QCD Corrections to Higgs Pair Production in Bottom Quark Fusion

Chung Kao [高 鐘]
University of Oklahoma
Norman Oklahoma USA

†Presented at the National Tsing Hua University, May 31 (Thursday), 2007.

QCD Corrections to Higgs Pair Production in Bottom Quark Fusion

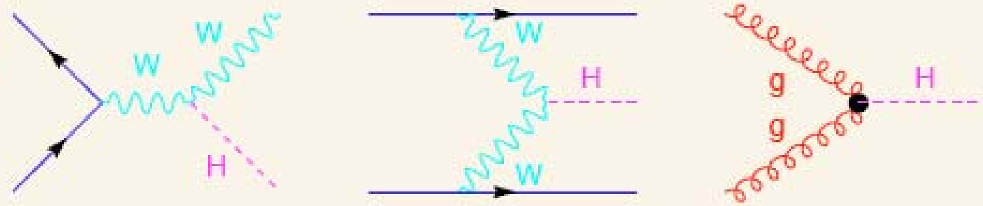
- Introduction: The Standard Higgs Model
- Leading-order cross section for $b\bar{b} \rightarrow hh$
- NLO Corrections to $b\bar{b} \rightarrow hh$
  - the α_s corrections
  - the $1/\Lambda$ corrections ($bg \rightarrow b\,hh$), where $\Lambda \equiv \ln(M_h/m_b)$
- Two-cutoff phase space slicing method
- Results for Higgs pair production
- Conclusions

[†]S. Dawson, C. Kao, Y. Wang and P. Williams, hep-ph/0610284, to be published in Phys. Rev. D.

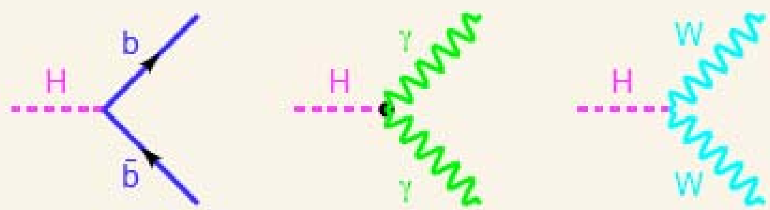
The Standard Model Higgs Boson

- In the SM, there is one Higgs doublet and a spin-0 particle: the Higgs boson (H).

It can be produced at colliders:



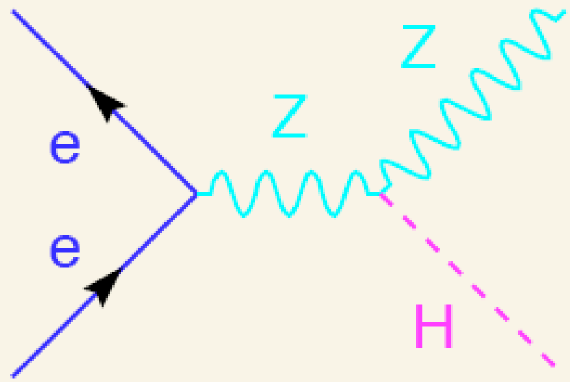
Its decays are well known:



Why has't it been discovered yet?

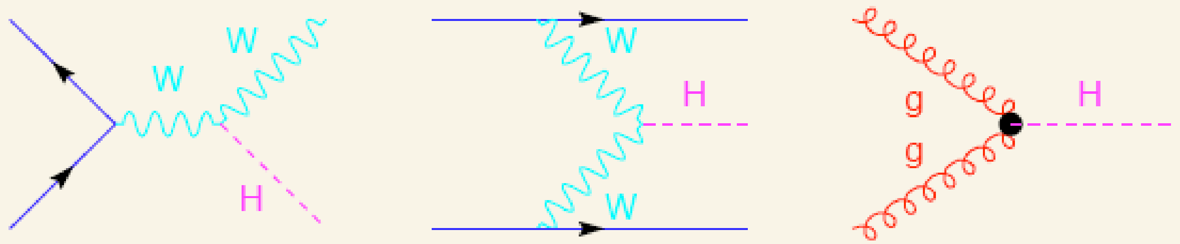
We need higher energy and higher luminosity!

The Search for the SM Higgs boson



- Mass limit from LEP 2
  With a CM energy up to $\sqrt{s} = 209\,\mathrm{GeV}$
  and $L = 100\ \mathrm{pb}^{-1}$ per experiment,
  a stringent mass limit for the Higgs boson
  at 95% C.L. is $M_H > 114\ \mathrm{GeV}/c^2$

