Lepton flavor violation in Tau decays

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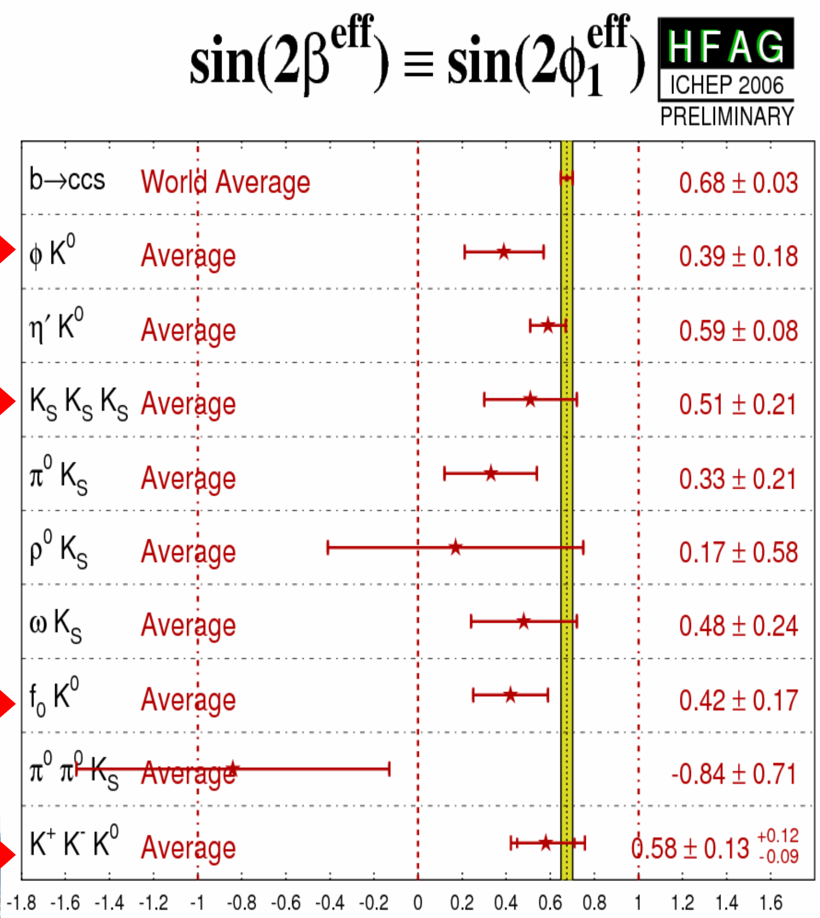
- Introduction & Motivation
- Higgs-mediated LFV
- Implications of LFV in tau decays
- Speculation of Higgs-mediated effects

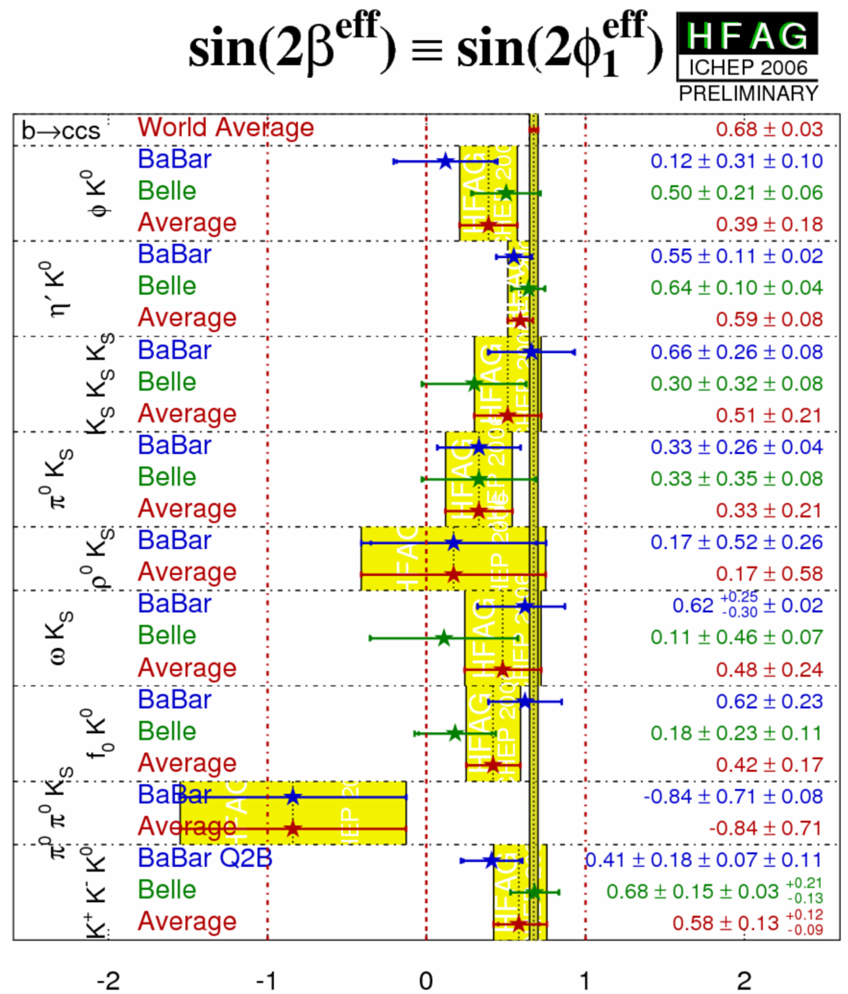
Introduction & Motivation

What we can learn from B factories:

- CP violations: time-dependence CP asymmetry & direct CP violation

ICHEP2006: ϕ_1 with $b \to s$ Penguins





	BaBar		Belle	
	$BR \times 10^{-6}$	A_{cp}	$BR \times 10^{-6}$	A_{cp}
$B^+ \to \pi^+\pi^0$	$5.12 \pm 0.47 \pm 0.29$	$-0.019 \pm 0.088 \pm 0.014$	$6.6 \pm 0.4^{+0.4}_{-0.5}$	$+0.07 \pm 0.06 \pm 0.01$
$B^0 \to \pi^+\pi^-$	$5.8 \pm 0.4 \pm 0.3$	$+0.16 \pm 0.11 \pm 0.03$	$5.1 \pm 0.2 \pm 0.2$	$+0.55 \pm 0.08 \pm 0.05$
$B^0 \to \pi^0\pi^0$	$1.48 \pm 0.26 \pm 0.12$	$+0.33 \pm 0.36 \pm 0.08$	$1.1 \pm 0.3 \pm 0.1$	$+0.44^{+0.73+0.04}_{-0.62-0.06}$
$B^0 \to K^+\pi^-$	$19.7 \pm 0.6 \pm 0.6$	$-0.108 \pm 0.024 \pm 0.007$	$20.0 \pm 0.4^{+0.9}_{-0.8}$	$-0.093 \pm 0.018 \pm 0.008$
$B^0 \to K^0\pi^0$	$10.5 \pm 0.7 \pm 0.5$	$-0.20 \pm 0.16 \pm 0.03$	$9.2^{+0.7+0.6}_{-0.6-0.7}$	$-0.05 \pm 0.14 \pm 0.05$
$B^+ \to K^+\pi^0$	$13.3 \pm 0.56 \pm 0.64$	$+0.016 \pm 0.041 \pm 0.010$	$12.4 \pm 0.5^{+0.7}_{-0.6}$	$+0.07 \pm 0.03 \pm 0.01$
$B^+ \to K^0\pi^+$	$23.9 \pm 1.1 \pm 1.0$	$-0.029 \pm 0.039 \pm 0.010$	$22.9^{+0.8}_{-0.7} \pm 1.3$	$+0.03 \pm 0.03 \pm 0.01$
$B^0 \to K^0\overline{K^0}$	$1.08 \pm 0.28 \pm 0.11$	$0.40 \pm 0.41 \pm 0.06$	$0.86^{+0.24}_{-0.21} \pm 0.09$	$-0.57^{+0.72}_{-0.65} \pm 0.13$
$B^0 \to K^+K^-$	< 0.40		< 0.25	
$B^+ \to \overline{K^0}K^+$	$1.61 \pm 0.44 \pm 0.09$	$0.10 \pm 0.26 \pm 0.03$	$1.22^{+0.33+0.13}_{-0.28-0.16}$	$+0.13^{+0.23}_{-0.24} \pm 0.02$

