Axion Cosmology & Finite Temp. Effects

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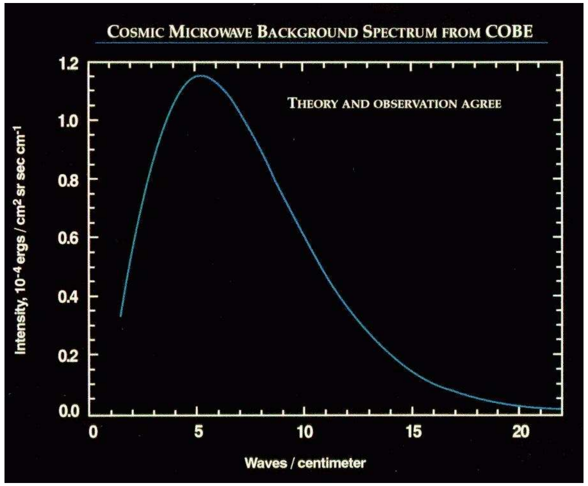
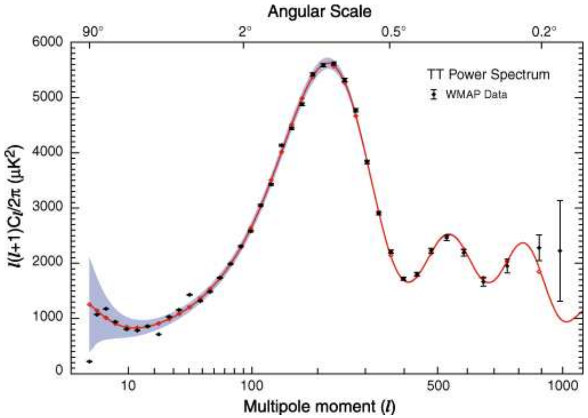
Outline

- Basic FRW Cosmological setup
- Finite temp. corrections in cosmology
- Axions in cosmology
- Finite temp. effects in axion cosmology
- Results
- Possible related issues





$\Longrightarrow$

2006 Nobel Prize - Mather and Smoot (COBE)

Opened the doors of Precision Cosmology

FRW Cosmology

- Spherically symmetric, homogeneous and isotropic universe
- $ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$
- K - spatial curvature, $a(t)$ - scale factor (dim. length), r - comoving coordinates - dimensionless ($k = +1, -1, 0$ corresponding to the spherical (closed), negative-curvature (open) and flat cases)
- Redshift parameter $1 + z = \frac{a(t_0)}{a(t)}$
- $\frac{a(t)}{a(t_0)} = 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots.$
- $H = \frac{\dot{a}(t)}{a(t)}$ $\qquad q_0 = -\frac{\ddot{a}(t)}{\dot{a}^2(t)} a(t) = -\frac{\ddot{a}(t)}{a(t) H^2}$
- $\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} \equiv \mathcal{G}_{\mu\nu} = 8\pi G \mathcal{T}_{\mu\nu} + \Lambda g_{\mu\nu}$ Einstein's eq. (put $\Lambda = 0$)

$$\mathcal{R}_{00} = -3\frac{\ddot{a}}{a} \qquad \mathcal{R}_{ij} = -\left[\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2}\right] g_{ij} \qquad \mathcal{R} = -6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right]$$

•Specify the type of matter content $\Longrightarrow$ $\mathcal{T}_{\mu\nu}$ consistent with isotropy and homogeneiety $\mathcal{T}^{\mu}_{\nu} = diag.(\rho, -p, -p, -p)$ (EOM: $\mathcal{T}^{\mu\nu}{}_{;\nu} = 0$)

• Trivially satisfied for $\mu = 1, 2, 3$, for $\mu = 0$ $\quad \frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}(a^3)$

• $0 - 0$ component of Einstein's eq.$(\Lambda = 0)$ $\quad \underbrace{\frac{\dot{a}^2}{a^2}}_{H^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$

• $i - j$ component of Einstein's eq. $\quad 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi p\, G$

• Equation of state: $\quad p = \omega\rho \Longrightarrow \rho \propto a^{-3(1+\omega)}$ and $a \propto t^{2/3(1+\omega)}$

$\omega = 0$ NR matter $\quad \omega = 1/3$ Radiation $\quad \omega = -1$ Cosmo. Const.

