SUSY breaking by a metastable ground state: Why the early Universe preferred the non-supersymmetric vacuum

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base on collaboration with Steven A. Abel, Joerg Jaeckel and Valentin V. Khoze, hep-th/0610334

also closely related works:

N. J. Craig, P. J. Fox, J. G. Wacker, Reheating Metastable O'Raifeartaigh Models, hep-th/0611006

W. Fischler, V. Kaplunovsky, C. Krishnan, L. Mannelli, M. Torres, Meta-Stable Supersymmetry Breaking in a Cooling Universe, hep-th/0611018.

S. A. Abel, J. Jaeckel, V. V. Khoze, Why the early Universe preferred the non-supersymmetric vacuum: Part II. hep-th/0611130.

Outline

- Basics of SUSY gauge theory
- Review of ISS model: zero temperature
- Dynamical evoluion at finite temperature
- Summary and discussions

I. Basics of SUSY gauge theory

SUSY gauge theory, SQCD and Seiberg duality

- $\mathcal{N} = 1$ superfields: V vector superfield, gauge group G φ chiral superfields in reps r or the gauge group (with generators T^a), the Lagrangian for the $\mathcal{N} = 1$ supersymmetric gauge theory is given by

$$\mathcal{L} = \int d^4\theta K(\varphi^\dagger, e^{V\cdot T}\varphi) + \frac{1}{g^2}\int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + H.c. + \int d^2\theta W(\varphi) + H.c.$$

- Kahler potential K is a real function. It gives the kinetic terms for the matter fields, and gauge interactions for the chiral superfields.
- The superpotential W is a holomorphic function of the chiral superfields.
- g^2 in front of the kinetic term for the gauge multiplet is the Wilsonian gauge coupling.

- for the tree level Lagrangian,

$$K = \varphi^\dagger e^{V\cdot T}\varphi,$$

W has up to cubic terms in a renormalizable theory.

- Nonrenormalization theorem

- the superpotential is not renormalized in perturbation theory. Moreover using holomorphy, symmetries of the theory and known weakly coupled limits, it is often possible to determine the superpotential exactly.

- similarly, the kinetic term for the gauge multiplet is also a holomorphic function, thus one has exact result of the renormalization of the Wilsonian gauge coupling. (Novikov-Shifman-Vainshtein-Zakharov β function).

- the Kahler potential receives both perturbative and non-perturbative corections.

SQCD

- Consider $SU(N_c)$ SQCD with N_f flavors of quarks Q_f, $\tilde{Q}_g$ in fundamental and anti-fund reps. of the gauge group, $f, g = 1, \cdots, N_f$. And with a tree level superpotential

$$W = m \mathrm{Tr} M, \quad M_{fg} = Q_f \cdot \tilde{Q}_g$$

- The NSVZ (exact) β function is given by

$$\beta(g) = -\frac{g^2}{16\pi^2} \frac{3N_c - N_f + N_f \gamma(g)}{1 - N_c(g^2/8\pi^2)}$$

$$\gamma(g) = -\frac{g^2}{16\pi^2} \frac{N_c^2 - 1}{N_c} + \mathcal{O}(g^4)$$

Thusfor $N_f < 3N_c$, , the theory is asymptotically free and make sense in the UV. But the theory is strongly coupled in the IR.

The interesting thing is that there is a dual description of the IR fixed point in terms of Seiberg duality.

Seiberg duality for $N_c + 1 < N_f < 3N_c$

- The SQCD admits a dual, magnetic description in terms of a SQCD with gauge group $SU(N_f - N_c)$ with N_f flavors of q and $\tilde{q}$ transforming as fundamentals and ant-fundamentals, and a gauge singlet field M, corresponding to the mesons of the original theory. The magnetic theory has a tree-level superpotential

$$W = \tilde{q} M q$$

- The magnetic theory has beta function

$$3(N_f - N_c) - N_f = 2N_f - 3N_c$$

Thus

- for $\frac{3N_c}{2} < N_f < 3N_c$, the magnetic theory is asymptotically free. i.e. both electric and magnetic description make sense in the UV

- for $N_f < \frac{3N_c}{2}$, the magnetic theory is IR free. In particular vacuum structure is realiable in this description.

II. Review of ISS model: zero temperature

ISS model: Lagrangian

(Intriligator-Seiberg-Shih)

- The ISS model is given by the SQCD with fields content:

- $SU(N)$ gauge fields
N_f flavours of chiral superfields φ_i^c and $\tilde{\varphi}_c^i$ transforming in the fundamental and the anti-fundamental representations of the gauge group; $c = 1, \ldots, N$ and $i = 1, \ldots, N_f$.
$N_f \times N_f$ chiral superfield Φ_j^i which is a gauge singlet.

- tree level Kahler potential is

$$K = \mathrm{Tr}_{N_f} \varphi^\dagger e^V \varphi + \mathrm{Tr}_{N_f} \tilde{\varphi}^\dagger e^{-V} \tilde{\varphi} + \mathrm{Tr}_{N_f} \Phi^\dagger \Phi$$

tree level superpotential is

$$W = h \mathrm{Tr}_{N_f} \varphi \Phi \tilde{\varphi} - \mu^2 \mathrm{Tr}_{N_f} \Phi$$

where h, μ are constants.

- The Wilsonian gauge coupling is

$$e^{-8\pi^2/g^2(E)} = \left(\frac{E}{\Lambda_L} \right)^{N_f - 3N}$$

- We will consider

$$N_f > 3N.$$

such that the theory is IR free and strongly coupled in the UV.