- **Test QCD theories: PQCD, QCDF, final state interactions etc.**

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- **Determine the parameters of CKM matrix elements, or f_B**

$$V_{cb} \Rightarrow B \to X_c l \nu_l$$

$$V_{ub} \Rightarrow B \to X_u l \nu_l$$

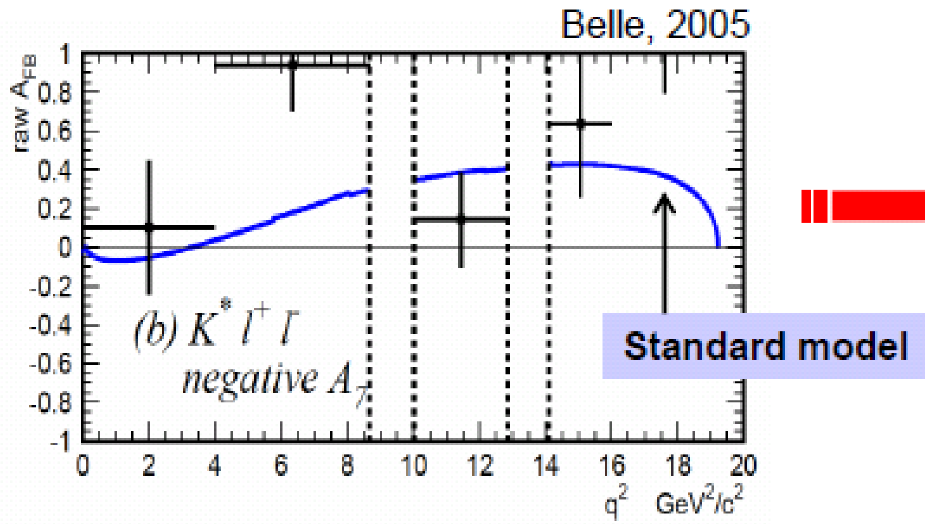
$$V_{td} / V_{ts} \Rightarrow B_d - \bar{B}_d \,/\, B_s - \bar{B}_s$$

$$V_{ub} f_B \Rightarrow B \to \tau \nu_\tau$$

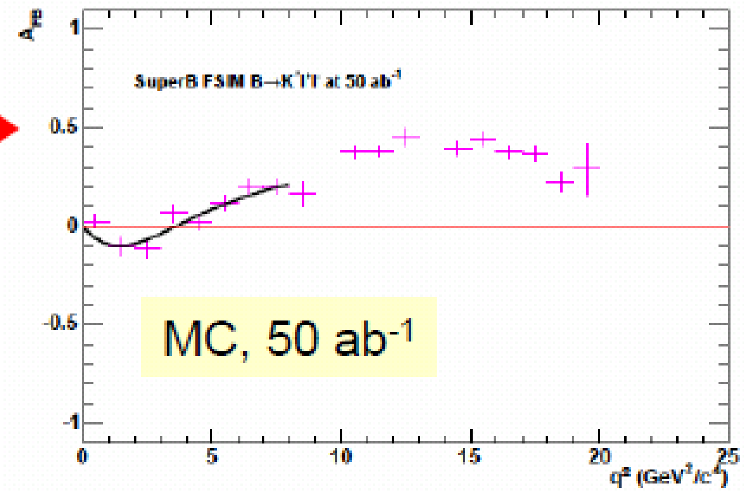
- **Probe flavor structures:**

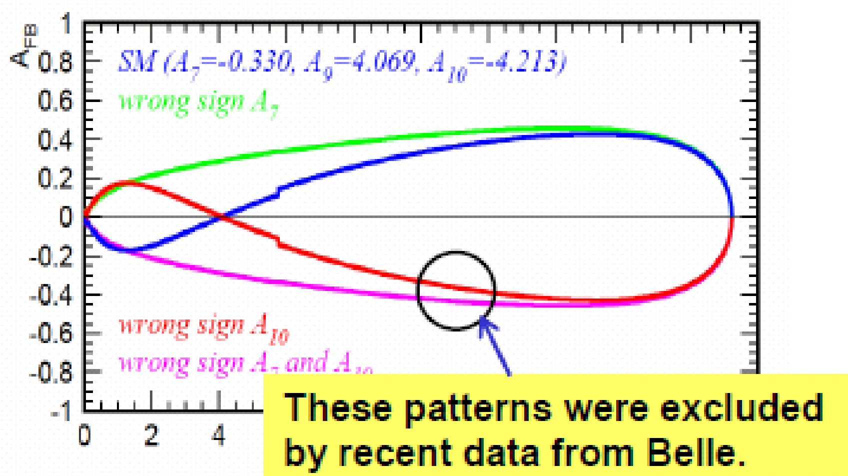
$$1.\ B \to K^{*}\gamma, B \to K^{(*)} l^{+} l^{-}\ 2.\ B \to l^{+} l^{-}, l^{+} l^{-}\gamma$$