• Recast $0 - 0$ component (Friedmann's eq.) $\frac{k}{H^2 a^2} = \Omega - 1$

• For radiation, Stephan's law gives $\rho \propto T^4 \Longrightarrow T \propto a^{-1}$

Thermodynamics of the early universe

- For a given species A we have $\left(f_A(\vec{p}) = \frac{1}{e^{(E_A-\mu_A)/T} \pm 1}\right)$

$$n_A = g_A \int \frac{d^3p}{(2\pi)^3} f_A(\vec{p})$$

$$\rho_A = g_A \int \frac{d^3p}{(2\pi)^3} E(\vec{p}) \; f_A(\vec{p}) \qquad\qquad g_A\text{: DOF}$$

$$p_A = g_A \int \frac{d^3p}{(2\pi)^3} \frac{|\vec{p}|^2}{3E} \; f_A(\vec{p})$$

- Zero chem. pot. and non-degenerate case

- High temp. regime $(T >> m_A)$ $\quad p_A = \frac{1}{3}\rho_A$

$$n_A = \begin{cases} \frac{\zeta(3)}{\pi^2} g_A T^3 & (BE) \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_A T^3 & (FD) \end{cases} \qquad \rho_A = \begin{cases} \left(\frac{\pi^2}{30}\right) g_A T^4 \\ \frac{7}{8}\left(\frac{\pi^2}{30}\right) g_A T^4 \end{cases}$$

- Low temp. regime $m_A >> T$, both behave similarly

$$\rho_A = m_A n_A \qquad p_A = n_A T << \rho_A \qquad n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} e^{-(m_A-\mu_A)/T}$$

• At high temp. all species are in thermal equilibrium and interact with each other

• As universe expands, temp. drops and also the interparticle distance becomes bigger than the effective interaction length

• Interaction rate $(\Gamma) \geq$ Hubble expansion rate $(H) \longrightarrow$ Species coupled else decouple from plasma

•ν-decoupling $\Gamma = n\sigma|v| \sim (n \sim T^3) \otimes (G_F^2 T^2) \otimes (v = 1) \sim G_F^2 T^5$

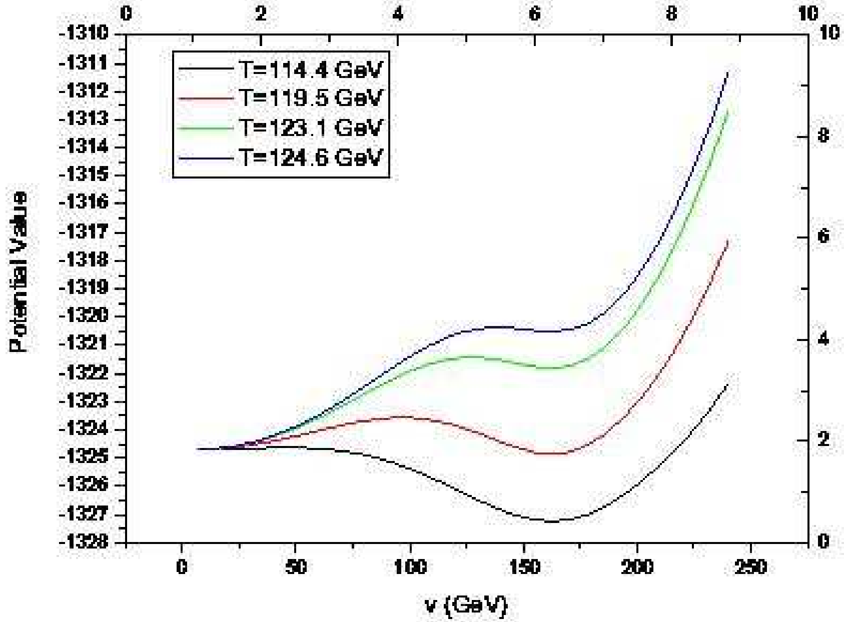
From Friedmann's eq. (neglecting Λ, k terms), $H \sim T^2/M_{Pl}$

$\Longrightarrow \frac{\Gamma}{H} \sim \left(\frac{T}{1\ MeV}\right)^3$ Weak int. decouple around MeV temp.