Being IR free allows us to have a good control of its low-energy dynamics as well as the vacuum structure. Important for studying SUSY breaking.

However, the theory is not well defined as in the short distance. Not satisfactory as fundamental theory. Can we improve it?

- Fortunately, this theory also admits a dual decription in terms of SQCD with gauge group $SU(N_c)$, $N_c = N_f - N$ and N_f flavors of quarks and anti-quarks. The dual description is precisely UV free but IR divergent (like QCD).

- The two descriptions is complementary!

- The original SQCD is a $macroscopic$ description. It is IR free and strongled coupled in UV.

- The dual SQCD is the opposite, i.e. UV free and strongly coupled in the IR. It is a $microcospic$ description.

We will not need anything of the dual description except knowing that it is there and provides a UV completion.

ISS model: Vacuum structure

- SUSY is spontaneously broken at tree level due to a *rank condition*: the F-term of Φ

$$F_{\Phi_{ij}} \sim \tilde{\varphi}^{jc}\varphi^i_c - \mu^2\delta^{ij}$$

cannot all vanish, because δ^{ij} has rank N_f, but $\tilde{\varphi}^{jc}\varphi^i_c$ only has rank $N < N_f$.

- The scalar potential is minimized at

$$\langle\varphi\rangle = \langle\tilde{\varphi}^T\rangle = \mu \begin{pmatrix} \mathbb{1}_N \\ 0_{N_f-N} \end{pmatrix} \; , \quad \langle\Phi\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi'_0 \end{pmatrix}$$

where Φ'_0 is an arbitry $(N_f - N) \times (N_f - N)$ matrix. The corresponding vacuum energy is

$$V_+ = (N_f - N)|h^2\mu^4|$$

meaning SUSY is broken.

Let us call this vacuum $|\mathrm{vac}\rangle_+$

- Analysis of mass spectrum gives:

- Moduli fields (Goldstone bosons) are massless
most fields has tree level masses $\sim |h\mu|$
pseudo-moduli fields receive mass at one-loop of order $|h^2\mu|$

- In particular the vacuum is non-tachyonic and is a stable one.

- A puzzle: The UV completion of the theory is provided by another SQCD with gauge group $SU(N_c)$, where $N_c = N_f - N$ (Seiberg duality).

This SQCD allows us to compute the Witten index reliably, giving N_c supersymmetric vacua.

Q. Where are they?

- Could it be that SUSY is restored by loops?

No. The usual holomorphicity arguments imply that the superpotential receives no corrections in perturbation theory. Thus SUSY is broken in to all order in perturbation theory.

- Could it be that SUSY is restored by nonperturbatively?

Yes!

Resolution: there is a non-perturbative contribution to the full superpotential of the theory,

$$W = W_{\rm cl} + W_{\rm ADS},$$

where the Affleck-Dine-Seiberg superpotential is

$$W_{\rm ADS} = N \left(h^{N_f} \frac{\det_{N_f} \Phi}{\Lambda_L^{N_f - 3N}} \right)^{\frac{1}{N}}$$

This dynamical superpotential is exact, its form is uniquely determined by the symmetries of the theory and it is generated by instanton-like configurations.

ISS model: Nonperturbative restoration of SUSY

• The F-term constraints for φ and $\tilde{\varphi}$ can be satisfied with $\varphi = \tilde{\varphi} = 0$. This also solve the D-term constraints for $\varphi, \tilde{\varphi}, \Phi$. Thus we only need to look at Φ. Including the ADS superpotential,

$$W = N \left(h^{N_f} \frac{\det_{N_f} \Phi}{\Lambda_L^{N_f - 3N}} \right)^{\frac{1}{N}} - h\mu^2 \mathrm{Tr}\Phi.$$

The scalar potential is extermized at

$$\Phi = \mu\gamma_0,$$

where

$$\gamma_0 := \frac{1}{h\epsilon^{\frac{N_f-3N}{N_f-N}}} , \quad \text{and} \quad \epsilon := \frac{\mu}{\Lambda_L} \ll 1$$

• Thus we get the SUSY preserving vacuum $|\mathrm{vac}\rangle_0$

$$\langle\varphi\rangle = \langle\tilde{\varphi}^T\rangle = 0 , \quad \langle\Phi\rangle = \mu\gamma_0 \, \mathbb{1}_{N_f} , \qquad V_0 = 0$$

There are $N_f - N$ of them, differing from each other by the phase factor $e^{2\pi i N/(N_f - N)}$.

