (2009).

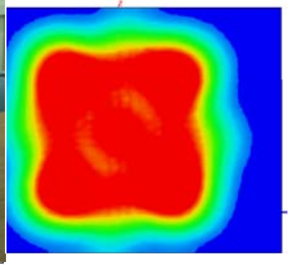
Applications: Circulator/Isolator

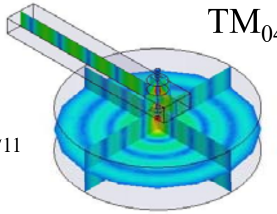
Optimal choice: TM_{11} mode



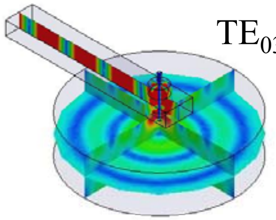
Plasma applicator: TE_{11}, TM_{040}, TE_{031}

Novel distributive type:
2×2 circular polarized TE_{11}





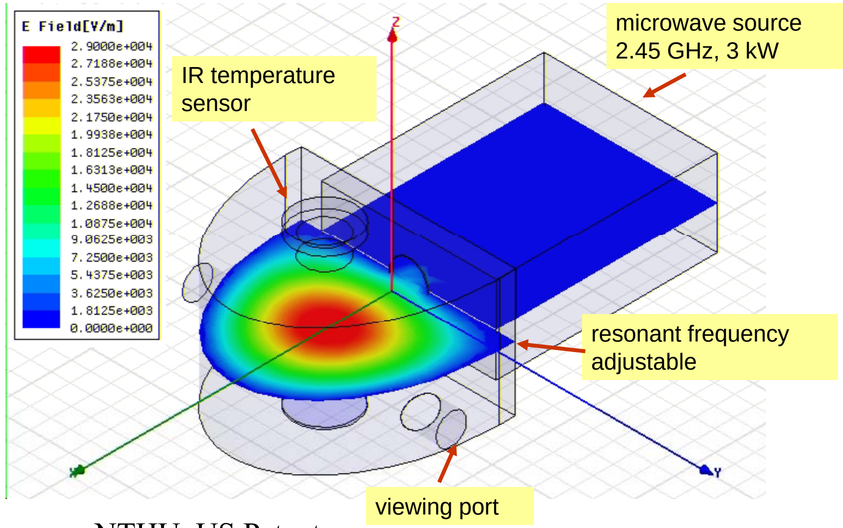
TM_{040}



TE_{031}

NTHU Patent pending

Microwave /material applicator
Example: TM_{110}



NTHU US Patent

The End of Chap. 3