Experimental result with 0.35 ab^{-1}



Sensitivity at Super KEKB



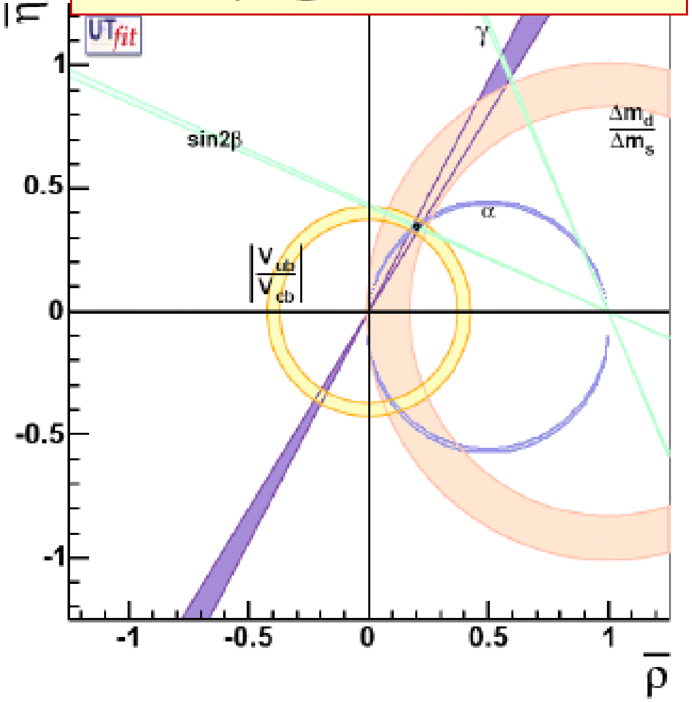


- Zero-crossing q^2 for A_{FB} will be determined with 5% error with 50ab^{-1}

Super B/Flavor Factories
$>10^{10}$ B's/year

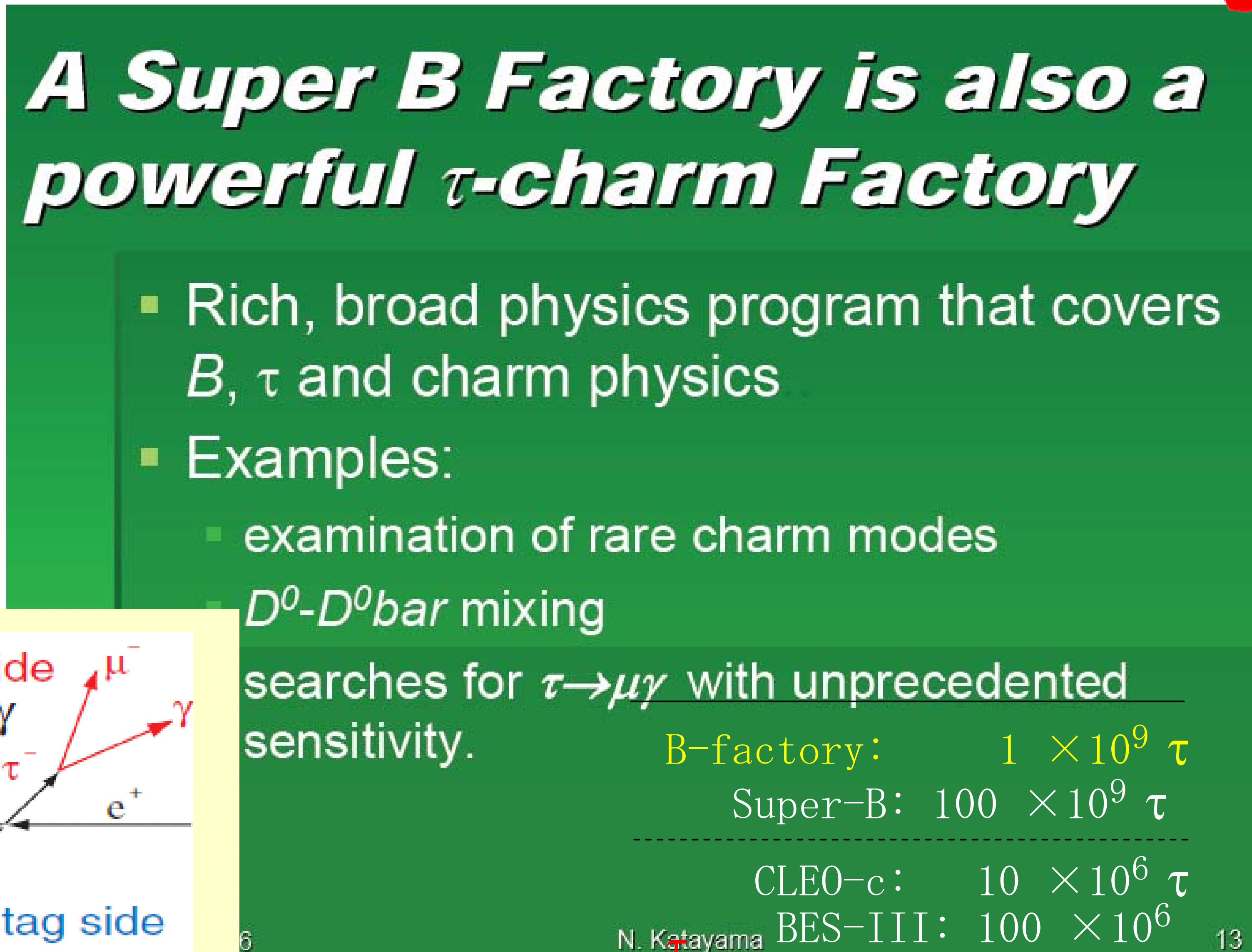
- LHCB @ $L_{int} > 10^{32}$ (~10^{10} B's/year)
- Super-Flavor Factories
  - e+e- superB: KEKB @ $L_{int} = 2\times10^{35}$ /cm^2/s (N. Katayama's talk this session)
  - linear Super-B factory @ $L_{int} > 10^{36}$ /cm^2/s(J. Seeman)

Hope@5×10^{10} BB's



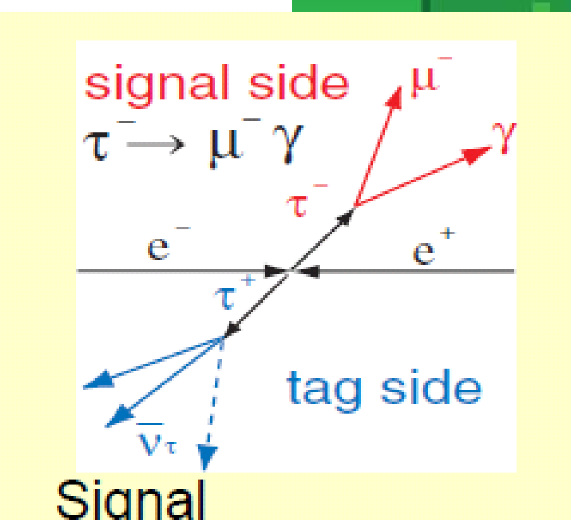
CPV in Rare Decays		e^+e^- Precision		
Measurement	**Goal**	**3/ab**	**10/ab**	**50/ab**
$S(B^0 \to \phi K^0_S)$	$\approx 5\%$	16%	8.7%	3.9%
$S(B^0 \to \eta' K^0_S)$	$\approx 5\%$	5.7%	3%	1%
$S(B^0 \to K^0_S \pi^0)$		8.2%	5%	4%
$S(B^0 \to K^0_S \pi^0 \gamma)$	SM: $\approx 2\%$	11%	6%	4%
$A_{CP}(b \to s\gamma)$	SM: $\approx 0.5\%$	1.0%	0.5%	0.5%
$A_{CP}(B \to K^* \gamma)$	SM: $\approx 0.5\%$	0.6%	0.3%	0.3%

Future tau (τ) physics



B-factory: 1 $\times 10^9$ τ
Super-B: 100 $\times 10^9$ τ

CLEO-c: 10 $\times 10^6$ τ
BES-III: 100 $\times 10^6$ τ



- results up to 2005

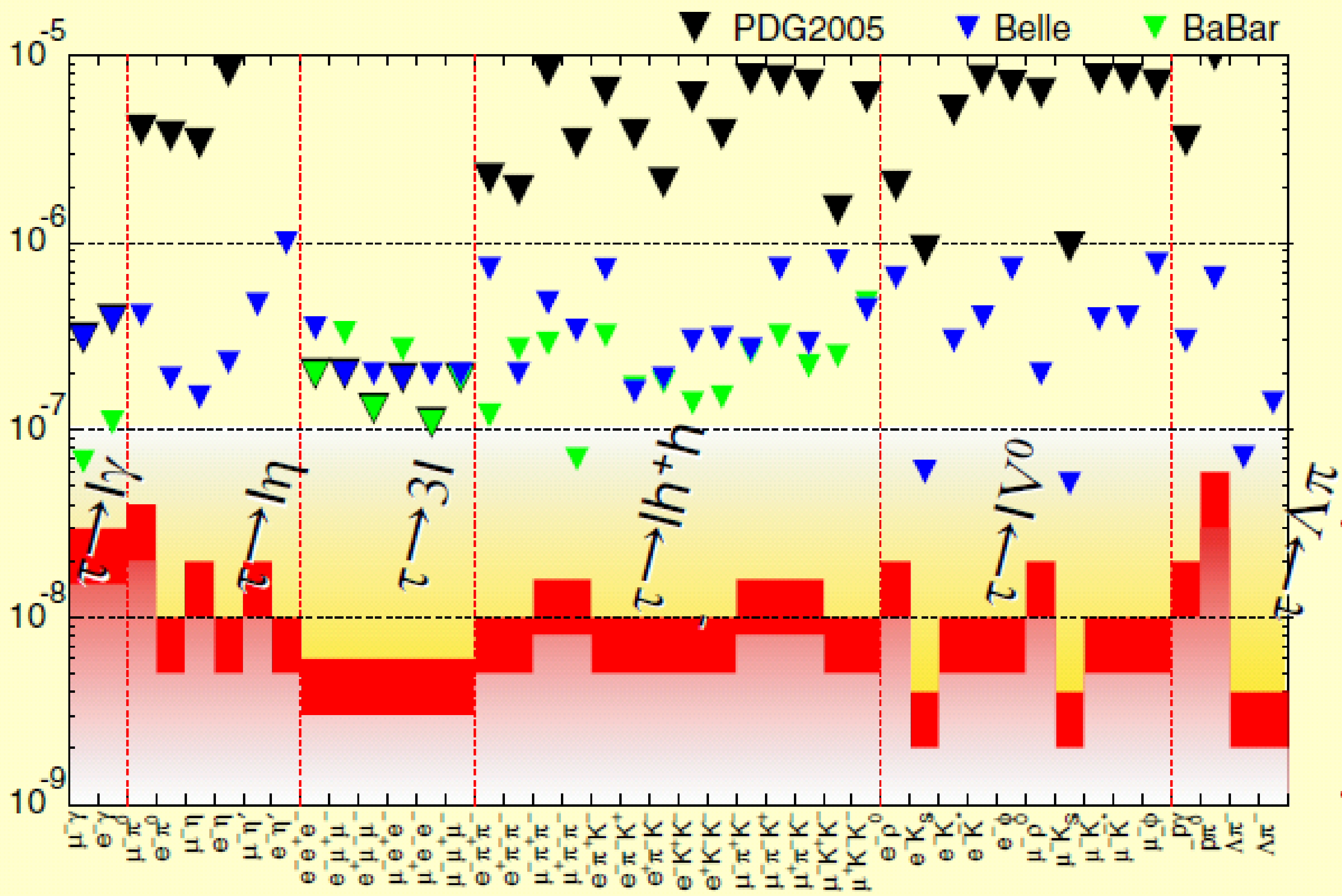
mode		$x10^{-7}$		mode		$x10^{-7}$
$\tau\to\mu\gamma$	BaBar	0.68		$\tau\to lll$		1.1-3.5
$\tau\to e\gamma$	Belle	3.1		$\tau^-\to\mu^+e^-e^-$	BaBar	1.1
$\tau\to\mu\eta$	Belle	1.5		$\tau^-\to\mu^-e^-e^+$	Belle	1.9
$\tau\to e\eta$	Belle	2.4		$\tau\to lhh$	BaBar	0.7-4.8
$\tau\to\mu\eta'$	Belle	10		$\tau^-\to\mu^+\pi^-\pi^-$	BaBar	0.7
$\tau\to e\eta'$	Belle	4.7		$\tau\to lV^0$	CLEO	20-75
$\tau\to\mu\pi^0$	Belle	4.1		$\tau\to e\rho^0$	CLEO	20
$\tau\to e\pi^0$	Belle	1.9				
$\tau\to\mu Ks$	CLEO	9.5		$\tau^-\to\Lambda\pi^-$	Belle	1.4
$\tau\to eKs$	CLEO	9.1		$\tau^-\to\bar{\Lambda}\pi^-$	Belle	0.72