ISS model: Metastable SUSY breaking vacuum

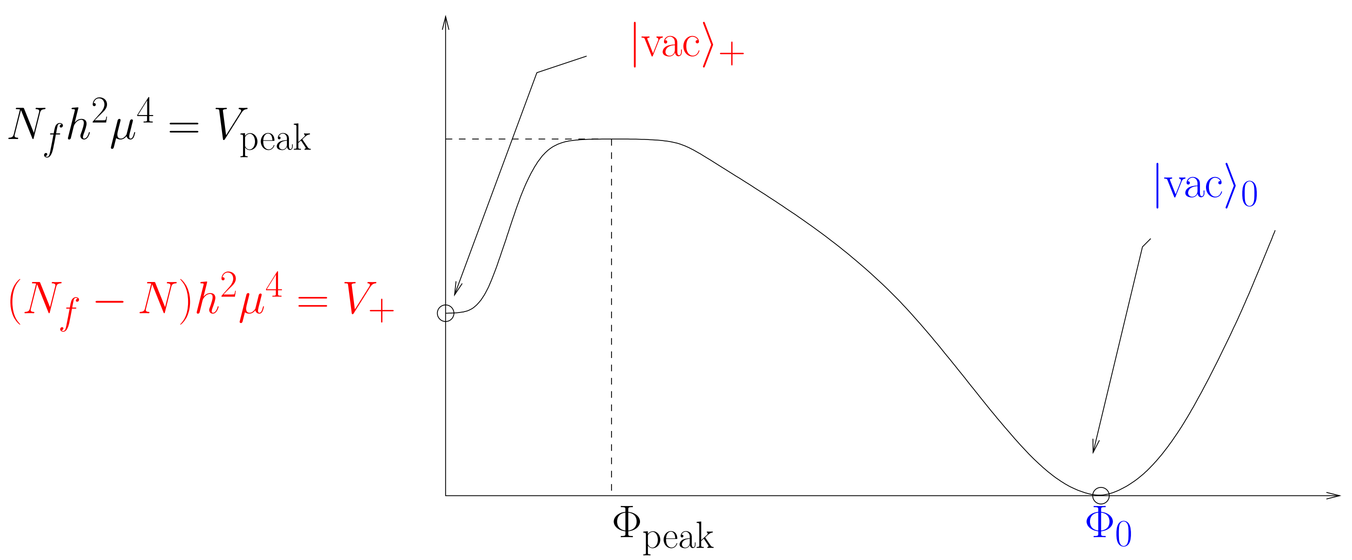
- Thus we get two set of vacua in the ISS model. The vacuum

$$|\mathrm{vac}\rangle_0 : \qquad \langle\varphi\rangle = \langle\tilde{\varphi}^T\rangle = 0 \ , \quad \langle\Phi\rangle = \Phi_0 = \mu\gamma_0 \, \mathbb{1}_{N_f} \ , \qquad V_0 = 0$$

is SUSY preserving and has zero energy. The vacuum

$$|\mathrm{vac}\rangle_+ : \qquad \langle\varphi\rangle = \langle\tilde{\varphi}^T\rangle = \mu \begin{pmatrix} \mathbb{1}_N \\ 0_{N_f-N} \end{pmatrix} \ , \quad \langle\Phi\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0' \end{pmatrix} , \quad V_+ = (N_f - N)|h^2\mu^4|$$

has positive energy and breaks SUSY.



The key features of this effective potential are:
(1) the large distance between the two vacua, $\gamma_0 \gg 1$,
(2) the slow rise of the potential to the left of the SUSY preserving vacuum.

- We think of $\epsilon := \mu/\Lambda_L$ as a small parameter. Thus

$$\langle h\Phi \rangle = \frac{\mu}{\epsilon^{\frac{N_f - 3N}{N_f - N}}} = \Lambda_L \epsilon^{2N/(N_f - N)}$$

is parametrically much smaller than the Landau pole, thus our low energy analysis is realiable.

- Conclusion of analysis:

1. Although SUSY is broken at tree level, SUSY is restored dynamically in the ISS model.

2. The existence of these supersymmetric vacua elsewhere in field space implies that the non-supersymmetric vacua become only meta-stable.

ISS model: Tunnelling

- Phenomenologically, our world is nonsupersymmetric, thus we should be sitting at the vacuum $|\mathrm{vac}\rangle_+$, at least at the present time. However this vacuum is unstable and will decay. Disaster?

- ISS has estimated the tunnelling rate from the metastable $|\mathrm{vac}\rangle_+$ to the supersymmetric vacuum $|\mathrm{vac}\rangle_0$. The false vacuum decay rate per unit volume is given by

$$\Gamma_4/V = D_4 e^{-S^{4D}_{\mathrm{bounce}}}. \tag{1}$$

where S^{4D}_{bounce} is the Euclidean action of the bounce and D_4 is a determinantal coefficient.

ISS approximated the potential in terms of a triangle one and obtianed

$$S^{4D}_{\mathrm{bounce}} = \frac{2\pi^2}{3h^2}\frac{N^3}{{N_f}^2}\left(\frac{\Phi_0}{\mu}\right)^4 \sim \frac{h^2}{\epsilon^{4(N_f-3N)/(N_f-N)}} \gg 1 \qquad \text{for} \quad \epsilon \ll 1$$

The life time can be made to arbitrary large as long as ϵ is small enough. Thus

- From observed life time of the universe, one can get a more precise estimate

$$S^{4D}_{\mathrm{bounce}} \gtrsim 400.$$

This translates into a lower bound on Φ_0,

$$\frac{\Phi_0}{\mu} \gtrsim 3\sqrt{h}\left(\frac{N_f^2}{N^3}\right)^{\frac{1}{4}},$$

- Note that the bound is on the relative width to height of the bump. Thus it is the flatness of the potential which protects the metastable vacuum against decay. i.e. nonperturbative effect.

- Note that this bound is very weak. For example, $N = 2$, $N_f = 7$,

$$\Phi_0/\mu \gtrsim 5\sqrt{h}$$

or in terms of ϵ,

$$\epsilon \lesssim 4.3 \cdot 10^{-4} \left(\frac{1}{h}\right)^{\frac{15}{2}} .$$

These are easy to satisfy and are not very constraining.

A few basic questions:

- Q1.: Is SUSY breaking by ISS similar or distinctively different from other mechanisms of dynamical SUSY breaking (e.g. spontaneous SUSY breaking)?