Discovery potential of hadron colliders



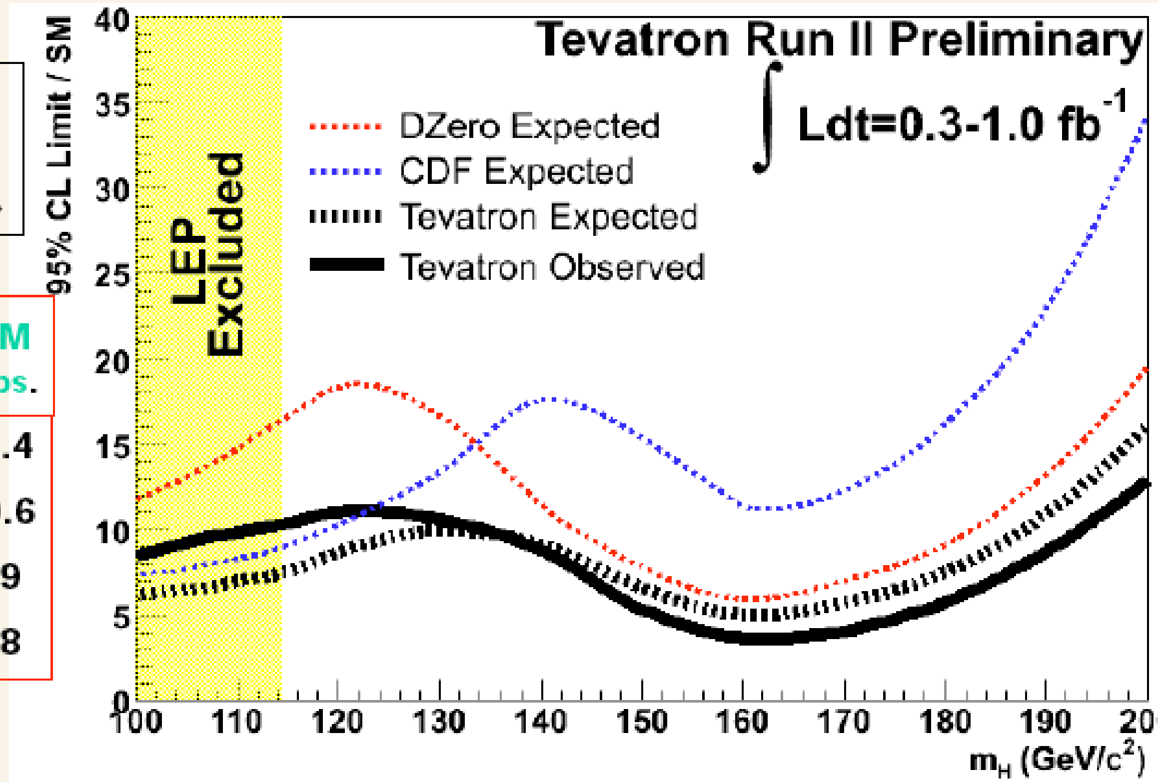
- The Tevatron Run II will be able to discover a SM Higgs boson up to 190 GeV with 30 fb^{-1}, or it will exclude the Higgs boson at 95% C.L. with 10 fb^{-1}.
- The LHC will be able to observe a SM Higgs boson with a mass up to approximately 1 TeV.

Stange, Marciano, and Willenbrok (1994); Han and Zhang (1998). CMS Technical Proposal (1994); ATLAS Technical Proposal (1994); ATLAS Technical Design Report (1999).

Tevatron SM Higgs Combination

All CDF and DØ results now combined for the first time ➔

m_H (GeV)	Limit/SM Exp.	Obs.
115	7.6	10.4
130	10.1	10.6
160	5.0	3.9
180	7.5	5.8

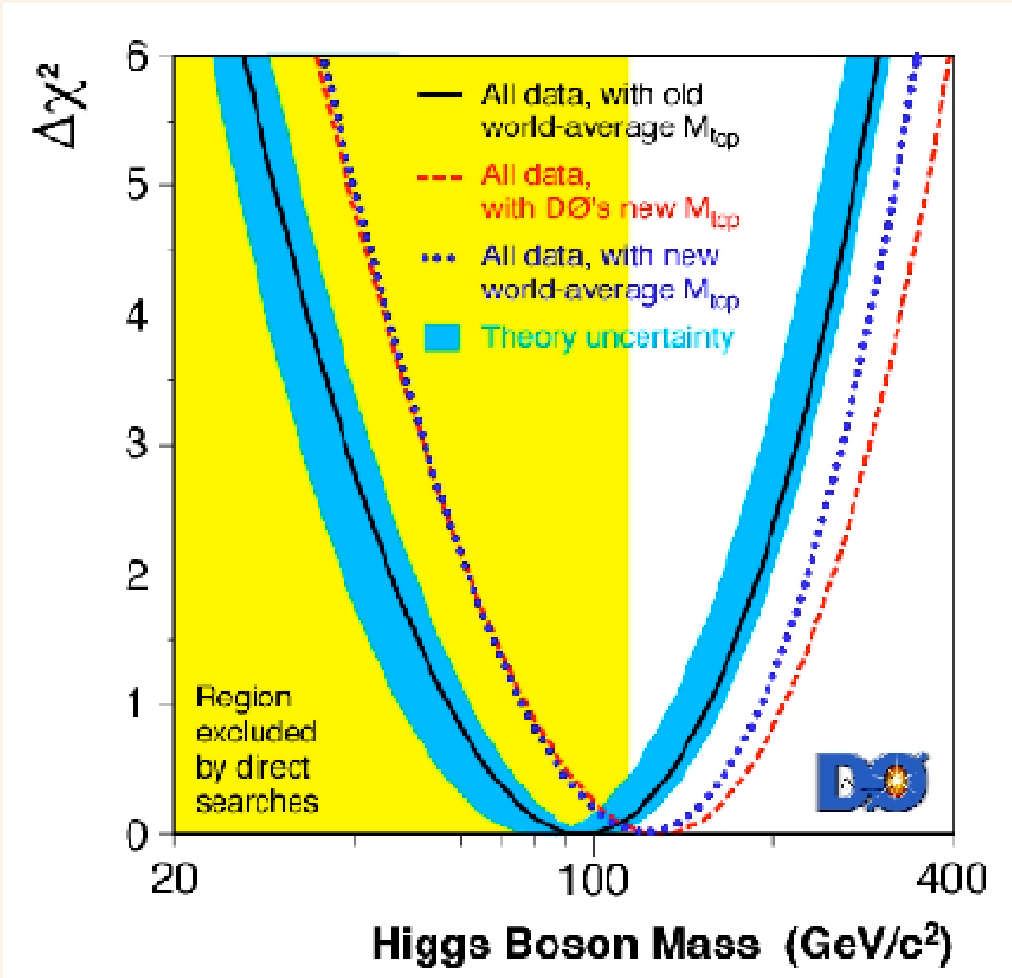


Note: the combined result is essentially equivalent to one experiment with 1.3 fb^{-1}, since both experiments have "complementary" statistics at low and high mass