• For careful estimate, solve Boltzmann's eq. in FRW background

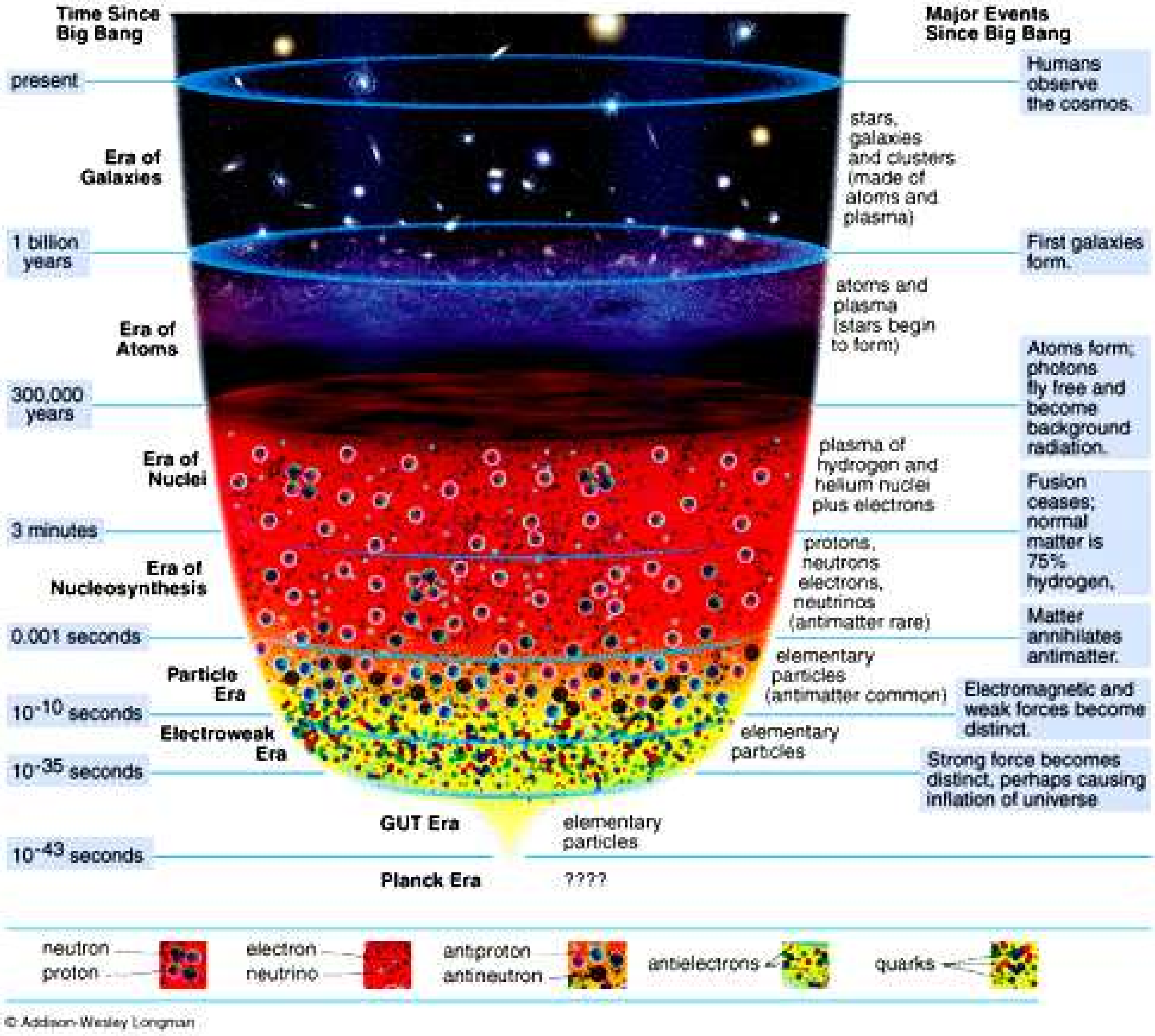
• Boltzmann's eq. for species ψ $(\psi + a + b..... \leftrightarrow i + j + ...)$

