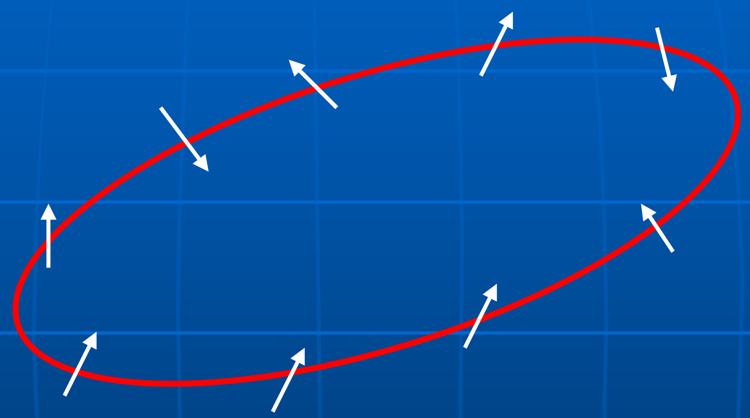
Spin chain from marginally deformed $AdS_3 \times S_3$



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hep-th/0610147

hep-th/0702xxx

string/spin chain correspondence

classical spinning string on S3

FAST STRING LIMIT

M. Kruczenski, Phys. Rev. Lett. 93

SU(2) Heisenberg spin chain $XXX_{1/2}$ Landau-Lifshitz equation

AdS_3/SL(2) S.Bellucci, P.Y.Casteill, J.F.Morales and C.Sochichiu, Nucl. Phys. B 707

SU(3) R.~Hernandez and E.~Lopez, JHEP 0404

string/spin chain correspondence

classical spinning string on deformed S3

FAST STRING LIMIT
+
SMALL DEFORMATION LIMIT

anisotropic SU(2) Heisenberg spin chain $XXY_{1/2}$

generalized Landau-Lifshitz equation

TARGET SPACE

- start with $AdS_5 \times S_5$ background

$$ds^2 = \frac{k}{4}\left(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_3\right)$$

$$+\frac{k}{4}\left(d\theta^2 + \sin^2 \theta d\varphi^2 + \cos^2 \theta d\tilde{\Omega}_3\right)$$

- marginally deformed S_3 from WZW sigma model + marginal operator

  D.Israel, C.Kounnas, D.Orlando and P.M.Petropoulos, Fortsch. Phys. 53

- 1-parameter deformed U(1) fiber in Hopf fibration of S_3.

$S_3 \xleftarrow{\;H\;} U(1)$

$S_3 \downarrow S_2$

- to be explicit,

$$d\tilde{\Omega}_3 = d\beta^2 + \sin^2 \beta d\alpha^2 + (1-2H^2)(d\gamma + \cos \beta d\alpha)^2$$

S_2 ($d\beta^2 + \sin^2 \beta d\alpha^2$), U(1) ($(d\gamma + \cos \beta d\alpha)^2$)

STRING WORLDSHEET

- embedding $\rho = \theta = 0 \quad \alpha(\tau,\sigma) \;\; \beta(\tau,\sigma) \;\; \gamma(\tau,\sigma)$
- gauge fixing $t = \kappa\tau$
- spinning $\alpha \to \alpha + t$
- induced Polyakov action

$$S = \frac{\sqrt{\lambda}k}{16\pi}\iint d\tau d\sigma - 2H^2\cos^2\beta\kappa^2 - (1-2H^2\cos^2\beta)\alpha'^2 - \beta'^2 - (1-2H^2)\gamma'^2$$
$$-2(1-2H^2)\cos\beta\alpha'\gamma'$$
$$+2(1-2H^2\cos^2\beta)\kappa\dot{\alpha} + 2(1-2H^2)\cos\beta\kappa\dot{\gamma}$$
$$+(1-2H^2\cos^2\beta)\dot{\alpha}^2 + \dot{\beta}^2 + (1-2H^2)\dot{\gamma}^2 + 2(1-2H^2)\cos\beta\dot{\alpha}\dot{\gamma}$$

- FAST STRING LIMIT $\kappa \to \infty \quad \dot{X}^{\mu} \to 0 \quad \kappa X^{\mu'}$ finite