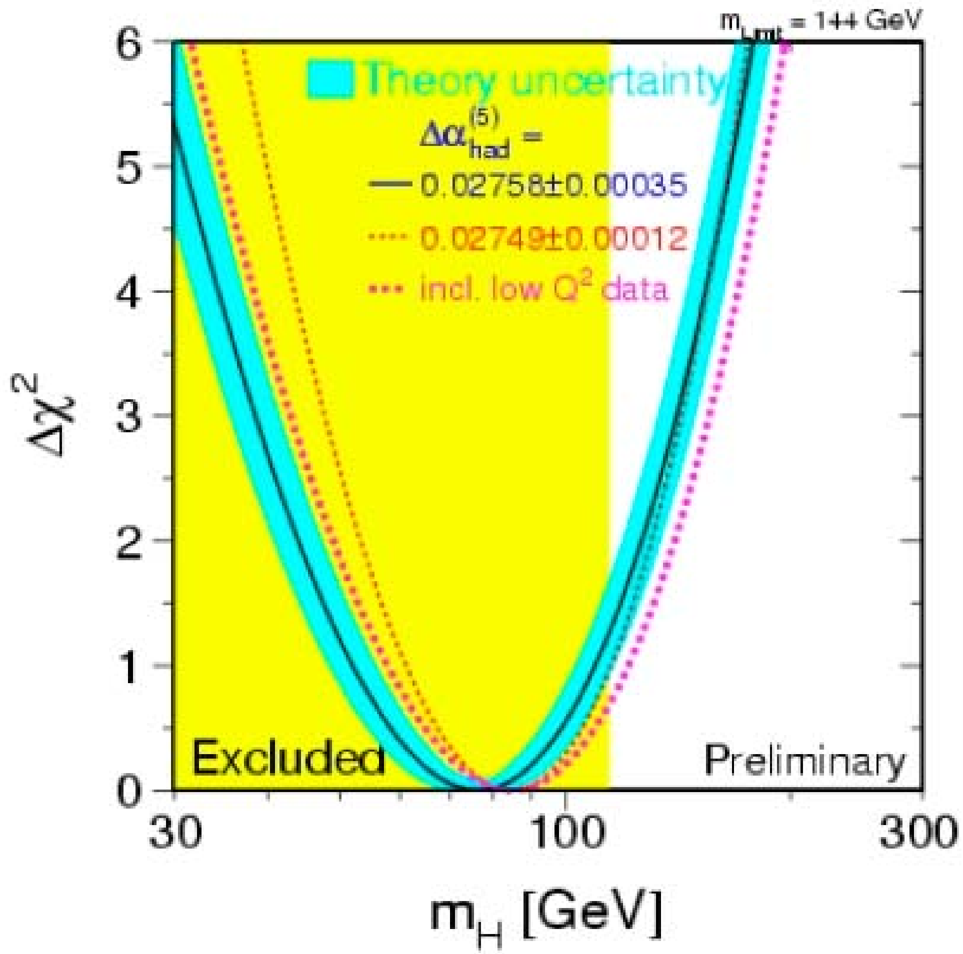
➔ we are indeed already close to the sensitivity required to exclude or "evidence" the higgs at the Tevatron

Gregorio Bernardi, ICHEP06, Moscow

Implications of Electroweak Precision Data for Higgs Mass with New m_t



M.W. Grunewald (2003); The D0 Collaboration (2004)



Top quark mass = 170.9 +- 1.8 GeV/c²

M_H < 144 GeV or M_H < 182 GeV @95% C.L.

High Energy Frontier in HEP

M. Lamont
Tev4LHC meeting
@ CERN (April)

Next projects on the HEP roadmap

- Large Hadron Collider LHC at CERN: pp @ 14 TeV
  - LHC will be closed and set up for beam on 1 July 2007
  - First beam in machine: August 2007
  - First collisions expected in November 2007
  - Followed by a short pilot run
  - First physics run in 2008 (starting April/May; a few fb^{-1}?)
- Linear Collider (ILC) : e+e- @ 0.5-1 TeV
  - Strong world-wide effort to start construction earliest around 2009/2010, if approved and budget established
  - Turn on earliest 2015 (in the best of worlds)
  - Study groups in Europe, Americas and Asia (→World Wide Study)