$$\dot{n}_\psi + \underbrace{3Hn_\psi}_{dilution} = \underbrace{-\int [d\Pi]_{all} (2\pi)^4 |\mathcal{M}|^2 \delta^4(i - f)[f_\psi f_a f_b .... - f_i f_j ....]}_{interaction}$$



• Another example of finite temp. effects - effective potential changes and symmetry can be restored at high temp.

• Our aim To identify/incorporate finite temp. corrections relevant for axion interactions



Thermal History of Universe

Axions & Current Status

• Rich vacuum structure of QCD leads to another term in action

$\mathcal{L}_{new} = \theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$ $\tilde{G}^{a\mu\nu} \sim \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}$ (dual strength)

•New term is total derivative term $\Longrightarrow$ No change in EOM but violates CP, T and P

• Neutron dipole moment $d_n \leq 10^{-25} e\ cm \rightarrow \theta \sim 10^{-10}$ Strong CP problem

• Form of new term is similar to anomaly term

•Peccei-Quinn solution/proposal Global chiral symmetry $(U(1)_{PQ})$ spontaneously broken at some scale (f_{PQ})

• Weinberg and Wilczek: spontaneous breakdown of global symmetry should bring massless particle Axion

• Due to chiral anomaly, axion gets small mass

• Also, due to anomaly a term in QCD arises $C_A \frac{A}{f_{PQ}} \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$
A axion field C_A model dependent constant

• These terms generate potential for axion which is minimised by $\langle A \rangle = -\frac{\theta C_A}{f_{PQ}}$ such that the total coeff. of $G\tilde{G}$ term vanishes

• $$\mathcal{L}_{axion} = \frac{1}{2}\partial^\mu A \partial_\mu A + C_A \frac{A}{f_{PQ}} \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + i\frac{g_{Aff}}{2m_f}\partial_\mu A(\bar{f}\gamma^\mu\gamma_5 f) + g_{A\gamma\gamma} A(\vec{E}.\vec{B})$$

• Hadronic axions - if the coupling to electrons is absent Changes the constraints on couplings of axion

• Upto factors of order unity, various sources suggest $f_A \geq 0.6 \times 10^9$ GeV and $m_A \leq 0.01$ eV

• Supernova obs. $m_A \leq 0.01$ eV or $f_A \geq 0.6 \times 10^9$ GeV

• Cosmological constraints from CMB, structure formation, distance measurements $m_A < 1.05$ eV or equivalently $f_A > 5.7 \times 10^6$ GeV

Axions in Cosmology

- Consider a complex scalar ϕ charged under globar $U(1)_{PQ}$. For $T \leq f_{PQ}$ $\quad V(\phi) = \lambda(|\phi|^2 - f_{PQ}^2/2)^2$ minimised for $\langle|\phi|\rangle = f_{PQ}/2$

- Breakdown of global $U(1)_{PQ}$ leads to massless axion field - coupling with matter $\propto f_{PQ}^{-1}$ (we use f_{PQ} and f_A interchangably)

- At high temp. axion is massless and gets small mass due to anomaly at small temp.