STRING WORLDSHEET

- virasora constraints

$$2(1-2H^2\cos^2\beta)\kappa\alpha' + 2(1-2H^2)\cos\beta\kappa\gamma' = 0$$

- resulting action

$$S = \frac{\sqrt{\lambda}k}{16\pi}\iint d\tau d\sigma \left[-2H^2\cos^2\beta\kappa^2 - \beta'^2 - \Delta_H(\beta)\gamma'^2\right.$$
$$\left. + 2(1-2H^2\cos^2\beta)\kappa\dot{\alpha} + 2(1-2H^2)\cos\beta\kappa\dot{\gamma}\right]$$

$$\Delta_H(\beta) \equiv \frac{(1-2H^2)\sin^2\beta}{1-2H^2\cos^2\beta}$$

Hamiltonian density

conjugated angular momentum density

H=0

- no deformation

$$\Delta_H = \sin^2\beta$$

- Hamiltonian density

$$H = \beta'^2 + \sin^2\beta\gamma'^2 = \vec{S}'\cdot\vec{S}'$$

$$\vec{S} = (\sin\beta\cos\gamma, \sin\beta\sin\gamma, \cos\beta)$$

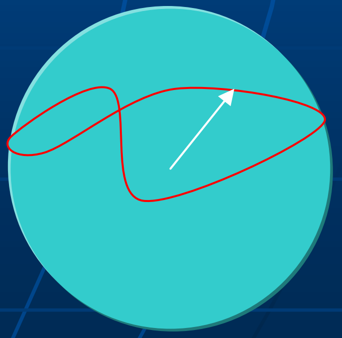
- conjugate angular momentum

$$\pi_\alpha = \frac{\sqrt{\lambda}k}{8\pi}\kappa\int d\sigma = J_0$$

length of spin chain

$$\pi_\gamma = \frac{\sqrt{\lambda}k}{8\pi}\kappa\int d\sigma\cos\beta$$

Wess-Zumino term



$H \neq 0$

- Hamiltonian density

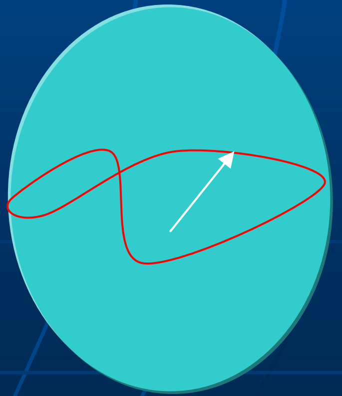
$$H = \beta'^2 + \Delta_H(\beta)\gamma'^2 + 2\kappa^2 H^2 \cos^2\beta \approx \vec{\tilde{S}}' \cdot \vec{\tilde{S}}' + (H \cdot \tilde{S}_z)^2$$

$$\vec{\tilde{S}} = (\sqrt{\Delta_H}\cos\gamma, \sqrt{\Delta_H}\sin\gamma, \tilde{S}_z)$$