mode		$\times 10^{-7}$
$\tau \to \mu\gamma$	Belle	0.45
$\tau \to e\gamma$	BaBar	1.1
$\tau \to \mu\eta$	Belle	0.65
$\tau \to e\eta$	Belle	0.92
$\tau \to \mu\eta'$	Belle	1.3
$\tau \to e\eta'$	Belle	1.6
$\tau \to \mu\pi^0$	Belle	1.2
$\tau \to e\pi^0$	Belle	0.80
$\tau \to \mu Ks$	Belle	0.52
$\tau \to eKs$	Belle	0.60

mode		$\times 10^{-7}$
$\tau \to lll$		1.1-3.5
$\tau^- \to \mu^+ e^- e^-$	BaBar	1.1
$\tau^- \to \mu^- e^- e^+$	Belle	1.9
$\tau \to lhh$	BaBar	0.7-4.8
$\tau^- \to \mu^+ \pi^- \pi^-$	BaBar	0.7
$\tau \to lV^0$	Belle	2.0-7.7
$\tau \to \mu\rho^0$	Belle	2.0
$\tau^- \to \Lambda\pi^-$	BaBar	0.59
$\tau^- \to \bar{\Lambda}\pi^-$	BaBar	0.58
$\tau^- \to \Lambda K^-$	BaBar	0.72
$\tau^- \to \bar{\Lambda} K^-$	BaBar	1.5

■ Possible sensitivity with Super B-factory

- Red band for 5000~10,000fb^{-1}



Can probe O(10^{-8}-10^{-9}) region by Super B-factory

Lepton flavor violation (LFV)

- quark flavor changing via charged current; charged weak current is written as

$$J_\mu = (\bar{u}, \bar{c}, \bar{t})\gamma_\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \bar{U}_i \gamma_\mu P_L D_j \qquad P_L = \frac{1-\gamma_5}{2}$$

In terms of physical eigenstates

$$J_\mu = \bar{u}_p \gamma_\mu P_L \underbrace{V^{U_L} V^{D_L^\dagger}}_{= V_{CKM}} d_p \qquad u_p = V^{U_L} U_w, d_p = V^{D_L} D_w$$

By Wolfenstein parametrization

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$A \approx 0.82, \quad \lambda \approx 0.224$$

- In the standard model (SM), the flavor changing neutral currents (FCNCs) arise from loop

- Rare decays induced from loop will be sensitive to new physics.

- In the SM, since neutrinos are massless, there are no flavor violations in lepton sector

$$J_{\mu}^{l} = \bar{\nu}\gamma_{\mu}P_{L}U_{\nu_L}U_{l_L}^{\dagger}l_{p} = \bar{\nu}\gamma_{\mu}P_{L}l_{p}$$

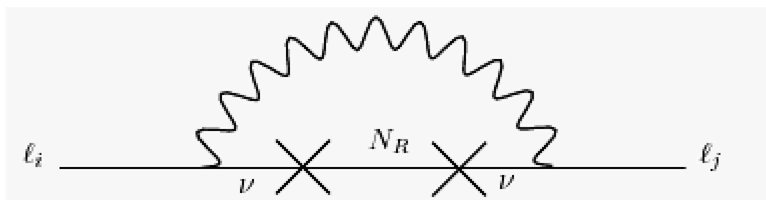
- According to observed neutrino oscillations, we know that neutrinos have masses so that $|\nu_i> = \sum_{\alpha} U_{\alpha i}|\nu_\alpha>$

$$U = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}}_{\text{Atmospheric}} \times \underbrace{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}}_{\text{Cross-Mixing}} \times \underbrace{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Solar}} \times \underbrace{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Majorana CP-violating phases}} .$$

Maki-Nakagawa-Suzuki-Pontecorvo (MNSP)

Higgs-mediated LFV

- Nonzero neutrino masses are definite.
- It is found that ***seesaw mechanism***, by introducing singlet right-handed Majorana neutrinos, is one of natural ways to solve the tiny neutrino masses which are less than eV.
- In non-SUSY models, the effects of LFV are suppressed by $1/M_R$ which is around GUTs scale 10^{15} GeV.



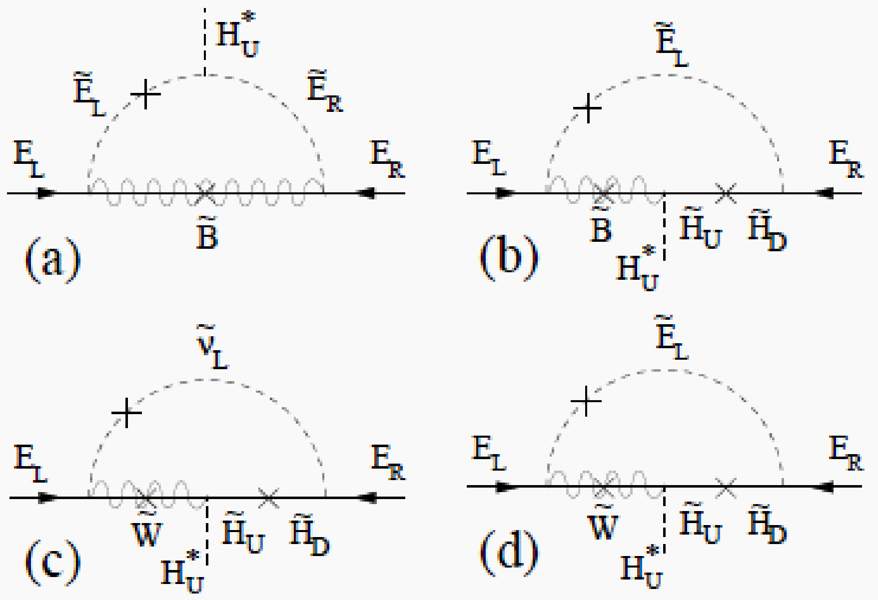
- However, in models with SUSY, due to the right-handed Majorana introduced, the flavor conservation in the slepton sector at unified scale will be violated at the M_R scale via renormalization.