- Q2.: Why did the universe today resides in the metastable minimum, instead of the global supersymmetric minimum, which is energetically more favorable?

For Q1, not clear.

For Q2:

- ISS provided a heuristic landscape-favoured answer:

The ISS theory naturally admits a finite number ofisolated supersymmetric vacua (as determined by the Witten index) along with a larger moduli space of metastable SUSY breaking vacua

$$\mathcal{M} = \frac{U(N_f)}{SU(N_f - N_c) \times SU(N_c)}$$

Since the latter is a much bigger configuration space, ISS suggested that it is more favorable for the Universe to be populated in the metastable SUSY breaking vacua.

- This explanation is based on probability equal a priori and is not satisfactory. However we do have an understanding of the physics behind and one should be able to distinguish and tell which set of vacuum is more favorable.

We give a "thermal distribution" by including the thermal effects of the early universe.

III. Dynamical evolution of ISS model at finite temperature

Effective potential at zero temperature

- We are interested in the effective potential between the two vacua $|\mathrm{vac}\rangle_+$ and $|\mathrm{vac}\rangle_0$. Parametrize the fields as

$$\varphi = \tilde{\varphi}^T = \sigma\mu \begin{pmatrix} \mathbb{1}_N \\ 0_{N_f-N} \end{pmatrix} , \quad \Phi(\gamma) = \gamma\mu\mathbb{1}_{N_f} ,$$

- The classical potential V_{cl} ignoring the non-perturbative contribution W_{dyn} is

$$\frac{1}{|h|^2}V_{\mathrm{cl}}(\gamma,\sigma) = \mathrm{Tr}_{N_f}|\Phi\varphi|^2+\mathrm{Tr}_{N_f}|\tilde{\varphi}\Phi|^2+\mathrm{Tr}_{N_f}|\tilde{\varphi}_i\varphi_j-\mu^2\delta_{ij}|^2 = |\mu|^4(2N\gamma^2\sigma^2+N(\sigma^2-1)^2+N_f-N)$$

This is extremized for $\sigma = \sqrt{1-\gamma^2}$ or $\sigma = 0$.

- We substitute the first solution for σ into V_{cl} for the range of $0 \le \gamma \le 1$, and the second solution, $\sigma = 0$ for $1 \le \gamma \le \gamma_0$. We thus obtain

$$\frac{1}{|h^2\mu^4|}V_{\mathrm{cl}}(\gamma) = \begin{cases} N_f - N + 2N\gamma^2(1-\frac{1}{2}\gamma^2) & 0 \le \gamma \le 1 \\ N_f & 1 \le \gamma \le \gamma_0 \end{cases}$$

• This potential is accurate for $\gamma \leq 1$, where the potential is rising. In order to address the flat regime $1 \leq \gamma \leq \gamma_0$ we need to include the contribution of the non-perturbative $W_{\rm ADS}$. This gives

$$V = \mathrm{Tr}_{N_f} \left| \frac{\partial}{\partial \Phi_{ij}} N \left(h^{N_f} \frac{\det_{N_f} \Phi}{\Lambda_L^{N_f - 3N}} \right)^{\frac{1}{N}} - h\mu^2 \delta_{ij} \right|^2 = |h^2\mu^4|\, N_f \left((\frac{\gamma}{\gamma_0})^{\frac{N_f - N}{N}} - 1 \right)^2$$

Combining these expressions, we have the effective potential interpolating between the two vacua:

$$\hat{V}_{T=0}(\gamma) \equiv \frac{1}{|h^2\mu^4|} V(\gamma)_{T=0} \quad = \quad \begin{cases} N_f - N + 2N\gamma^2(1 - \frac{1}{2}\gamma^2) & 0 \leq \gamma \leq 1 \\ \\ N_f \left((\frac{\gamma}{\gamma_0})^{\frac{N_f - N}{N}} - 1 \right)^2 & 1 \leq \gamma \leq \gamma_0 \end{cases}$$

This will be our basis for including finite temperature effecs.

Effective potential at finite temperature

- The effective potential at finite temperature along the Φ direction is governed by the well-known expression of Dolan and Jackiw

$$V_T(\Phi) = V_{T=0}(\Phi) + \frac{T^4}{2\pi^2}\sum_i \pm n_i \int_0^\infty \mathrm{d}q\, q^2 \ln\left(1 \mp \exp(-\sqrt{q^2 + m_i^2(\Phi)/T^2})\right)$$

The first term is the zero temperature effective potential; The second term is the purely thermal correction (which vanishes at $T = 0$) and it is determined at one-loop in perturbation theory.

- n_i denote the numbers of degrees of freedom present in the theory and the summation is over all of these degrees of freedom. The upper sign is for bosons and the lower one for fermions.

- $m_i(\Phi)$ denote the masses of these degrees of freedom induced by the vevs of the field Φ.

Degree of freedom

•

- In the metastable vacuum $|\mathrm{vac}\rangle_+$, $\varphi, \tilde{\varphi}$ are massless, Φ has mass $h\mu$.

- In the susy preserving vacuum $|\mathrm{vac}\rangle_0$, $\varphi, \tilde{\varphi}$ has mass $h\mu\gamma_0$, Φ is massless.

• Since $\epsilon \ll 1$, we have a natural separation of scales:

$$\mu \qquad \ll \qquad \Phi_0 = \mu\gamma_0 \qquad \ll \qquad \Lambda_L.$$

The masses of order $h\Phi_0$ will be treated as heavy and all other masses suppressed with respect to $h\Phi_0$ by a positive power of ϵ will be refered to as light.

