De Sitter, Lorentzian, and Noncommutative Geometries *in Quantum Relativity*

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Symmetry of "Space-Time" : -

Galilean Relativity

Einstein Relativity — special and general

Special Quantum Relativity

***or* Doubly (Triply) / Deformed Special Relativity**

······ General Quantum Relativity $\Longrightarrow$ Quantum Gravity +

Fundamental Constants —

$$c\,, \qquad \hbar\,, \qquad G\ \ (\ell_{\rm Pl} \text{ or } M_{\rm Pl})\,, \qquad \Lambda\ \ (\ell_C)\,.$$

What is the **Planck Scale** ?

$$\Delta M(L) = \frac{\hbar}{c\,L}$$

— there is a maximum (Schwarzchild) mass for L, $\frac{c^2\,L}{G}$

Set the two equal $\longrightarrow$ $\ell_{\rm Pl}$

...... when measured in different reference frames ?

special (Einstein) relativity makes c frame independent

quantum relativity makes $\ell_{\rm Pl}$ or $M_{\rm Pl}$, (also ℓ_c) frame independent

Bibliographical Background :-

H. Synder — PR 71 (1947), 38

also C.N. Yang — PR 72 (1947), 874

G. Amelino-Camelia — *since* PL B510 (2001), 255

L. Smolin (& Magueijo) — PRL 88 (2002), 190403 + ...

J. Kowalski-Glikman — LNP 669 (2004), 131 + ...

Girelli & Livine — gr-qc/0412004, gr-qc/0407098

E. Witten / A. Strominger — quantum de-Sitter gravity/duality

H.Y. Guo *et.al.* — de Sitter special relativity

My Basic Perspectives :-
— on issues related to Quantum Space-Time

- need new understanding of "Space-Time" (relativity principle)
- within Space-Time Geometry

  Non-Commutative Geometry *is to* Quantum Gravity
  as Non-Eucliean Geometry *is to* (Einstein) Gravity
- I don't share the *faith (Penrose)* — Quantum Field Theory, on (Minkowski/Einstein) Space-Time, perspective would be valid way beyond the experimentally probed energy/length scale

— Linear (Vs Nonlinear) Realization

- an exploration into **new** ways to think about fundamental physics
- withhold phenomenological discussions

A Stable Symmetry :-

R.V. Mendes (1994)
Chryssomalakos & Okon (2004)

- perturbations (deformations) of a symmetry algebra
- stable if deformed algebras isomorphic to original

— structure constants perturbations (experimental errors)

- Examples —

  1/ Galilean to Einstein Relativity ($1/c^2 \neq 0$)

  2/ classical to quantum mechanics ($\hbar \neq 0$)

  Poisson bracket $\Longrightarrow$ Moyal bracket

- deformation parameters $\longrightarrow$ fundamental constants

Space-Time + Quantum Symmetry

‘Poincaré + Heisenberg’ algebra $\Longrightarrow$ SO(1,5)

Quantum Relativity $SO(1,5)$:-

$$[J_{AB}\,, J_{LN}] = i\,(\eta_{BL}J_{AN} - \eta_{AL}J_{BN} + \eta_{AN}J_{BL} - \eta_{BN}J_{AL})$$

with $\eta^{AB} = (1,-1,-1,-1,-1,-1)$.

- linear realization on 6-geometry — $J_{\mathcal{AB}} = i\,(x_{\mathcal{A}}\partial_{\mathcal{B}} - x_{\mathcal{B}}\,\partial_{\mathcal{A}})$
- J_{AB} for $0 \to 3$ are the 10 generator for Lorentz symmetry $M_{\mu\nu}$
- NOT necessarily means extra space-time dimensions
- 3 deformations — $\boldsymbol{ISO(3)} \;\longrightarrow\; \boldsymbol{SO(1,3)} \;\hookrightarrow\; \boldsymbol{ISO(1,3)}$ $\;\longrightarrow\; \boldsymbol{SO(1,4)} \;\hookrightarrow\; \boldsymbol{ISO(1,4)} \;\longrightarrow\; \boldsymbol{SO(1,5)}$
- 3 invariant quantities : $c \quad \kappa \quad \ell$ (triply SR)
- curved momentum space $\Longrightarrow$ non-commutative space-time