- Mechanisms for axion production in universe: Thermal vs Non-thermal

- Non-thermal include production via decay of topological defects and the so called misalignment mechanism

- It is possbile that enough axions are produced in thermal equilibrium in the early universe when temp. is very high

- Our interest is in thermal production of axions
- Important processes for thermal production and thermalization

Early stages $ag \leftrightarrow gg$, $aq \leftrightarrow \gamma(g)q$, $ag \leftrightarrow q\bar{q}$, $aq \leftrightarrow gq$

Late stages Axion interaction with pions and nucleons (relevant after quark-hadron transition)

- Initially Turner considered only $aq \leftrightarrow \gamma q$ to conclude that for thermal axions to be more than non-thermal ones

$f_A < 4 \times 10^{-8}$ GeV - in conflict with experimental bounds/limits

- But when all the processes listed are considered $f_A < 1.2 \times 10^{-12}$ GeV
- The reason for such a big difference is the effect of colour and flavour factors and the increase in T dependence from other processes
- Axions are also possible Dark Matter candidates Depending on the dominant production process & time/temp. of freeze-out, they can form substantial amount of Cold Dark Matter

Axions & Isocurvature perturbations

• Usual density perturbations are the adiabatic/curvature pert. ie the number density does not change but it is fluctuation in the energy density

• Isocurvature (entropic) pert. arise due to local change in the EOS or the number density, such that the total energy density is unperturbed

• Isocurvature pert. require multiple fields present

• Axions produced via non-thermal mechanisms can induce large isocurvature pert. ie tight constraints can be obtained almost ruling out axion

• However, in most analyses in literature, the careful treatment is missing and thus drawing clear and conclusive inference is not easy

• Also for multiple fields, treatment of interactions in deSitter brings issues of QFT in curved spacetime which are overlooked

Analogy with pions

• Being pseudoscalars, axions and pions have similar properties $\Longrightarrow$ General structure of Results obtained in case of pions can be borrowed

• Pion-photon-photon coupling is provided by the Axial Anomaly term - the axion coupling to photons/gluons also has the same form

• $\mathcal{L}_{\pi\gamma\gamma} = \left(\frac{e^2 N_c}{48\pi^2}\right)\frac{1}{f_\pi}\pi F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{\pi\gamma\gamma}\pi F_{\mu\nu}\tilde{F}^{\mu\nu}$

• It is known that the coefficient of axial anomaly is independent of temp. BUT the amplitude for $\pi^0 \rightarrow \gamma\gamma$ is modified

• For $T << f_\pi$, one finds the following results

$$f_\pi(T) = \left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2}\right) f_\pi \qquad \text{Decreases}$$

$$m_\pi(T) = \left(1 + \frac{1}{6}\frac{T^2}{f_\pi^2}\right) m_\pi \qquad \text{Increases}$$

• Expect from the zero temp. case that $g_{\pi\gamma\gamma}(T) \sim 1/f_\pi(T)$ ie the decay amplitude to be enhanced compared to the zero temp. case

BUT

• Heat bath provides a preferred frame and therefore there are additional contributions to the tensor decomposition of amplitude in comparison with the zero temp. result

• The amplitude therefore changes and the result is

$$g_{\pi\gamma\gamma}(T) = \left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2}\right) g_{\pi\gamma\gamma}$$

• At the effective Lagrangian level, there is an extra non-local contribution at non-zero temp. which is responsible for the modification

• Adler-Bardeen theorem remains valid but "Sutherland-Veltman" theorem (after including the anomaly part) fails at non-zero temp.

• We use the above forms for the axion case as well and study the effects due to these modifications

Putting it all together

• We consider the modifications due to temp. as described above

• Note that these do not exhaust all the temp. dependent modifications

• Also in the late time era, we correct for the pion mass but leave axion mass uncorrected - can be justified as the corrections are insignificant

• For example, we ignore here the temp. dependent corrections arising due to thermal loops for other couplings – should be eventually considered

• In the present case, the matrix elements therefore get modified by the overall factors as described above

• In practical cosmology calculations, it is convenient to scale out the effect of expansion – consider the evolution of number of particles in comoving volume

• Define a new variable $Y \equiv \frac{n}{s}$ (s is the entropy density)