• Therefore:

- in the metastable vacuum $|\mathrm{vac}\rangle_+$, all degrees of freedom are light.

- in the susy preserving vacuum $|\mathrm{vac}\rangle_0$, the N_f flavours of φ and $\tilde{\varphi}$, are heavy, they get the tree-level masses $m_\varphi = h\mu\gamma_0$. All other degrees of freedom in $|\mathrm{vac}\rangle_0$ are light.

• Hence the SUSY preserving vacuum $|\mathrm{vac}\rangle_0$ has fewer light degrees of freedom than the SUSY breaking vacuum $|\mathrm{vac}\rangle_+$. This is an important feature of the ISS model.

Back to the finite temperature potential.

- As we interpolate from the metastable vacuum to the supersymmetric one, the φ and $\tilde{\varphi}$ acquire masses $m_\varphi = h\Phi = h\mu\gamma$ and become heavy. To a good approximation all other degrees of freedom can be counted as essentially massless.

- As we are not interested in the overall additive (T dependent, but field independent) constant in the thermal potential, we need to include only those variables whose masses vary from being light in one vacuum to being heavy in the other, i.e. the N_f flavours of φ's and $\tilde{\varphi}$'s.

We have

$$\hat{V}_\Theta(\gamma) = \hat{V}_{\Theta=0}(\gamma) + \frac{h^2}{2\pi^2}\Theta^4 \sum_{\pm} \pm 4NN_f \int_0^\infty \mathrm{d}q\, q^2 \ln\left(1 \mp \exp(-\sqrt{q^2+\gamma^2/\Theta^2})\right).$$

where $\qquad \gamma := \Phi/\mu, \quad \Theta := T/|h\mu| \qquad$ are dimensionless variables.

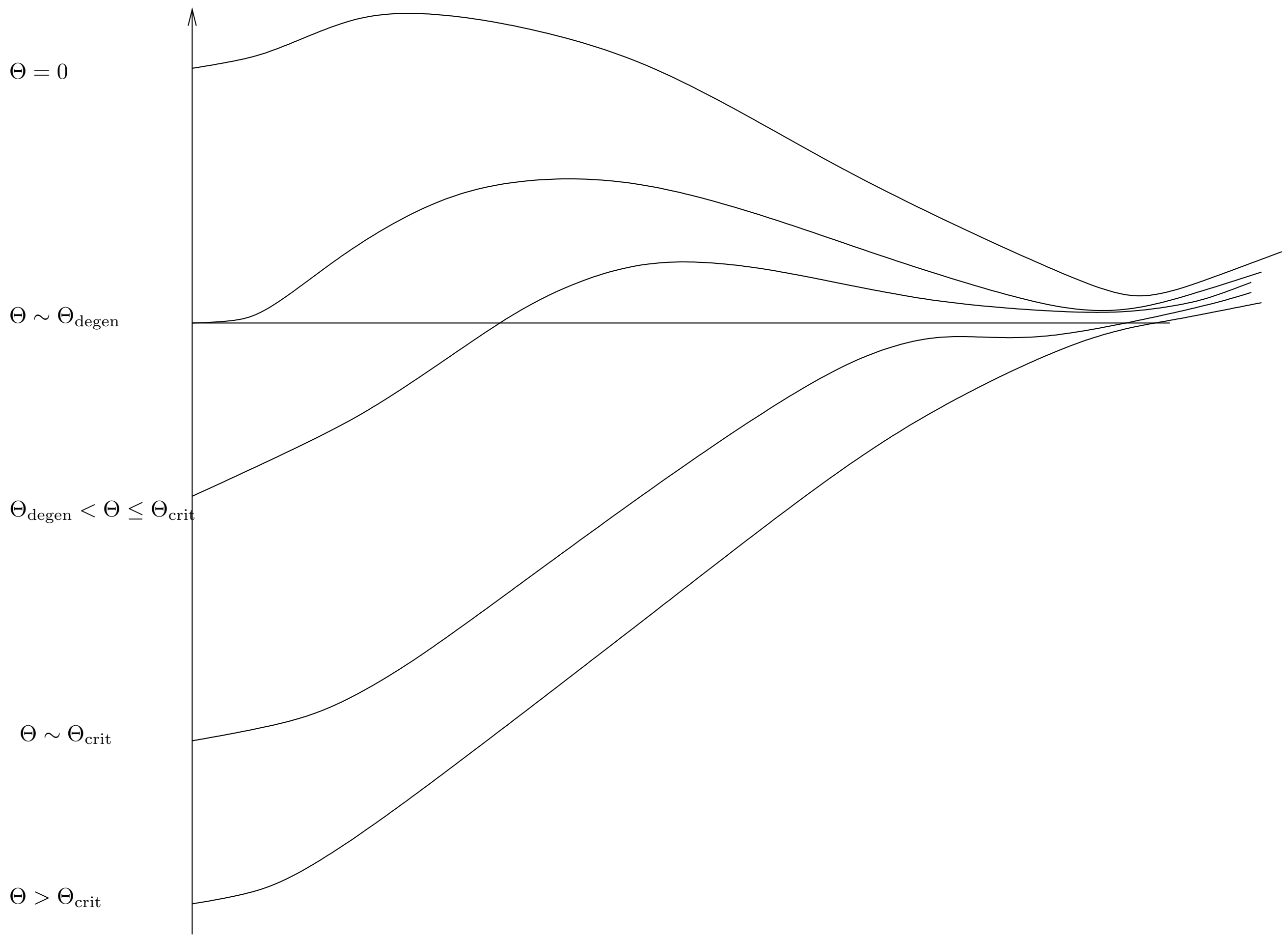
Various important temperatures

- As temperature changes, the shape of the potential changes also. We will first state the various temperatures which are going to be important for our analysis of the thermal history.