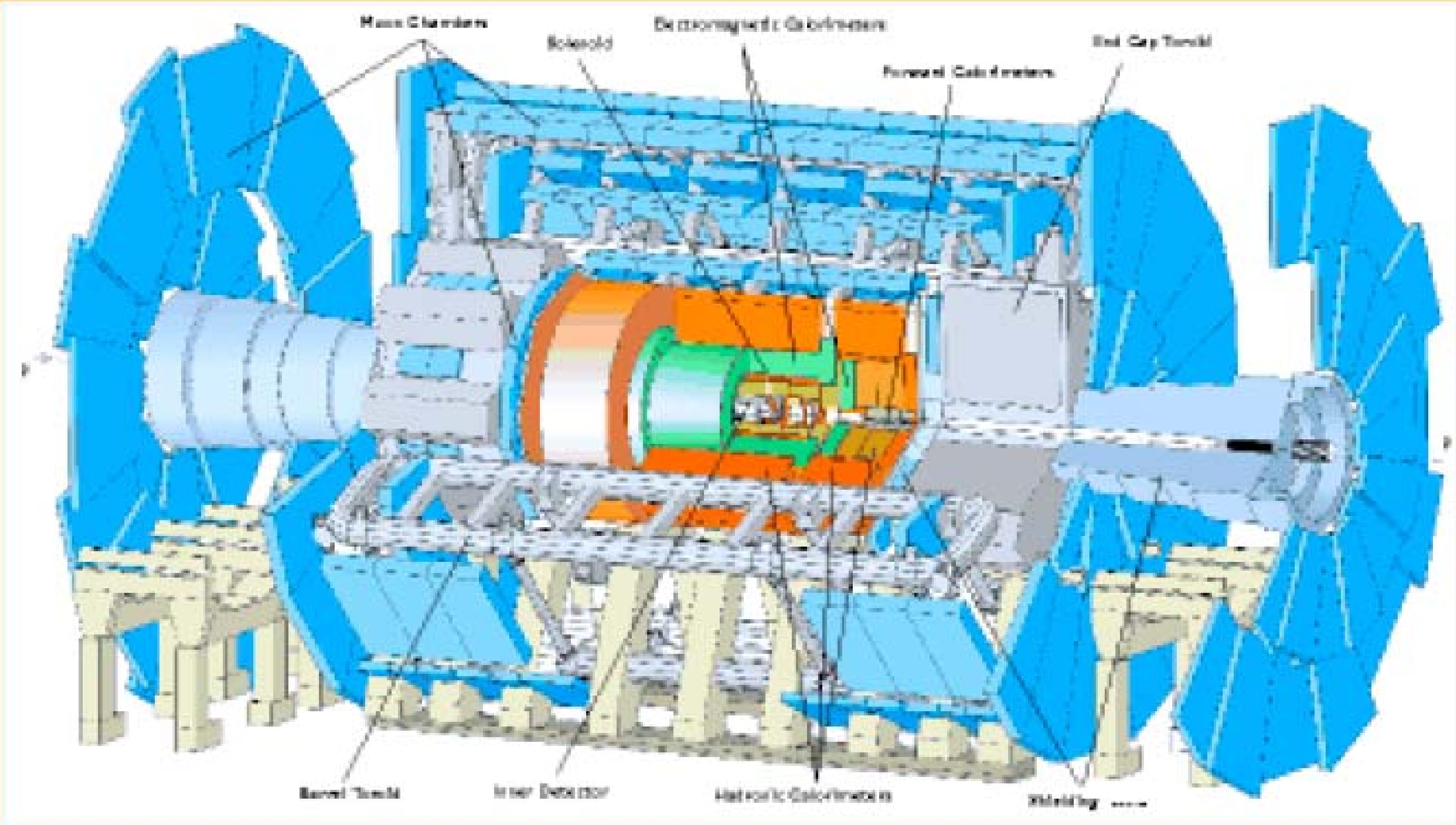
Quest for the Higgs particle is a major motivation for these new machines

The Search for New Particles at Hadron Colliders

- We need accelerators: Fermilab Tevatron Collider near Chicago and CERN Large Hadron Collider (LHC) in Geneva.
- We need detectors: D0 and CDF (Tevatron), as well as ATLAS and CMS (LHC).
- We look for e, μ, γ (photon), jets, and hadrons (mesons or baryons).
- A jet = a quark, an anti-quark, or a gluon.