Borzumati, Masiero, PRL57,961; Hall, Rattazzi, Sarid, PRD50,7048; Babu, Kolda, PRL89,241802

$$\frac{d}{d\log Q}(m_L^2)_{ij} = \left(\frac{d}{d\log Q}(m_L^2)_{ij}\right)_{\rm MSSM} + \frac{1}{16\pi^2}\left[m_L^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_L^2 + 2(Y_\nu^\dagger m_{\tilde{\nu}_R}^2 Y_\nu + m_{H_u}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu)\right]_{ij} \quad (3)$$

$$-\mathcal{L} = \overline{E}_R Y_E L_L H_d + \overline{\nu}_R Y_\nu L_L + \frac{1}{2}\nu_R^\top M_R \nu_R$$

Babu & Kolda’s mechanism for lepton flavor violation



$$\left(\Delta m_{\tilde{L}}^2\right)_{ij} \simeq -\frac{\log(M/M_R)}{16\pi^2}\left(6m_0^2(Y_\nu^\dagger Y_\nu)_{ij} + 2\left(A_\nu^\dagger A_\nu\right)_{ij}\right)$$

$$\xi = -\frac{\log(M/M_R)}{16\pi^2}(6+2a^2)m_0^2.$$

nonholomophic terms

$$-\mathcal{L} = \overline{E}_R Y_E E_L H_d^0 + \overline{E}_R Y_E\left(\epsilon_1 \mathbf{1} + \epsilon_2 Y_\nu^\dagger Y_\nu\right) E_L H_u^{0*} + h.c.$$

$$\epsilon_{2a} \simeq \frac{\alpha'}{4\pi}\xi\mu M_1 f_2\left(M_1^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\tau}_L}^2, m_{\tilde{\ell}_R}^2\right)$$

$$\epsilon_{2b} \simeq -\frac{\alpha'}{8\pi}\xi\mu M_1 f_2\left(\mu^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\tau}_L}^2, M_1^2\right).$$

$$\epsilon_{2c} \simeq \frac{\alpha_2}{4\pi}\xi\mu M_2 f_2\left(\mu^2, m_{\tilde{\nu}_\ell}^2, m_{\tilde{\nu}_\tau}^2, M_2^2\right).$$

$$\epsilon_{2d} \simeq \frac{\alpha_2}{8\pi}\xi\mu M_2 f_2\left(\mu^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\tau}_L}^2, M_2^2\right).$$

$$-f_2(a,b,c,d) \equiv \frac{a\log(a)}{(a-b)(a-c)(a-d)} + \frac{b\log(b)}{(b-a)(b-c)(b-d)} + (a \leftrightarrow c, b \leftrightarrow d).$$

$$
\begin{aligned}
\epsilon_1 &= \frac{\alpha'}{8\pi}\mu M_1\left[2f_1\left(M_1^2, m_{\tilde{\ell}_L}^2, m_{\tilde{\ell}_R}^2\right) - f_1\left(M_1^2, \mu^2, m_{\tilde{\ell}_L}^2\right) + 2f_1\left(M_1^2, \mu^2, m_{\tilde{\ell}_R}^2\right)\right] \\
&+ \frac{\alpha_2}{8\pi}\mu M_2\left[f_1\left(\mu^2, m_{\tilde{\ell}_L}^2, M_2^2\right) + 2f_1\left(\mu^2, m_{\tilde{\nu}}^2, M_2^2\right)\right]
\end{aligned}
$$

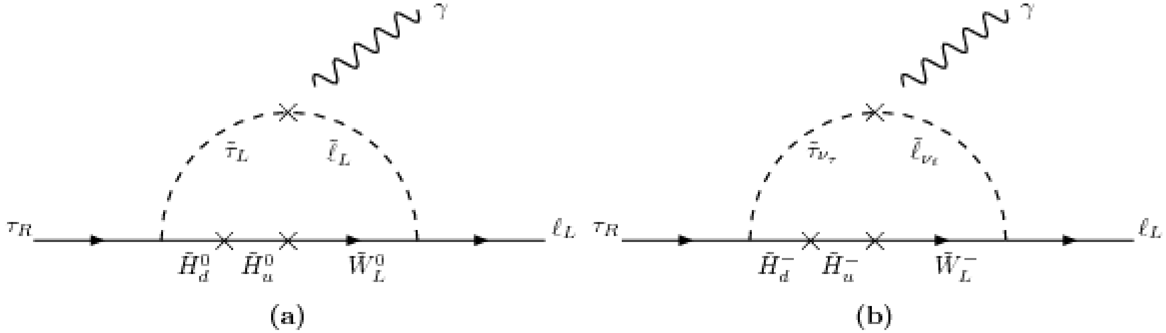
$$
-\mathcal{L} \simeq (2G_F^2)^{1/4}\frac{m_\tau \kappa_{32}}{\cos^2\beta}\left(\overline{\tau}_R\, \mu_L\right)\left[\cos(\beta-\alpha)h^0 - \sin(\beta-\alpha)H^0 - iA^0\right] + h.c.
$$

$$
\kappa_{ij} = -\frac{\epsilon_2}{\left[1+\left(\epsilon_1+\epsilon_2(Y_\nu^\dagger Y_\nu)_{33}\right)\tan\beta\right]^2}\left(Y_\nu^\dagger Y_\nu\right)_{ij}.
$$

- At large tanβ ~ mt/mb, or cosβ << 1, the Higgs-mediated LFV will be enhanced

Implications of LFV in τ decays

- $\tau \to \mu$ (γ, 3μ, ϕ, KK) induced by slepton mixings with large $\tan\beta$



Due to slepton mixings

$$\left(\Delta m^2_{\tilde{L}}\right)_{ij} \approx -\frac{1}{(4\pi)^2}\left(6 m_0^2 Y_\nu^\dagger Y_\nu + 2A^\dagger A\right)_{ij} \ln\left(\frac{M_U}{M_R}\right)$$

$$T = \sqrt{2} G_F\, e\, m_\tau \epsilon^{\mu *}(k) \bar{\ell}(p-k) i\sigma_{\mu\nu} k^\nu A_R P_R \tau(p)\,,$$

$$A_R = \frac{M_2 \mu}{(4\pi)^2} \frac{m_W^2}{M_2^2} \tan\beta (\Delta m^2_{\tilde{L}})_{\tau\ell} \sum_{S=\tilde{\ell},\tilde{\nu}_\ell} G_S\,,$$

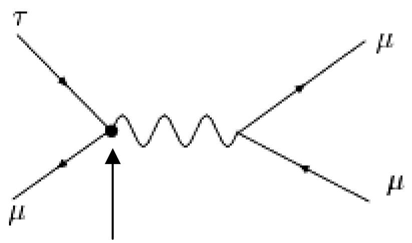
$$\Gamma(\tau \to \ell\gamma) = \frac{\alpha_{em}}{2} G_F^2 m_\tau^5 |A_R|^2.$$

$$G_{\tilde{\ell}} = -\frac{1-\tan^2\theta_W}{m^2_{\tilde{\ell}_L} - m^2_{\tilde{\tau}_L}}\left[\frac{f_n(x_{\tilde{\ell}_L})}{m^2_{\tilde{\ell}_L}} - \frac{f_n(x_{\tilde{\tau}_L})}{m^2_{\tilde{\tau}_L}}\right],$$

$$G_{\tilde{\nu}_\ell} = \frac{4}{m^2_{\tilde{\nu}_\ell} - m^2_{\tilde{\nu}_\tau}}\left[\frac{f_c(x_{\tilde{\nu}_\ell})}{m^2_{\tilde{\nu}_\ell}} - \frac{f_c(x_{\tilde{\nu}_\tau})}{m^2_{\tilde{\nu}_\tau}}\right],$$

$$f_n(x) = \frac{1}{(1-x)^3}\left(1 - x^2 + 2x\ln x\right),$$

$$f_c(x) = -\frac{1}{2(1-x)^3}\left(3 - 4x + x^2 + 2\ln x\right),$$

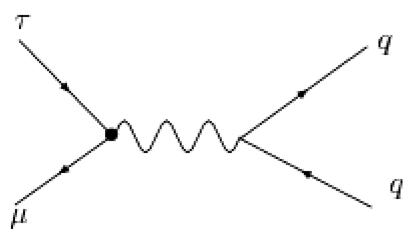


Dipole operators

$$R_\mu = \frac{\mathrm{BR}(\tau \to 3\mu)}{\mathrm{BR}(\tau \to \mu\gamma)} \simeq \frac{\alpha_{em}}{3\pi}\left(2\ln\frac{m_\tau}{2m_\mu} - \frac{7}{12}\right).$$

$$R_e = \frac{\mathrm{BR}(\tau \to e\mu^+\mu^-)}{\mathrm{BR}(\tau \to e\gamma)} \simeq \frac{\alpha_{em}}{3\pi}\left(2\ln\frac{m_\tau}{2m_\mu} - \frac{4}{3}\right).$$

$$R_\mu \sim R_e \sim O(10^{-3})$$



$$\Gamma(\tau \to \ell\phi) \simeq 2\pi m_\tau^5 \alpha_{em}^2 G_F^2 Q_s^2 \frac{f_\phi^2}{m_\phi^2}|A_R|^2 \left(1 - \frac{m_\phi^2}{m_\tau^2}\right)^2 \left(1 + \frac{m_\phi^2}{2m_\tau^2}\right)$$