• Also define another variable $x \equiv \frac{M}{T}$ (M is the relevant scale. Here take it as f_A)

• The Boltzman eq. take the form

$x\frac{dY}{dx} = \frac{\Gamma}{H}(Y^{eq} - Y)$ (Γ is the thermally av. rate)

•In the radiation dominated era, $H = \left(\frac{4\pi^3 g_{eff}}{45}\right)^{1/2} \frac{T^2}{M_P}$ (g_{eff} is the effective DOF at temp. T)

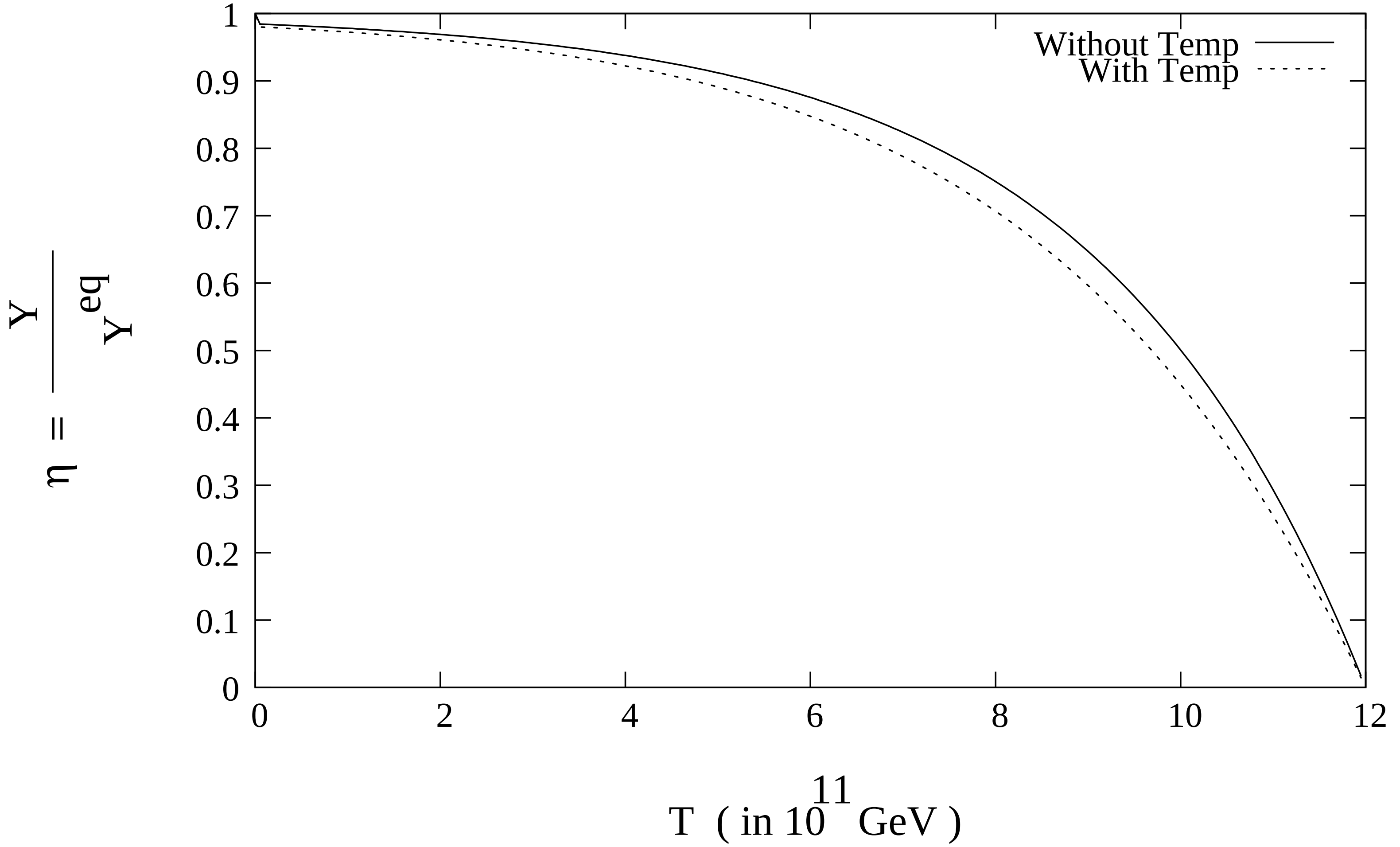
$$\Gamma \simeq 7.1 \times 10^{-6} \frac{T^3}{f_a^2} \left(1 - \frac{T^2}{12 f_a^2}\right)^2 = \Gamma_0 \left(1 - \frac{T^2}{12 f_a^2}\right)^2$$