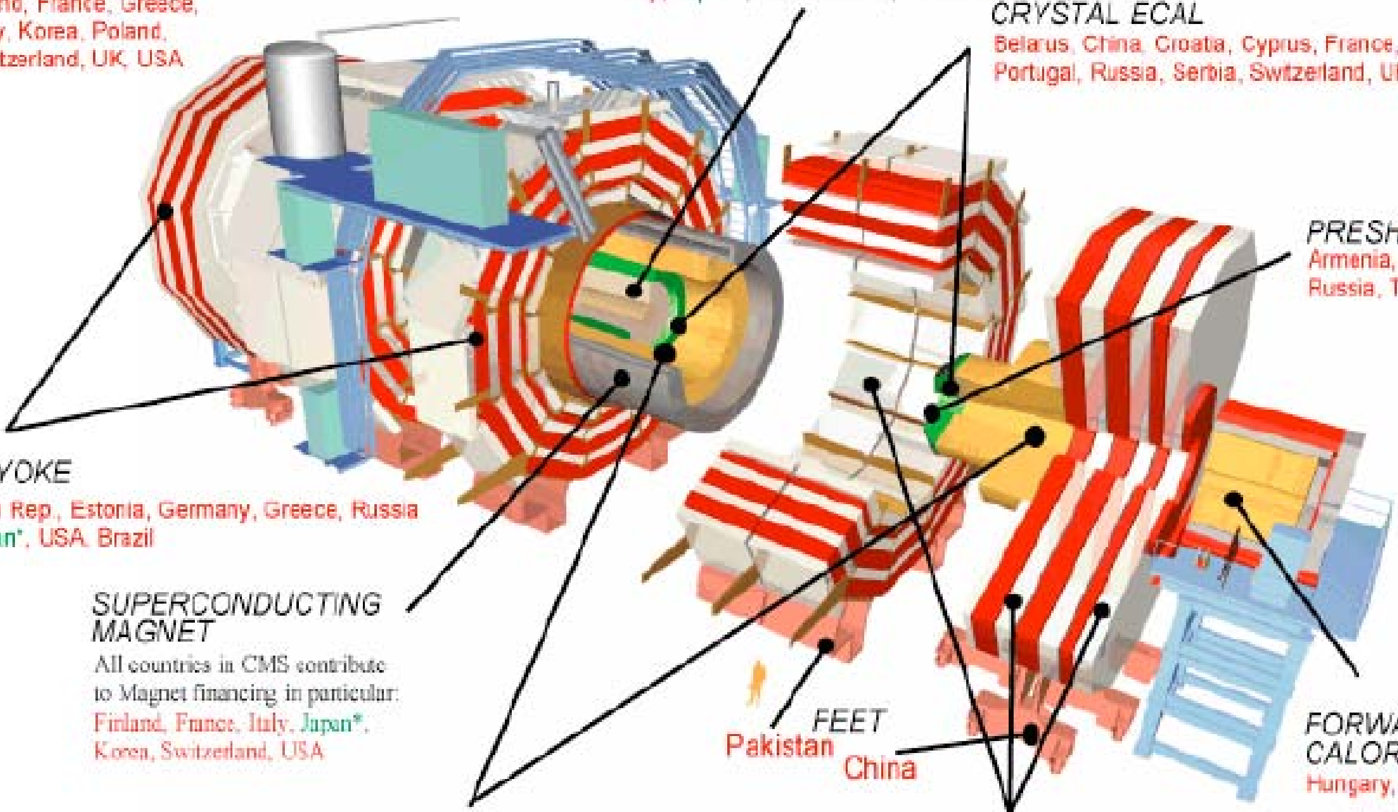
ATLAS

A Toroidal LHC ApparatuS



CMS Collaboration

36 Nations, 159 Institutions, 1940 Scientists (February 2003)



Total weight	: 12500 T
Overall diameter	: 15.0 m
Overall length	: 21.5 m
Magnetic field	: 4 Tesla

* Only through industrial contracts

Production of Higgs Bosons

A. Gluon Fusion: $gg \to \phi^0$ ($\tan\beta < 7$).

B. Bottom Quark Fusion: $b\bar{b} \to \phi^0$ ($\tan\beta > 7$)

- $\sigma(gg \to \phi^0 b\bar{b})[m_b(M_b)]$
  $\approx 3\sigma(gg \to \phi^0 b\bar{b})[m_b(M_\phi)]$, $M_\phi = 200$ GeV
- $\sigma(gg \to \phi^0 b\bar{b}) \approx \sigma(b\bar{b} \to \phi^0)$, $\mu_F = M_\phi/4$

S. Dawson, C.B. Jackson, L. Reina, D. Wackeroth (2003 & 2004);
V. Ravindran, J. Smith, and W.L. van Neerven (2003);
R.V. Harlander & W.B. Kilgore (2002); C. Anastasiou & K. Melnikov (2002).
M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas (1995).
T. Plehn (2002); F. Maltoni, Z. Sullivan and S. Willenbrock (2003);
E. Boos and T. Plehn (2003); R.V. Harlander and W.B. Kilgore (2003).
B. Plumper, DESY-THESIS-2002-005.
J. Campbell *et al.*, arXiv:hep-ph/0405302.

Higgs Boson Production via Bottom-Quark Fusion

- The dominant subprocess for the production of a Higgs boson in association with bottom quarks is bottom-quark fusion $b\bar{b} \to \phi^0$.
- If we require one bottom quark at high p_T from the production process, the leading-order subprocess should become $bg \to b\phi^0$.
- For the production of the Higgs boson accompanied by two high p_T b quarks, the leading subprocess should be $gg, qq \to b\bar{b}\phi^0$.

Campbell, Ellis, Maltoni and Willenbrock (2003);
S. Dawson, C.B. Jackson, L. Reina, D. Wackeroth (2003 & 2004);
Hou, Ma, Zhang, Sun, and Wu (2003); C.S. Huang and S.H. Zhu (1999); Choudhury, Datta and Raychaudhury (1998).

Higgs Boson Production via Bottom-Quark Fusion

There were two puzzling aspects in the NLO calculations of bottom quark fusion:

- The independent corrections of order α_s and $1/\ln(m_h/m_b)$ are both large and of opposite sign.
- The cross section in hadron collisions via $gg \to b\bar{b}\phi^0$ is an order of magnitude smaller than that obtained from $b\bar{b} \to \phi^0$.

One simple solution: $\mu_{Factorization} = m_{\phi/4}$.