- $\Theta_{\rm crit} > \Theta_* > \Theta_{\rm degen}$

- At high temperature, there is only the minimum $|{\rm vac}\rangle_+$.
- As we decrease down to $\Theta_{\rm crit}$, the second minimum $|{\rm vac}\rangle_+$ form
- At temperatures below $\Theta_{\rm crit}$ the SUSY breaking minimum at the origin still has lower free energy then $|{\rm vac}\rangle_0$. One would have to calculate the bubble nucleation rate to the true vacuum $|{\rm vac}\rangle_+$.
- $\Theta_{\rm degen}$ is the temperature when the SUSY preserving vacuum $|{\rm vac}\rangle_0$ becomes degenerate with the non-supersymmetric vacuum $|{\rm vac}\rangle_+$.
- For a generic potential one would expect that somewhere between these two scales there is a temperature Θ_* ($\Theta_{\rm degen} < \Theta_* \leq \Theta_{\rm crit}$) where the rate of bubble nucleation turns from large to small.

Thermal effective potential for different values of the temperature:



Critical temperature $T_{\rm crit}$

- In the limit of very high temperatures $\Theta \gg \gamma$, the thermal potential grows as

$$\hat{V}_\Theta - \hat{V}_{\Theta=0}(\gamma) \sim \, N N_f \, \Theta^2 \gamma^2 - \, {\rm const} \, \Theta^4 \ , \qquad \Theta \gg \gamma = \Phi/\mu$$

Thus there is only one minimum at the origin, and the SUSY preserving minimum disappears. This is caused by N_f flavours of φ and $\tilde{\varphi}$ becoming heavy away from the origin.

- Thus there is a critical temperature $\Theta_{\rm crit}$ such that at $\Theta > \Theta_{\rm crit}$ there is only one minimum, and at $\Theta < \Theta_{\rm crit}$ the supersymmetry preserving second minimum starts to materialise.

- Using the approximation,

$$\pm \int_0^\infty {\sf d}q \, q^2 \ln\left(1 \mp \exp(-\sqrt{q^2+\gamma^2/\Theta^2})\right) \sim \Theta^4 \left(\frac{\gamma}{2\pi\Theta}\right)^{\frac{3}{2}} \exp\left(-\frac{\gamma}{\Theta}\right) , \quad {\rm for} \quad \gamma \gg \Theta$$

one can get the estimate,

$$\Theta_{\rm crit} \approx \frac{\gamma_0}{\log\left(\gamma_0^4/C\right)} \ , \qquad C = \frac{\sqrt{2}\pi^{\frac{3}{2}}}{N h^2}\left(1 - \frac{N_f}{N}\right)^2$$

- We remark that for the ISS model, the global minimum at zero temperature is the SUSY preserving state. And the global minimum at high temperature is the SUSY breaking state.

This is in sharp contrast to the situation in usual finite temperature field theory where symmetry is usually restored at high temperature! e.g. restoration of electroweak symmetry in standard model.

- This is a very special (unique?) feature of the ISS model guarantees and is a consequence of the nonperturbative nature of the SUSY preserving vacua. This feature is not necessary shared by other possible metastable susy breaking sceranio or dynamical SUSY breaking mechanism.

initial condition

- Now, at high temperature, the susy breaking vacua is energtically favorable. However for smaller temperature below $T_{\rm cric}$, the second vacuum appears; and if the initial condition (right after inflation) was such that the universe sat at the susy preserving vacuum initally, then the universe might got stuck there.