• In terms of variables $\eta = \frac{Y}{Y^{eq}}$ and $k = x\frac{\Gamma_0}{H}$,

$$x^2 \frac{d\eta}{dx} = k\left(1 - \frac{1}{12x^2}\right)^2 (1 - \eta) \quad \Longrightarrow \eta(x) = 1 + C\; exp\left[\frac{k}{x} - \frac{k}{18x^3}\right]$$

C determined by boundary condition, $\eta = 0$ for $\frac{f_a(T)}{T} = 1$.

In the present case, bdry. conditions is at $\eta = 0$ at $x \sim 1.08$ instead of at $x = 1$ (a difference of $\mathcal{O}(10\%)$)



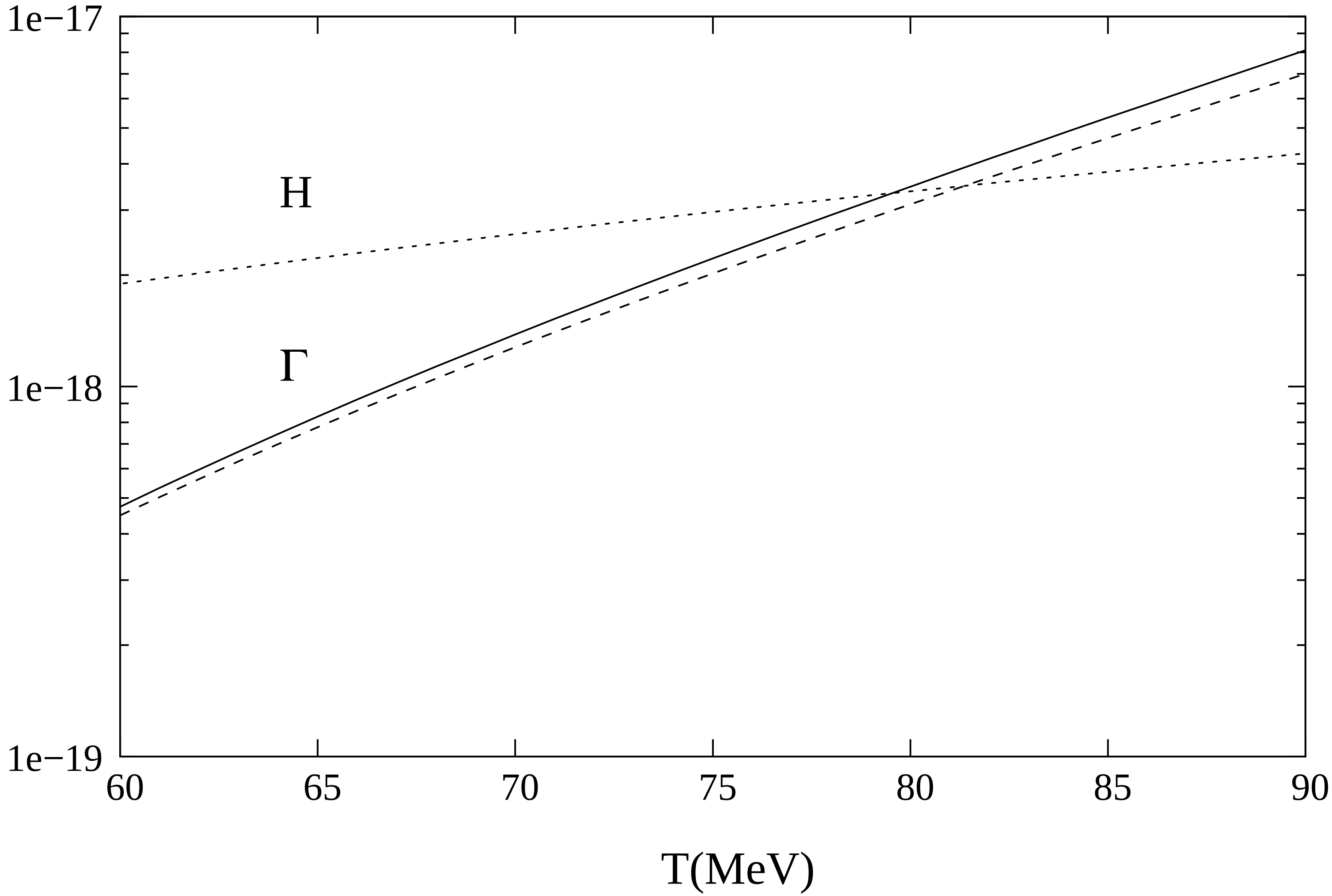
Abundance of axions for $f_a = 1.2 \times 10^{12}$ GeV – Change in the abundance for the temperature range $(6 - 10) \times 10^{11}$ GeV