F. Maltoni, Z. Sullivan, and S. Willenbrock, Phys. Rev. D **67**, 093005 (2003).

Order Counting for Bottom Quark Fusion

Dicus, Stelzer, Sullivan and Willenbrock (1999)

Leading-order contribution: $b\bar{b} \to H$: $\mathcal{O}[\alpha_s^2 \ln^2(M_H/m_b)]$

$\mathcal{O}(\alpha_s)$ correction:
(1) $b\bar{b} \to H$ with virtual gluon, and
(2) $b\bar{b} \to Hg$: soft, hard/collinear, and hard/non-colinear

$\mathcal{O}[(1/\ln(M_H/m_b)]$ correction: $bg \to bH$

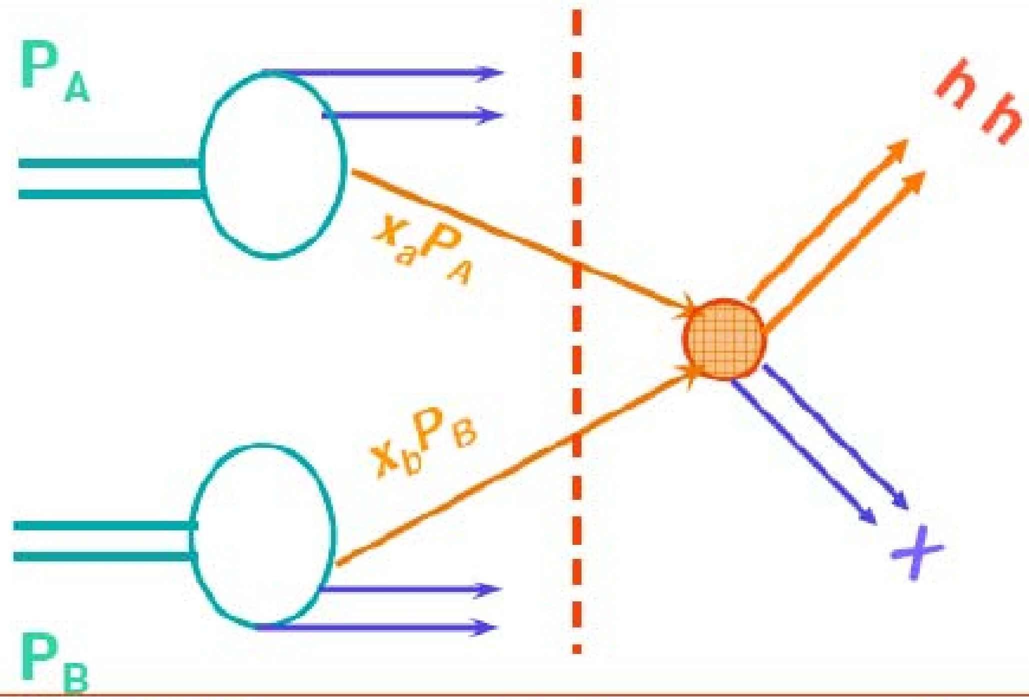
$\mathcal{O}[1/\ln^2(M_H/m_b)]$ corrections: $gg \to b\bar{b}H$

Next-to-leading order (NLO) correction =
$\mathcal{O}(\alpha_s)$ correction $+\mathcal{O}[(1/\ln(M_H/m_b)]$ correction.

Higgs Pair Production in Bottom Quark Fusion

- In the Standard Model, gluon fusion is the dominant process to produce a pair of Higgs bosons via triangle and box diagrams with quarks.
- Bottom quark fusion can also produce Higgs pairs at a lower rate.
- The rate for Higgs pair production at the LHC is small in the Standard Model.
- However, it can become significant in models in which the Higgs coupling to the bottom quark is enhanced.
- The high energy and high luminosity at the LHC might provide opportunities to detect a pair of Higgs bosons as well as to measure the trilinear Higgs couplings.

Parton Model



interference between different momentum scales are power suppressed

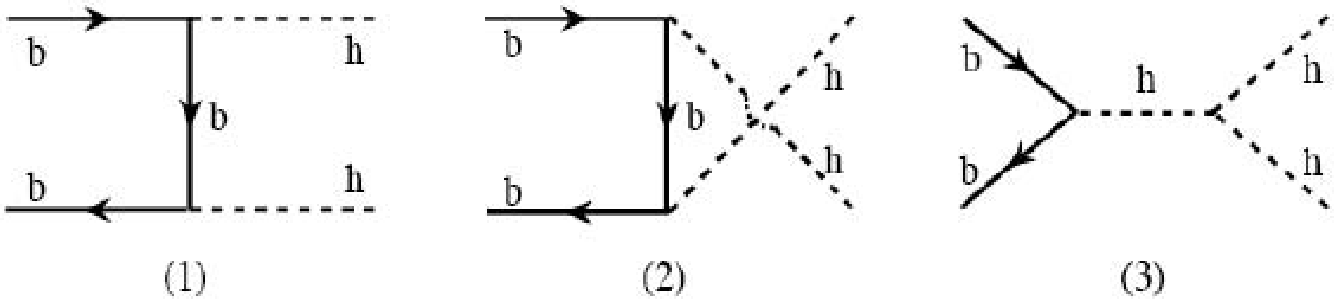
Parton distributions donot interfere with hard interaction. They are universal

$$\sigma = \sum_f \int dx_1 \varphi_{f/A}(x_1)\ dx_2 \varphi_{\bar{f}/B}(x_2)\ \hat{\sigma}(b\bar{b} \to hh)$$

Probability of finding a parton of flavor a in hardon A

Leading Order Cross Section

lowest order cross section for $b\,\bar{b} \rightarrow h\,h$:



$$b(p_1)\,\bar{b}(p_2) \rightarrow h(p_3)\,h(p_4)$$

$$\hat{\sigma}_{b\bar{b}} = \frac{1}{2}\frac{1}{2\hat{s}}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}$$

$$(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)\,|\overline{M_0}|^2$$

Final state identical

$$|\overline{M_0}|^2 = \left(\frac{1}{3}\cdot\frac{1}{3}\right)\left(\frac{1}{2}\cdot\frac{1}{2}\right)\sum_{\substack{\text{spin}\\\text{color}}}|M_0|^2$$