$$= 4\pi\alpha_{em} Q_s^2 \frac{f_\phi^2}{m_\phi^2}\left(1 - \frac{m_\phi^2}{m_\tau^2}\right)^2 \left(1 + \frac{m_\phi^2}{2m_\tau^2}\right)\Gamma(\tau \to \ell\gamma),$$

$$\frac{d\Gamma(\tau \to \ell K^+K^-)}{\Gamma(\tau \to \ell\gamma)dQ^2} \simeq \frac{\alpha_{em}}{6\pi}\frac{|F_1^{KK}(Q^2)|^2}{Q^2}\left(1 - \frac{Q^2}{m_\tau^2}\right)^2\left(1 + \frac{Q^2}{2m_\tau^2}\right)\left(1 - \frac{4m_K^2}{Q^2}\right)^{1/2}$$

$$\langle 0|\bar{q}\gamma^\mu q|\phi\rangle = i m_\phi f_\phi \epsilon_\phi^*(k)$$

$$\langle K^+(p_1)K^-(p_2)|\bar{q}\gamma^\mu q|0\rangle = (p_1^\mu - p_2^\mu)F_q^{K^+K^-}(Q^2).$$

$$\frac{\Gamma(\tau \to l\,\phi)}{\Gamma(\tau \to l\,\gamma)} \sim \frac{\Gamma(\tau \to l\,KK)}{\Gamma(\tau \to l\,\gamma)} \sim O(10^{-3})$$

LFV with Higgs-mediated

$$
\begin{aligned}
-\mathcal{L}_{\rm eff} &= \bar{E}_{Ri} Y_i \left[\delta_{ij} H_d^0 + (\epsilon_1 \delta_{ij} + \epsilon_2 I_{ij})\, H_u^{0*}\right] E_{Lj} + h.c.\,, \\
&= \bar{E}_R M_\ell^0 E_L + h.c.\,,
\end{aligned}
$$

nonholomophic terms

Y: diagonalized Yukawa matrix of leptons

$$I_{ij} = (\Delta m_{\tilde{L}}^2)_{ij} / m_0^2$$

- *Due to nonholomophic effects,* M_1^0 *is not diagonal matrix*

- *Since* ϵ_2 *are loop effects, they are much less than 1,*
  the LFV can be regarded as a leading expansion of $\epsilon_2 I_{ij}$

$$U M_\ell^0 U^\dagger \approx (1+\Delta) M_\ell^0 (1-\Delta) = M_\ell^{dia}\,, \qquad U_{L(R)} \approx 1 + \Delta_{L(R)}$$

$$(M_\ell^0)_{ii} \approx (M_\ell^{dia})_{ii}\,,\ \Delta_{ij} \approx \frac{(M_\ell^0)_{ij}}{(M_\ell^0)_{ii} - (M_\ell^0)_{jj}} \quad (\mathrm{i} \neq \mathrm{j}).$$

➢ *Employ the physical mass eigenstates of the Higgses*

$$H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}, \qquad H_u = \begin{pmatrix} H_u^{0*} \\ -H_u^- \end{pmatrix}$$

$$\begin{aligned} ReH_d^0 &= v_d + \frac{1}{\sqrt{2}}\left[\cos\alpha H^0 - \sin\alpha h^0\right] , \\ ReH_u^0 &= v_u + \frac{1}{\sqrt{2}}\left[\sin\alpha H^0 + \cos\alpha h^0\right] , \\ ImH_d^0 &= \frac{1}{\sqrt{2}}\left[\cos\beta G^0 - \sin\beta A^0\right] , \\ ImH_u^0 &= \frac{1}{\sqrt{2}}\left[\sin\beta G^0 + \cos\beta A^0\right] , \end{aligned}$$

➢ *The interactions for the LFV via the Higgs-mediated mechanism are expressed by*

$$\mathcal{H}_{\rm eff}^{i\neq j} = (\sqrt{2}G_F)^{1/2}\frac{m_{\ell i}C_{ij}}{\cos^2\beta}\bar{\ell}_{Ri}\ell_{Lj}\left[\sin(\alpha-\beta)H^0 + \cos(\alpha-\beta)h^0 - iA^0\right] + h.c.$$

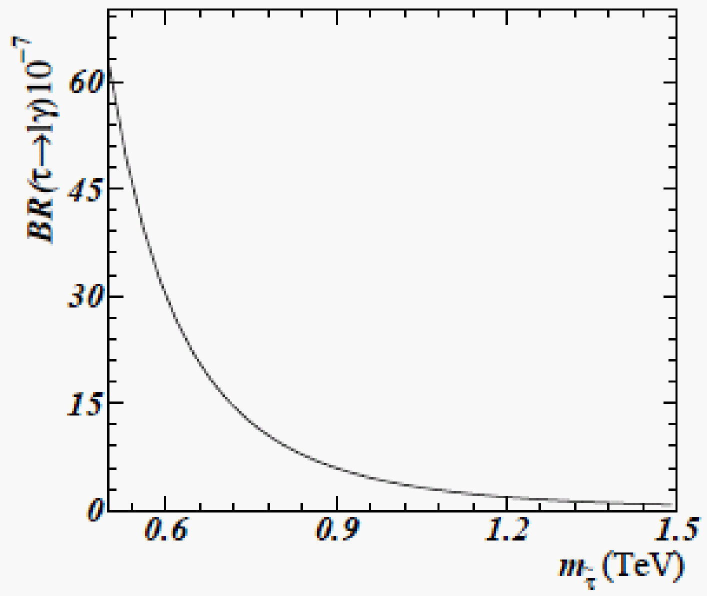
$$C_{ij} = \epsilon_2 I_{ij}/(1+(\epsilon_1+\epsilon_2 I_{ii})\tan\beta)^2. \qquad I_{ij} = (\Delta m_{\tilde{L}}^2)_{ij}/m_0^2$$

τ→ μ γ

- Since the LFVs in $\tau \to l\,\gamma$ and $\tau \to l\,X\,(X = l^{+}l^{-}, \eta^{(\prime)}, f_0(980), KK)$ decay are driving by the same mechanism, to simplify the discussions, we take $M_1 \sim M_2 \sim m_0 \sim \mu \sim m_{\tilde{l}}$

$$A_R \approx \frac{1}{6(4\pi)^2}\frac{m_W^2}{m_{\tilde{\tau}}^2}\frac{(\Delta m_{\tilde{L}}^2)_{\tau\ell}}{m_0^2}\tan\beta(1+\tan^2\theta_W),$$

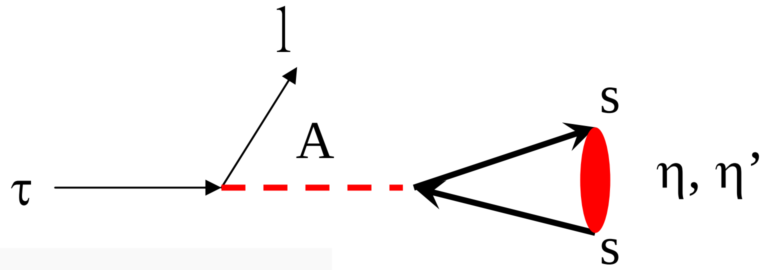
$$\epsilon_1 \approx \frac{3\alpha_{em}}{4\pi\sin^2(2\theta_W)}, \quad \epsilon_2 \approx \frac{\alpha_{em}}{16\pi}\left(\frac{1}{3\cos^2\theta_W}+\frac{1}{\sin^2\theta_W}\right),$$



$$(\Delta m_{\tilde{L}}^2)_{ij} \approx -\frac{1}{(4\pi)^2}\left(6m_0^2 Y_\nu^\dagger Y_\nu + 2A^\dagger A\right)_{ij}\ln\left(\frac{M_U}{M_R}\right)$$

- $M_U \sim 10^{19}$ GeV , $M_R \sim 10^{14}$ GeV
- $\tan\beta = 60$
- $BR_{exp} < (0.45, 0.68) \times 10^{-7}$