  $+$ curved space-time $\Longrightarrow$ non-commutative momentum space

The Three Deformations :-

$\Delta x^i(t) = v^i \cdot t$	$\Delta x^\mu(\sigma) = p^\mu \cdot \sigma$	$\Delta x^A(\rho) = z^A \cdot \rho$
$\lvert v^i \rvert \le c$ $-\eta_{ij} v^i v^j = c^2\left(1 - \frac{1}{\gamma^2}\right)$	$\lvert p^\mu \rvert \le \kappa\, c$ $\eta_{\mu\nu} p^\mu p^\nu = \kappa^2 c^2\left(1 - \frac{1}{\Gamma^2}\right)$	$\lvert z^A \rvert \le \ell$ $\eta_{AB}\, z^A z^B = \ell^2\left(1 - \frac{1}{G^2}\right)$
$M_{0i} \equiv N_i \sim P_i$ $[N_i, N_j] \longrightarrow -i\, M_{ij}$	$J_{\mu 4} \equiv O_\mu \sim P_\mu$ $[O_\mu, O_\nu] \longrightarrow i\, M_{\mu\nu}$	$J_{A5} \equiv O'_A \sim P_A$ $[O'_A, O'_B] \longrightarrow i\, J_{AB}$
$\vec{u}^4 = \frac{\gamma}{c}(c, v^i)$ $\eta_{\mu\nu} u^\mu u^\nu = 1$ $\mathbb{R}^3 \rightarrow SO(1,3)/SO(3)$	$\vec{\pi}^5 = \frac{\Gamma}{\kappa\, c}(p^\mu, \kappa\, c)$ $\eta_{AB}\pi^A \pi^B = -1$ $\mathbb{R}^4 \rightarrow SO(1,4)/SO(1,3)$	$\vec{X}^6 = \frac{G}{\ell}(z^A, \ell)$ $\eta_{\mathcal{MN}} X^{\mathcal{M}} X^{\mathcal{N}} = -1$ $\mathbb{R}^5 \rightarrow SO(1,5)/SO(1,4)$

- 4-momentum is *defined by* $p^\mu = \frac{dx^\mu}{d\sigma}$
- $z^A = \frac{dx^\mu}{d\rho}$ is *chosen as* a length
- without the p^μ deformation, $X^{\mathcal{M}}$ are just alternative coordinates of a de-Sitter space-time
- no $\hookrightarrow ISO(1,5)$: naive *translations* no longer symmetries

dS_5 as Hypersurface in Minkowski 6-Geometry :-

$x^{\mathcal{M}} \qquad (\mathcal{M} = 0, 1, 2, 3, 4, 5)$

- $x^0 = c\,t$
- $x^4 = \kappa\, c\, \sigma$ —$\sigma \sim \frac{[\text{time}]}{[\text{mass}]}$
- $x^5 = \ell\, \rho$ — ρ is a pure number

dS_5 : $\quad \eta_{\mathcal{MN}} x^{\mathcal{M}} x^{\mathcal{N}} = -\ell^2$

$$x^A = \ell \omega^A \sinh\zeta \; , \qquad x^5 = \ell \cosh\zeta \; , \qquad \eta_{AB}\omega^A\omega^B = 1$$

- $G = \cosh\zeta \; , \qquad z^A = \ell\omega^A \tanh\zeta \; , \qquad \gamma^A = \dfrac{z^A}{\ell} = \omega^A \tanh\zeta$
- Beltrami (5-)coordinates — $z^A = \ell \, \frac{x^A}{x^5}$

  on the Beltrami patch $x^5 = \ell G > 0 \qquad G^2(1-\gamma^2) = 1$

Lorentzian 5-vectors :-

- the 5D metric $g_{AB} = G^2 \eta_{AB} + \frac{G^4}{\ell^2} \eta_{AC} \eta_{BD} z^C z^D$
- Lorentzian 5-coordinate $Z_A^{(\mathcal{L})} = G^{-4} z_A = \eta_{AB} z^B$

- Lorentzian 5-momentum $[Z_A^{(\mathcal{L})}, q_B] = -i\,\eta_{AB}$, $(q_A \equiv i \frac{\partial}{\partial z^A})$

$$P_A^{(\mathcal{L})} = q_A - Z_A^{(\mathcal{L})} \frac{1}{\ell^2} \left(\eta^{BC} Z_B^{(\mathcal{L})} q_C \right)$$

- quantum relativity algebra —

$$J_{\mathcal{M}\mathcal{N}} = Z_{\mathcal{M}}^{(\mathcal{L})} P_{\mathcal{N}}^{(\mathcal{L})} - Z_{\mathcal{N}}^{(\mathcal{L})} P_{\mathcal{M}}^{(\mathcal{L})}$$

$(P_5^{(\mathcal{L})} \equiv 0)$ $\longrightarrow$ $J_{A5} = \ell\, P_A^{(\mathcal{L})}$ *noncommutative* !