Matrix Element Squared

Amplitudes for each diagram

$$\mathbf{M}_s^0 = \hat{\mathbf{M}}_s^0 \delta_{ji} = -\frac{3\overline{m}_b(\mu) M_h^2}{v^2 (s - M_h^2 + i M_h \Gamma_h)} \bar{v}(p_2) u(p_1) \delta_{ji}$$

$$\mathbf{M}_t^0 = \hat{\mathbf{M}}_t^0 \delta_{ji} = \frac{\overline{m}_b^2(\mu)}{v^2 t} \bar{v}(p_2) \not{p}_3 u(p_1) \delta_{ji}$$

$$\mathbf{M}_u^0 = \hat{\mathbf{M}}_u^0 \delta_{ji} = -\frac{\overline{m}_b^2(\mu)}{v^2 u} \bar{v}(p_2) \not{p}_3 u(p_1) \delta_{ji}$$

Matrix Element Squared

$$|\mathbf{M}_0|^2 = |\mathbf{M}_s^0|^2 + |\mathbf{M}_t^0|^2 + |\mathbf{M}_u^0|^2 + 2\mathrm{Re}\left(\mathbf{M}_t^0 \mathbf{M}_u^{0*}\right)$$

$$= \frac{3}{2}\left(\frac{\overline{m}_b^2(\mu)}{v^2}\right)\left(\frac{\hat{s}}{v^2}\right)\left|\frac{M_h^2}{(s - M_h^2 + i M_h \Gamma_h)}\right|^2 + \frac{1}{6}\left(\frac{\overline{m}_b^4(\mu)}{v^4}\right)\left(1 - \frac{M_h^4}{ut}\right)\frac{(u-t)^2}{ut}$$

Next-to-Leading Order Corrections

- α_s Corrections from $b\bar{b} \rightarrow hhg$
  - ❑ Corrections from virtual gluons.
    Infrared singularity: $p_g \rightarrow 0$,
    ultra-violet singularity: $p_g \rightarrow \infty$
  - ❑ Corrections from real gluon emission
    Infrared singularity: $p_g \rightarrow 0$
    collinear singularity:

p_g parallels to one of initial b or $\bar{b}$ momentums.

- 1/Λ Corrections from $bg \rightarrow bhh$

  only collinear singularities

gluon splits into a pair of collinear b

Infrared and Collinear Divergences

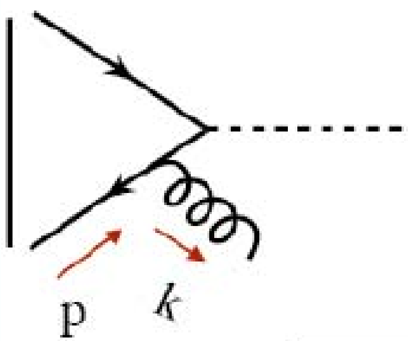
- **Relevant Lagrangian:** g = gauge coupling, T = SU(3) matrices

$$\mathcal{L} = \bar{\psi}(i\partial - gA\cdot T - m)\psi - \frac{1}{4}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} - \frac{m_{\Psi}}{v}H\bar{\Psi}\Psi - 3\,\frac{m_h^2}{v}HHH$$

Fields: Quark, ψ, gluon and Higgs, H.

- **Problems arise from parton level interactions**

Infrared (IR) and collinear (CO) singularities



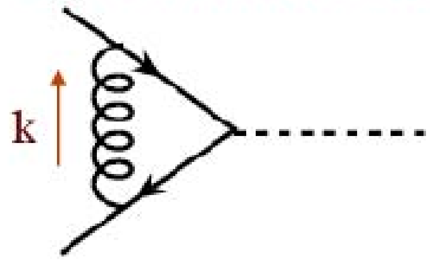
$$\left| \ \cdots \ \right|^2 \to \infty \quad as \quad \begin{cases} k^{\mu} \to 0 & \text{Infrared divergence} \\ k^{\mu} \parallel p^{\mu} & \text{Collinear divergence} \end{cases}$$

$$\frac{1}{(p-k)^2 - m^2} = \frac{1}{-2p\cdot k}$$

m = 0

Ultra-Violet Divergence

Ultra-violet singularity



$$\sim \int^{\infty} d^4k \frac{k^\mu k^\nu}{k^2 k^2 k^2} \rightarrow \infty \ \text{as} \ |k| \rightarrow \infty$$

- **Vertex with Yukawa coupling must be renormalized.**

Renormalization introduces a renormalization scale μ_R

In principle, μ_R is arbitrary

In practice, μ_R is chosen to be a physical scale Q or $\sqrt{\hat{s}}$

interaction at distance « $1/\mu_R$ or momentum scale » μ_R are integrated out.