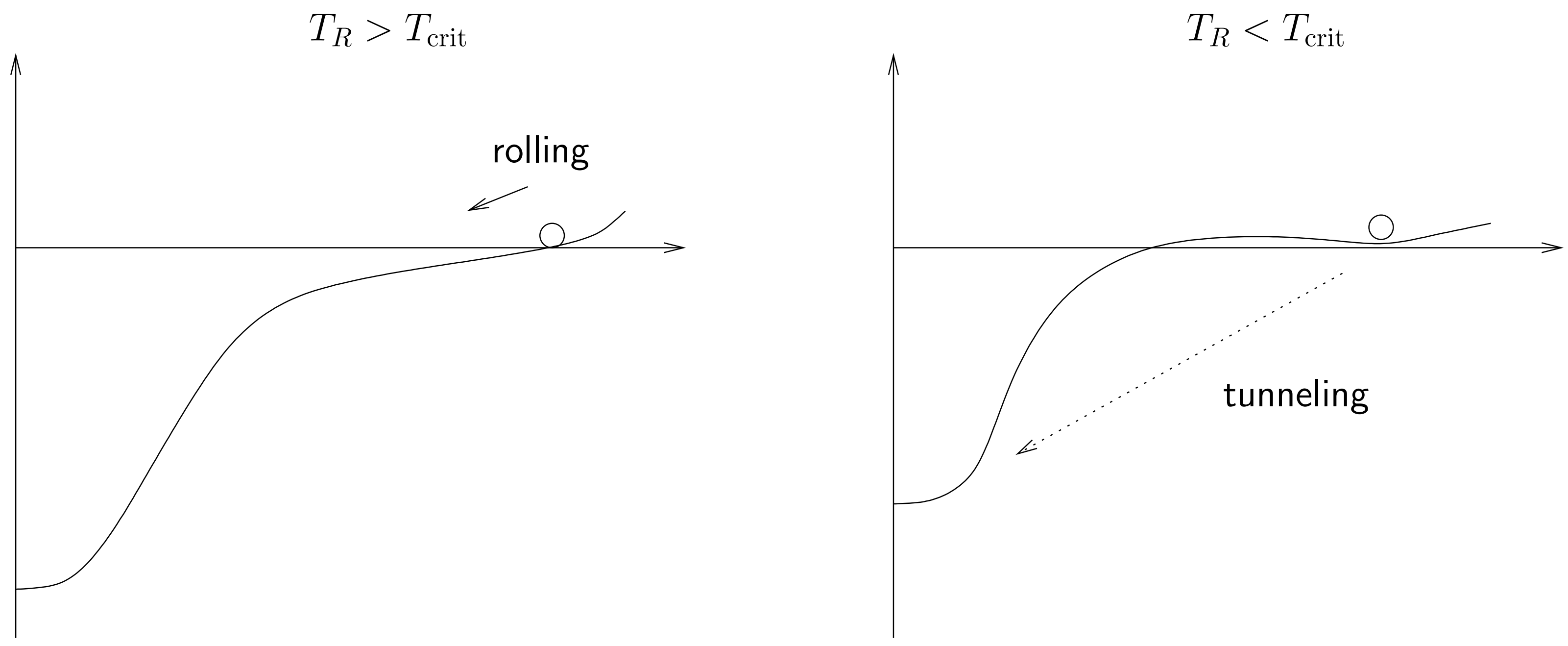
- At the end of inflation the Universe is in a state of very low temperature and one may assume that it is in the energetically preferred supersymmetric state $|{\rm vac}\rangle_0$. After inflation the Universe reheats to a temperature T_R.

- Thus the initial condition for us is that the universe is in the supersymmetric state $|{\rm vac}\rangle_0$ and at a tempearture T_R.

The universe may escape out of the supersymmetric state through the two processes:

i. by classical rolling to the vacuum $|\mathrm{vac}\rangle_+$ if $T_R > T_{\mathrm{cric}}$.

ii. by tunnelling to the vacuum $|\mathrm{vac}\rangle_+$ if $T_R < T_{\mathrm{cric}}$,



Classical rolling

We should check that the field has time enough to evolve to $|\mathrm{vac}\rangle_+$ before the temperature drops significantly.

- One can estimate the time for the field to roll down to the origin takes the time

$$\Delta t_{\mathrm{roll}} \sim \frac{\Phi_0}{T_R^2} \lesssim \frac{1}{T_{\mathrm{crit}}},$$

which is parametrically very small.

- Assuming the universe is radiation dominated with scale factor $a(t) \sim t^{1/2}$, then the universe cool down for a temperature of

$$\Delta T \sim T_R^2/M_{\mathrm{pl}} \ll T_R.$$

This means the field is able to roll down essentially instaneously to the origin.

Degenerate temperature $T_{\rm degen}$

- Bubble nucleation is possible once temperature drop below the critical temperature. However as temperature continues to drop, there is a certain minimum where the two minimum becomes degenerate, and bubble nucleation stops. This is the degeneate temperature.

- The effective potential at the origin is

$$\hat{V}_\Theta(\gamma = 0) = (N - N_f) - \frac{\pi^2 h^2 \Theta^4}{90}\left(n_B + \frac{7}{8} n_F\right),$$

where $n_B = n_F = 4NN_F$ counts the number of bosonic and fermionic degrees of freedom.

- The effective potential at the second vacuum remains approximately zero since the temperature $T_{\rm degen}$ is much below Φ_0.

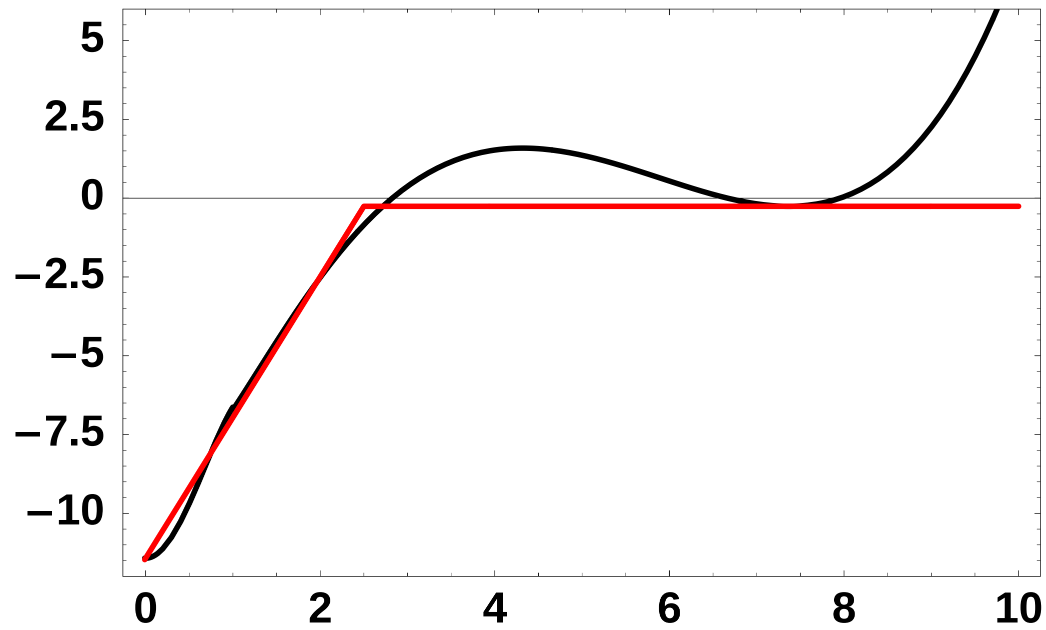
- One obtains the degenerate temperature

$$\Theta_{\rm degen} = \left(\frac{12(N_f - N)}{h^2 \pi^2 N N_f}\right)^{\frac{1}{4}}$$