$\longrightarrow$ $q_5 = -\frac{1}{\ell} \left(\eta^{BC} Z_B^{(\mathcal{L})} q_C \right)$ — scale transformation generator

Non-commutative (Space-Time) Geometry :-

cf. Kowalski-Glikman & Smolin
Chryssomalakos & Okon

- (four) space-time position operators $\hat{X}_\mu = -\frac{1}{\kappa c}\, i\,(x_\mu \partial_4 - x_4\, \partial_\mu)$

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{\kappa^2 c^2} M_{\mu\nu}$$

- (four) energy-momentum operators $\hat{P}_\mu = \frac{1}{\ell}\, i\,(x_\mu \partial_5 - x_5\, \partial_\mu)$

$$[\hat{P}_\mu, \hat{P}_\nu] = \frac{i}{\ell^2} M_{\mu\nu}$$

$$[\hat{X}_\mu, \hat{P}_\nu] = -i\,\eta_{\mu\nu} \hat{F}\ , \qquad [\hat{X}_\mu, \hat{F}] = +\frac{i}{\kappa^2 c^2} \hat{P}_\mu\ , \qquad [\hat{P}_\mu, \hat{F}] = -\frac{i}{\ell^2} \hat{X}_\mu$$

- $i\,\partial_\mu \ll \kappa c$ and $i\,\partial_4 = p_4 = -\kappa c$: $\quad \hat{X}_\mu \longrightarrow x_\mu$
- $x_\mu \ll \ell$ and $x_5 = -\ell$ $(\rho = 1)$: $\quad \hat{P}_\mu \longrightarrow i\,\partial_\mu = p_\mu$

De Sitter Momentum Boosts (with $\hat{X}_\mu$) :-

$$J_{\mu 4} = i\,(x_\mu \partial_4 - x_4\,\partial_\mu)$$

— rotations among x_μ's and $x_4 = -\kappa\, c\, \sigma$

⋆ $\boldsymbol{\sigma}$ is peculiar — (*doubt* space-time interpretation !)

- boost charaterized by p^μ, hence $\sim$ mass (quantum frames)
- $p^\mu = \frac{dx^\mu}{d\sigma}$ Vs $m\,c\,u^\mu = \gamma\, m\, c\, \frac{dx^\mu}{dx^0}$ (as Einstein/classical limit)
  $\sigma \longrightarrow \frac{\tau}{m}$ for an Einstein particle
- general transformation — on-shell condition not preserved
  classical to quantum frame, uncertainty : observer to observed
- preferred frame for a quantum state : $\vec{\pi}^{\,5} = (0, 0, 0, 0, 1)$
  the reference frame does not see its own 4-momentum/mass

Quantum Frame of Reference :-

Aharonov & Kaufherr, PR 30, 368
C. Rovelli, Class.Quantum.Grav. 8, 317

- context: nonrelativistic QM / gravitation
- characterized by *mass* (together with $v + \cdots$)
- interaction of frame (observer/device) with observed

— observation has nontrivial effect on observer

- uncertainty principle on frame (observer/device)

  $\longrightarrow$ uncertainty on observed

- grav. — local gauge inv. observables obtained after dynamic properties of frame (observer/device) taken into consideration

⋆ *Is there such a thing as a practical quantum measurement ?*

De Sitter Translational Boosts (with $P_a^{(\mathcal{L})}$) :-

$$J_{A5} = i\,(x_A \partial_5 - x_5\, \partial_A)$$

— rotations among x_A's and $x_5 = -\ell\,\rho$

- ρ may characterize the scale
- boost charaterized by z^A, a location vector

  sort of a translation
- preferred frame for a state : $\vec{X}^6 = (0,0,0,0,0,1)$

  the reference frame does not see its own location

  — $(0,0,0,0,0,1)$ characterizes the coordinate origin $z^A = 0$

★ **a reference frame does not observe itself** (no v^i, p^μ, $z^A = 0$)

- Beltrami description *preserves* physics of 5-momentum constraint

De Sitter, Lorentzian, Noncommutative, ...

- Beltrami (5-)coordinates give geodesic as inertial motion
- equivalent to $\frac{dP^A_{(\mathcal{L})}}{ds} = 0$ or $G^2 \frac{dz^A}{ds} = 0$
- $P^{(\mathcal{L})}_A = q_A - Z^{(\mathcal{L})}_A \frac{1}{\ell^2} \left(\eta^{BC} Z^{(\mathcal{L})}_B q_C\right) = \frac{1}{\ell} (x_A p_5 - x_5 p_A)$

$$\Longrightarrow \quad \hat{P}_\mu = P^{(\mathcal{L})}_\mu \quad \text{and} \quad \hat{F} = -i\,\eta_{\mu\nu} \left(-\frac{1}{\kappa\, c} P^{(\mathcal{L})}_4\right)$$

⋆ **dS$_5$ in 6D — linear realization of quantum relativity**

- **the 6th coordinate like a scale**
- **Beltrami 5-coordinate having some Lorentzian structure**
- **noncommutative “4D” description**

Remarks :-

- *(bottom line)* an interesting, radical but sensible, approach
- primitive stage — difficult to make (and identify) progress
- need creative but careful thinking about 'physics' *beyond* usual framework (*e.g.* new casuality-like conditions)
- job for Einstein — hope we can make minor steps

" The chief cause of my failure was my clinging to the idea that the variable t only can be considered as the true time and my local t' must ······ In Einstein's theory, ······ t' plays the same part as t ······."

The Theory of Electrons (1916 ed.) — Lorentz (*from* A.I. Miller)

THANK YOU !