satisfying uniform speed condition $(\sqrt{\Delta_H}')^2 + (\tilde{S}_z')^2 = 1$

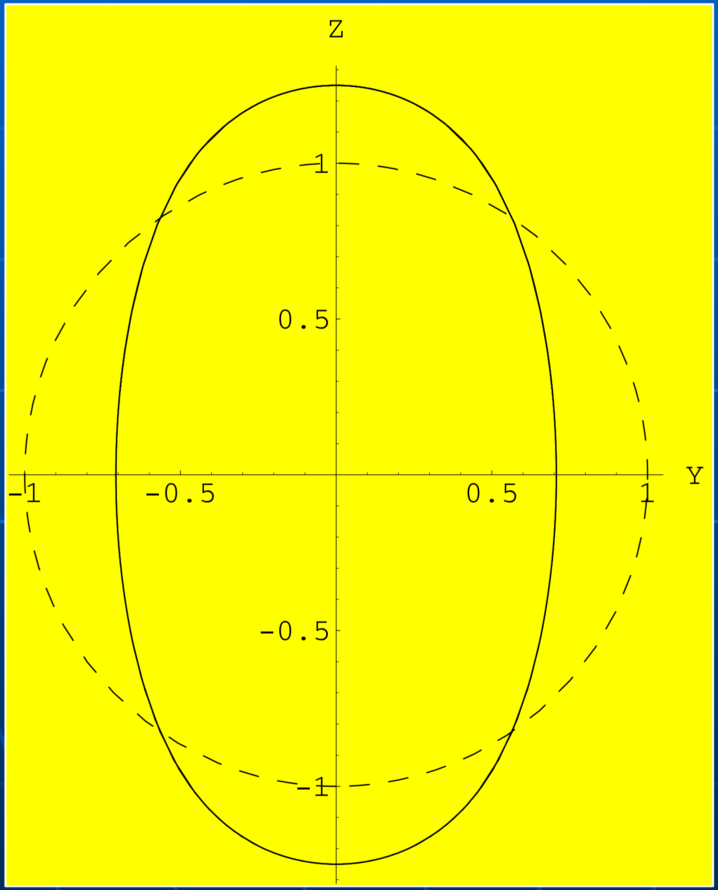
- conjugate angular momentum

$$J = J_0 + 2\kappa H^2 \int d\sigma \cos^2\beta$$

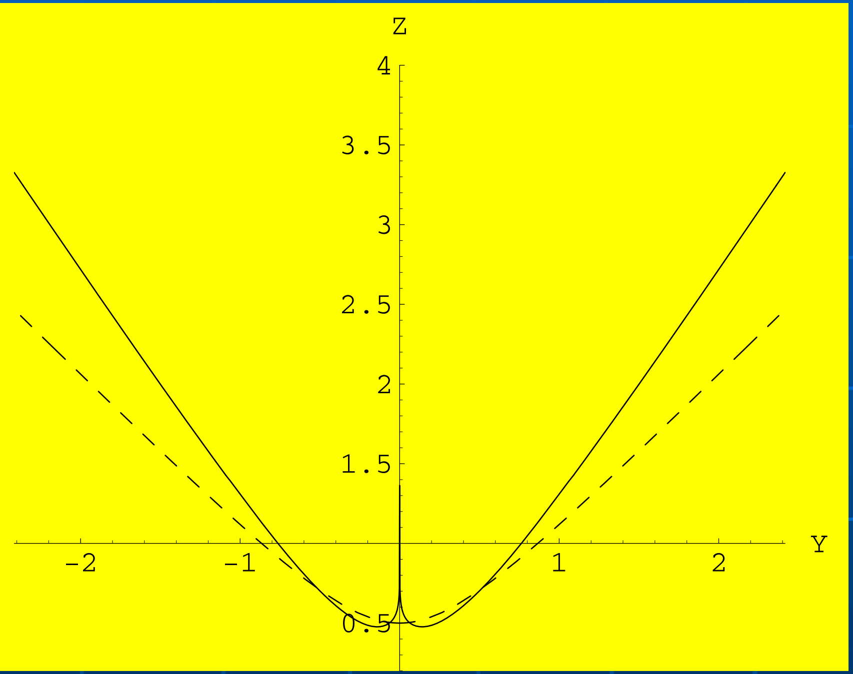


• no analytic solution for S were found in deformed background

deformed S3



hyperbolic deformed AdS3



SMALL DEFORMATION LIMIT

- to render analytic solution for S, but still keep nontrivial interaction term

$$H \to 0 \quad \kappa H \ \text{finite}$$

- the resulting action

$$S = \frac{\sqrt{\lambda}k}{16\pi}\iint d\tau d\sigma - 2H^2\cos^2\beta\kappa^2 - \beta'^2 - \sin^2\beta\gamma'^2 + 2\kappa\dot{\alpha} + 2\cos\beta\kappa\dot{\gamma}$$

- Hamiltonian density indicates the anisotropic spin chain system

$$H = \vec{S}' \cdot \vec{S}' + 2H^2 S_z^{\ 2}$$

Landau-Lifshitz equation

- equation of motion

$$\beta'' - \sin\beta\,\dot{\gamma} - \sin\beta\cos\beta(\gamma')^2 + H^2\sin 2\beta = 0$$

$$\sin\beta\,\dot{\beta} + (\sin^2\beta\,\gamma')' = 0$$

can also be written as (generalized) Landau-Lifshitz equation

$$\partial_t \vec{S} = \vec{S}\times\partial_\sigma^2\vec{S} + \vec{S}\times J\vec{S}$$

$$J = \begin{pmatrix} j_1 & & \\ & j_2 & \\ & & j_3 \end{pmatrix}$$

with

$$j_1 = j_2 = 1$$

$$j_3 = 1 - 2H^2$$

Conclusion

- Landau-Lifshitz equation is an integral system, which implies at least a sector of string theory in AdS3xS3 is also integrable.
- In principle, we may work out small deformation limit of deformed AdS_3 and explicitly show there is also an noncompact version of LL equation.
- We may also work backwards, to find out which spinning string solution corresponds to most general LL equation with all j's nontrivial.
- If we like, we may also include the interaction term which is linear to Sz.
- It is interesting to see if similar correspondence exists in higher dimensions.

Thank You