This is of order ϵ^0.

Bubble nucleation

• For temperatures in the range $T_{\rm degen} < T < T_{\rm crit}$, the vacuum $|{\rm vac}\rangle_0$ is already formed but still higher than $|{\rm vac}\rangle_+$, it is possible for the universe to tunnel from $|{\rm vac}\rangle_0$ to $|{\rm vac}\rangle_+$. We would like to know when bubble nucleation is fast enough to lead to a phase transition.



• characteristic features:

- the "false" vacua $|{\rm vac}\rangle_0$ is far away in the field space from $|{\rm vac}\rangle_+$

- the potential climbs very slowly from $|{\rm vac}\rangle_0$ to a shallow barrier and then descends steeply to the 'true' vacuum $|{\rm vac}\rangle_+$.

These features are a reflection of the fact that at zero temperature the vacuum $|{\rm vac}\rangle_0$ was generated non-perturbatively.

- Model this by a simple linear no-barrier potential,

$$\hat{V}_{\rm lin}(\gamma) = \theta(\eta - \gamma) \times N_f K (\gamma - \eta)$$

where K and η are constants, N_f is put due to Tr_{N_f}.

- The potential admits a simple analytic solution for the bounce configuration. The corresponding action is

$$\frac{1}{T} S^{3D}_{\rm bounce} \approx {\rm const} \left(\frac{\Phi_0}{T}\right)^3$$

- In order for the Universe to have undergone a phase transition by bubble nucleation one requires a sufficiently high nucleation probability of bubbles of metastable vacuum. The nucleation rate is given by

$$\Gamma_3 \sim T^4 e^{-S^{3D}/T}.$$

- From this we get the temperature T_* at which the tunnelling transitions are still possible is:

$$T_* \sim \Phi_0,$$

Below this temperature, bubble nucleation is slow and non-effective in bringing the universe out of the supersymmetry preserving minimum.

Bound on Reheating temperature

- Therefore,

- If T_R falls in the range $T_{\rm degen} < T_R \lesssim T_* \sim T_{\rm crit}$, the Universe will remain in the supersymmetric state $|\mathrm{vac}\rangle_0$ although $|\mathrm{vac}\rangle_+$ is energetically preferred. It is stuck there because the barrier makes classical evolution impossible and bubble nucleation is too slow.

- On the other hand, if $T_R > T_*$, then the bubble nucleation rate will be high and if $T_R > T_{\rm crit}$ the classical evolution becomes possible.

- Since $T_* \sim T_{\rm crit}$, we obtain that:

if the reheating temperature T_R fulfills

$$T_R \gtrsim \frac{\mu}{\epsilon^{1-2N/(N_f-N)}}$$

it is ensured that the Universe always ends up in the supersymmetry breaking ground state $|\mathrm{vac}\rangle_+$.

- the condition that the metastable vacuum $|\mathrm{vac}\rangle_+$ is long-lived at zero temperature does not require that ϵ to be very small.

For a low supersymmetry breaking scale μ of order of a few TeV the lower bound on the reheating temperature can be easily satisfied for reheating temperatures as low as around 10 or 100 TeV.

On the other hand, for models with a significantly higher SUSY breaking scale, this bound becomes more constraining.

IV. Summary and discussions

- We have considered thermal effects in the ISS model and have shown that, provided the reheating temperature is sufficiently large, the early Universe is always driven to metastability.

This phenomenon is a consequence of two distinguishing properties of the ISS theory which are not necessarily shared by other models with metastable SUSY breaking vacua:
- the metastable minima have more light degrees of freedom than the SUSY preserving vacua,
- the metastable vacua are separated from the SUSY preserving vacuum by a very shallow dynamically induced potential.

Both are due to the nonperturbative nature of the dynamically restored supersymmetry.

Some further directions:

1. • Key features of ISS which enable metastable SUSY breaking:

i. existence of nonperturbative ADS superpotential

ii. existence of dual QFT as UV completion because of Seiberg duality

- ii is useful, but not essential. One may also consider effective field theory in string theory, e.g. on D-brane construction, geometric engineering etc., where it is known that string theory provides the UV completion.

- The exisence of nonperturbative superpotential is physically more important.

Thus one could imagine other sceranio of metastable susy breaking where the UV completion is given by some other mechanism, however non-perturbative effect of some form is necessary.

2 • Phenomenologically it will be interesting to use ISS as a sector couple to the supersymmetric standard model (e.g. MSSM).
There could be distinctive effects different from usual mechanism of dynamical susy breaking due to the nonperturbative nature of the restored supersymmetry. It will be interesting to study and identify these effects.

3. • Construction of metastable SUSY breaking model with local supersymmetry?