- $\tau \to l\,(\eta, \eta')$

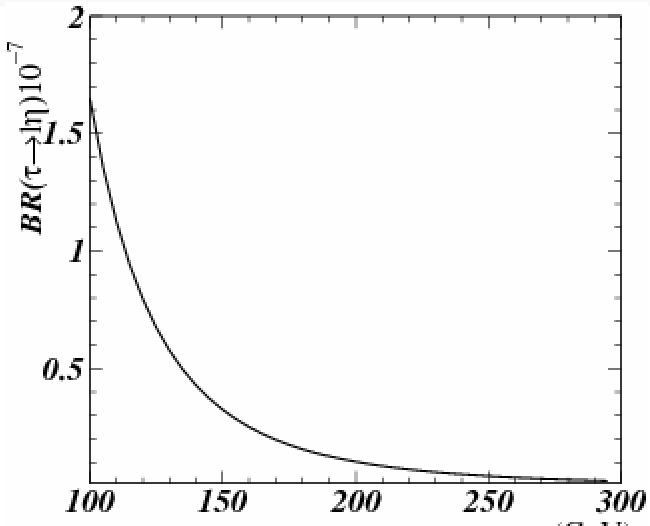


$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

$$\langle 0 | \bar{q}' \gamma_\mu \gamma_5 q' | \eta_{q'}(p) \rangle = f_{\eta_{q'}} p_\mu$$

$$m_{qq}^2 = \frac{\sqrt{2}}{f_q} \langle 0 | m_u \bar{u} \gamma_5 u + m_d \bar{d} \gamma_5 d | \eta_q \rangle, \quad m_{ss}^2 = \frac{2}{f_s} \langle 0 | m_s \bar{s} \gamma_5 s | \eta_s \rangle.$$

$$\Gamma(\tau \to \ell\eta) \simeq \frac{G_F^2 m_\tau^3 |C_{\tau\ell}|^2}{64\pi} \tan^6\beta \left(\sin\phi f_s \frac{m_{ss}^2}{m_A^2} \right)^2 \left(1 - \frac{m_\eta^2}{m_\tau^2} \right)^2 .$$

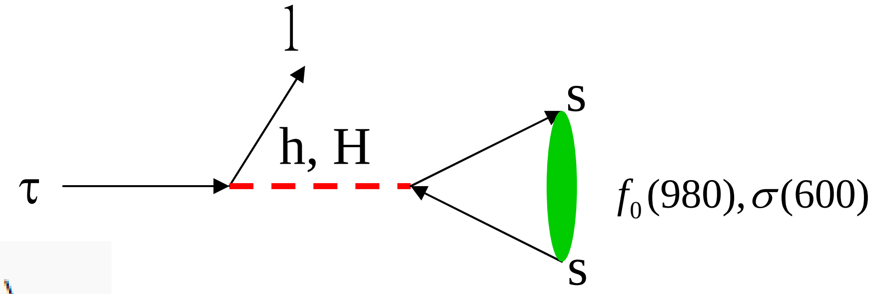


(a)

$$\frac{\Gamma(\tau \to \ell\eta')}{\Gamma(\tau \to \ell\eta)} = \cot^2\phi \left(\frac{1 - m_{\eta'}^2/m_\tau^2}{1 - m_\eta^2/m_\tau^2} \right)^2$$

- $\phi = 39.3^0$
- $f_s = 0.17$ GeV
- $m_{ss} = 0.69$ GeV
- $BR_{exp} < 0.65 \times 10^{-7}$

- $\tau \to l\,(f_0(980), \sigma(600))$



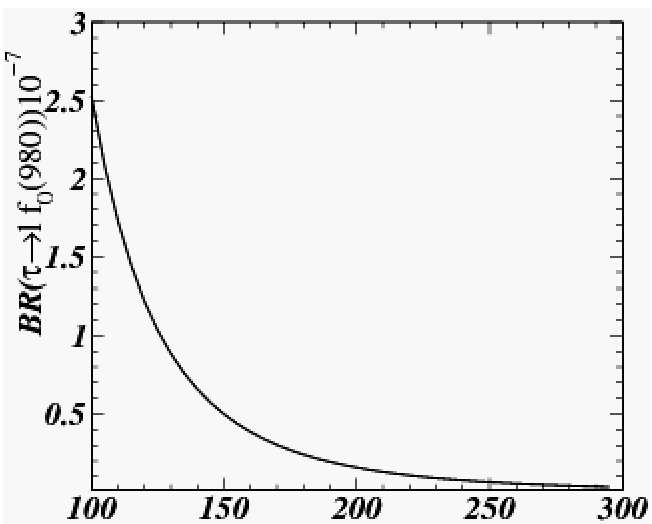
$$|f_0(980)\rangle = \cos\theta|s\bar{s}\rangle + \sin\theta|n\bar{n}\rangle\,,$$
$$|\sigma(600)\rangle = -\sin\theta|s\bar{s}\rangle + \cos\theta|n\bar{n}\rangle\,,$$

$$n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$$

$$\langle f_0^s|\bar{s}s|0\rangle = m_{f_0}\tilde{f}^s_{f_0}, \quad \langle \sigma^s|\bar{s}s|0\rangle = m_\sigma \tilde{f}^s_\sigma\,,$$

$$\Gamma(\tau \to \ell f_0(980)) \simeq \frac{G_F^2 m_\tau^3 |C_{\tau\ell}|^2}{16\pi\cos^6\beta}\left(m_s m_{f_0}\tilde{f}^s_{f_0}\cos\theta\right)^2 \left(\frac{cs}{m_h^2} - \frac{sc}{m_H^2}\right)^2 \left(1 - \frac{m_{f_0}^2}{m_\tau^2}\right)^2 .$$

- cs: cos(α-β)sin α ; sc=sin(α-β)cosα



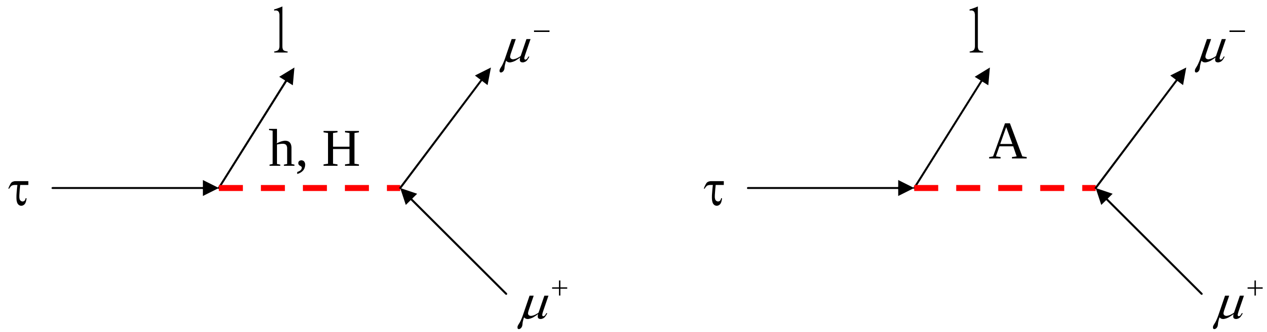
(b)

$$\frac{\Gamma(\tau \to \ell\sigma(600))}{\Gamma(\tau \to \ell f_0(980))} \simeq \left(\frac{m_\sigma \tilde{f}^s_\sigma \tan\theta}{m_{f_0}\tilde{f}^s_{f_0}}\right)^2 \left(\frac{1 - m_\sigma^2/m_\tau^2}{1 - m_{f_0}^2/m_\tau^2}\right)^2$$

- decoupling limit: $\alpha \to \beta - \pi/2$

- $\theta = 30^0$,
- $m_s = 0.15$ GeV,
- $\tilde{f}^s_\sigma \sim \tilde{f}^s_{f_0} = 0.33\,GeV$

$\tau \to l\, \mu^+ \mu^-$



$$\Gamma(\tau \to \ell \mu^+ \mu^-) \simeq c_\ell \frac{G_F^2 m_\mu^2 m_\tau^7 |C_{\tau\ell}|^2}{3 \cdot 2^9 \pi^3 \cos^6\beta} \left[\left(\frac{cs}{m_h^2} - \frac{sc}{m_H^2} \right)^2 + \left(\frac{\sin\beta}{m_A^2} \right)^2 \right] ,$$