Hadronic axion in the post QCD era

- The axion-pion interaction is of the form $(C_{A\pi} = \frac{1-z}{3(1+z)}; \, z = m_u/m_d = 0.56)$

$$L_{A\pi} = \frac{C_{A\pi}}{f_\pi f_A} \left(\pi^0\pi^+\partial_\mu\pi^- + \pi^0\pi^-\partial_\mu\pi^+ - 2\pi^+\pi^-\partial_\mu\pi^0\right)\partial_\mu A$$

- Relevant processes: $a\pi^\pm \to \pi^0\pi^\pm$ and $a\pi^0 \to \pi^+\pi^-$
- Nucleons being heavy and non-rel. have very low densities and therefore do not significantly affect the rates
- Here temp. is much much lower than f_A - so no need to include corrections to it
- f_π decreases, m_π increases with temperature – Int. rate increases
- Therefore, the axions decouple later in time or at lower temperatures



Hadronic axion reaction rate with(solid) and without(dashed) temperature effect for $f_A = 10^7$ GeV

f_A(GeV)	T_{D1}(MeV)	T_{D2}(MeV)
3 $\times 10^5$	26.43	26.43
1 $\times 10^6$	35.34	35.34
3 $\times 10^6$	49.84	49.5
1 $\times 10^7$	81.04	79.12
1.2 $\times 10^7$	87.61	85.48
1.3 $\times 10^7$	90.1	87.9

Decoupling temperature of axions T_{D1} (without) and T_{D2} (with) temperature effects for different values of f_A – note that the deviation starts becoming clear as $T \longrightarrow f_\pi$ – the approx. breaks here

Summary and main points

• Some very specific finite temp. effects considered in context of axion cosmology

• In both the interesting eras of axion cosmology, there are perceptible deviations

• However, the approx. breaks down for temp. where the effects are large

• More work required to go beyond the present approx.

• We have not considered some of the other possible corrections - can turn out to be important

• Recall from experience with neutrino decoupling that a shift from $(1.2 \rightarrow 1.4)$ MeV is seen by BBN

- As cosmological obs. get more and more precise, 10% change will not be tolerated
- Already, with the above changes, f_A for thermal axions in early universe goes down – allowed window squeezes, though by a small amount only
- Effect of chiral symmetry breaking corrections can also turn out to be imp.
- However, finite temp. effects of the type discussed will still be there
- Imp. to have a very precise estimate of axion density at various epochs for the issue of isocurvature perturbations as well
- Almost all models beyond SM bring along axions - so its imp. to have a clear picture