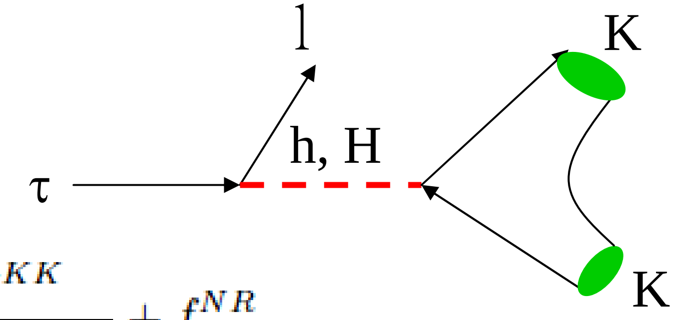
$$\Gamma(\tau \to \ell \mu^+ \mu^-) = \frac{c_\ell m_\mu^2 m_\tau^4}{3 \cdot 2^5 \pi^2} \left[\frac{\Gamma(\tau \to \ell\eta)}{C_\eta} + \frac{\Gamma(\tau \to \ell f_0(980))}{C_{f_0}} \right] ,$$

$$C_\eta = (\sin^2\beta \sin\phi f_s m_{ss}^2/2)^2 (1 - m_\eta^2/m_\tau^2))^2 \qquad C_{f_0} = (m_s m_{f_0} \tilde{f}_{f_0}^s \cos\theta)^2 (1 - m_{f_0}^2/m_\tau^2)^2.$$

- decoupling limit: $\alpha \to \beta - \pi/2$

$$\Gamma(\tau \to \ell f_0(980)) : \Gamma(\tau \to \ell \mu^+ \mu^-) : \Gamma(\tau \to \ell\eta) \approx 1.3 \ : \ 0.36 c_\ell \ : \ 1.$$

• $\tau \to l K^+ K^-$



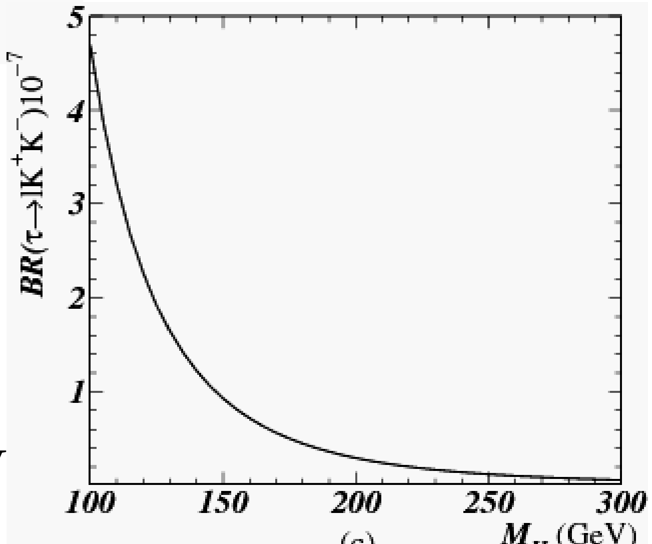
$$\langle K^+(p_1)K^-(p_2)|\bar{s}s|0\rangle \equiv f_s^{K^+K^-}(Q^2) = \sum_S \frac{m_S\, \tilde{f}_S^s\, g^{S\to KK}}{m_S^2 - Q^2 - i m_S \Gamma_S} + f_s^{NR}$$

Cheng,Chua, Soni, PRD72,094003

$$f_s^{NR} = \frac{v}{3}\left(3F_{NR}^1 + 2F_{NR}^2\right) + v\frac{\kappa}{Q^2}\left[\ln\frac{Q^2}{\Lambda^2}\right]^{-1}$$

$$\begin{aligned} F_u^{K^+K^-} &= F_\rho + 3F_\omega + \frac{1}{3}\left(2F_{NR}^1 - F_{NR}^2\right) \\ F_s^{K^+K^-} &= -3F_\phi - \frac{1}{3}\left(3F_{NR}^1 + 2F_{NR}^2\right), \\ F_V &= \frac{c_V}{m_V^2 - Q^2 - i m_V \Gamma_V}, \\ F_{NR}^{1(2)} &= \left(\frac{x_1^{1(2)}}{Q^2} + \frac{x_2^{1(2)}}{Q^4}\right)\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-1} \end{aligned}$$

$$\frac{d\Gamma(\tau \to \ell K^+K^-)}{dQ^2} \simeq \frac{G_F^2 m_\tau^3 |C_{\tau\ell}|^2}{2^8 \pi^3 \cos^6\beta}\left(m_s f_s^{K^+K^-}\right)^2 \left(\frac{cs}{m_h^2} - \frac{sc}{m_H^2}\right)^2 \times \left(1 - \frac{Q^2}{m_\tau^2}\right)^2 \left(1 - \frac{4m_K^2}{Q^2}\right)^{1/2}.$$



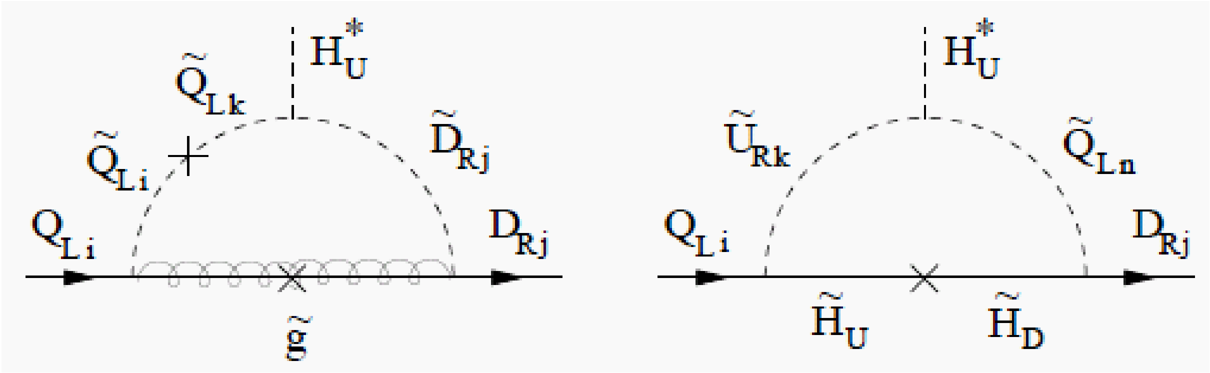
(c)

- v=2.87 GeV, κ=-10.4 GeV^4
- $\tilde{f}^s_{f_0(1530)} \sim \tilde{f}^s_{f_0(980)} = 0.33$ GeV
- $g^{f_0(980)\to KK} = 1.50,\ g^{f_0(1530)\to KK} = 3.18$ GeV

Speculation of Higgs-mediated effects

- In terms of squark mixings,

Babu,Kolda, PRL84,228(00)



$$-\mathcal{L}_{eff} = \overline{D}_R \mathbf{Y_D} Q_L H_d + \overline{D}_R \mathbf{Y_D} \left[\epsilon_g + \epsilon_u \mathbf{Y_U^\dagger Y_U}\right] Q_L H_u^* + h.c.$$

$$\mathcal{L}_{FCNC} = \frac{\overline{y}_b V_{tb}^*}{\sin\beta} \chi_{FC} \left[V_{td} \overline{b}_R d_L + V_{ts} \overline{b}_R s_L\right] \left(\cos\beta H_u^{0*} - \sin\beta H_d^0\right) + h.c.$$

- It is interesting to examine the effects on B decays

$$-\mathcal{L}_D^{qq} = \left(\sqrt{2}G_F\right)^{1/2} \frac{m_q}{\cos\beta(1+\tan\beta\epsilon_0)} \{[(-\sin\alpha + \cos\alpha\epsilon_0)h^0 + (\cos\alpha + \sin\alpha\epsilon_0)H^0]\bar{q}\,q - i\sin\beta A^0\bar{q}\gamma_5 q\} \,,$$

$$-\mathcal{L}_D^{b\to q} = \left(\sqrt{2}G_F\right)^{1/2} V_{tb}V_{tq}^* \frac{m_b C_3}{\cos^2\beta}\bar{q}_L b_R \left[\cos(\alpha-\beta)h^0 + \sin(\alpha-\beta)H^0 + iA^0\right] \,.$$

$$C_j = \epsilon_Y y_t^2/(1+\tan\beta\epsilon_j)/(1+\tan\beta\epsilon_0)$$

- Charged Higgs on $B \to \tau\nu$
- Higgs-mediated on $B_q - \bar{B}_q$
- Higgs-mediated on $B_q \to l^+ l^-, B \to K^{(*)} l^+ l^-$
- Higgs-mediated on $B \to K(\eta', \eta)$