Ultra-violet divergences are hidden into quantities which can be measured experimentally: mass, coupling

Running mass for Quarks

As a consequence of renormalization, just like the coupling constant, quark masses also depend on the momentum exchange and renormalization scheme

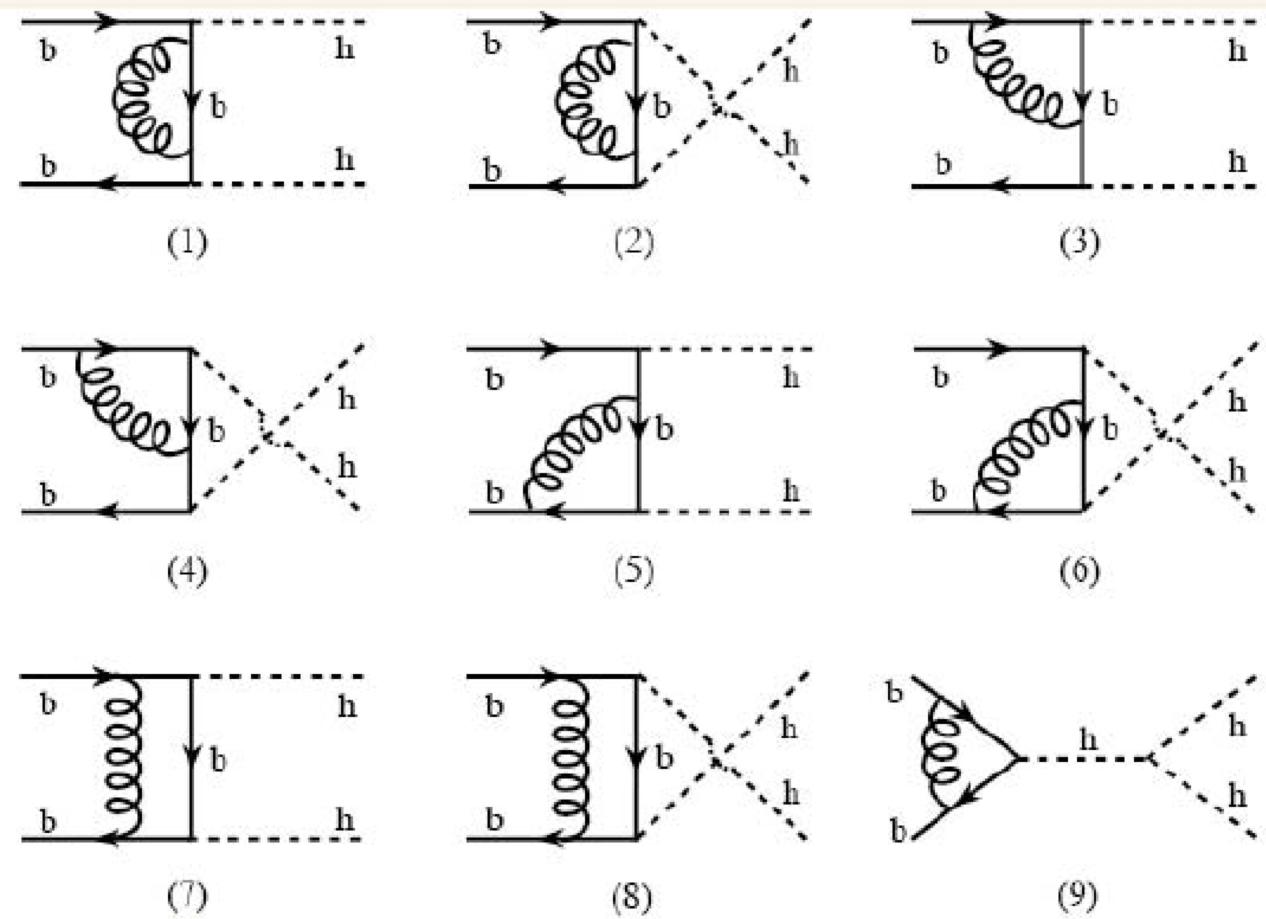
$$\overline{m}(\mu) = \overline{m}(\mu_0)\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\gamma_0/\beta_0} \frac{1 + a_1 \frac{\alpha_S(\mu)}{\pi}}{1 + a_1 \frac{\alpha_S(\mu_0)}{\pi}}$$

$$\gamma_0 = 1$$
$$\gamma_1 = \frac{1}{16}\left(\frac{202}{3} - \frac{20}{9} N_f\right)$$

$$a_1 = -\frac{b_1 \gamma_0}{b_0^2} + \frac{\gamma_1}{b_0}$$

Pole mass: $M_b = \overline{m}(M_b)\left(1 + C_F \frac{\alpha_S(M_b)}{\pi}\right)$

Diagrams with Virtual Gluons



$$M_i \equiv g_s^2 (T^a T^a)_{ji} \hat{M}_d^0 X_i$$

Amplitude of Loop Diagrams

- Amplitude for one loop virtual corrections.

$$M_{loop} = g_s^2 (T^a T^a)_{ji} (X_s \hat{M}_s^0 + X_t \hat{M}_t^0 + X_u \hat{M}_u^0)$$

$$X_s = X_9$$

$$X_t = X_1 + X_3 + X_5 + X_7$$

$$X_u = X_2 + X_4 + X_6 + X_8$$

Virtual corrections contain both UV and IR divergences
UV is removed by renormalization counter term.

- b quark Yukawa coupling is renormalized

$$\frac{\delta m_b}{m_b} = -A \frac{16 \pi \alpha_s}{\varepsilon}$$

$$A = \frac{1}{16 \pi^2} \Gamma(1+\varepsilon)(4 \pi \mu^2)^{\varepsilon}$$

Contributions from Virtual Gluons

Matrix element squared

$$|M_v|^2 = 2\mathrm{Re}(M_{loop} M_0^*) + |M_{CT}|^2$$
$$= A \frac{64 \pi \alpha_s}{3} \{ [-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \ln(\hat{s}) - \frac{3}{2\varepsilon}] |M_0|^2 - |M_D|^2 \}$$

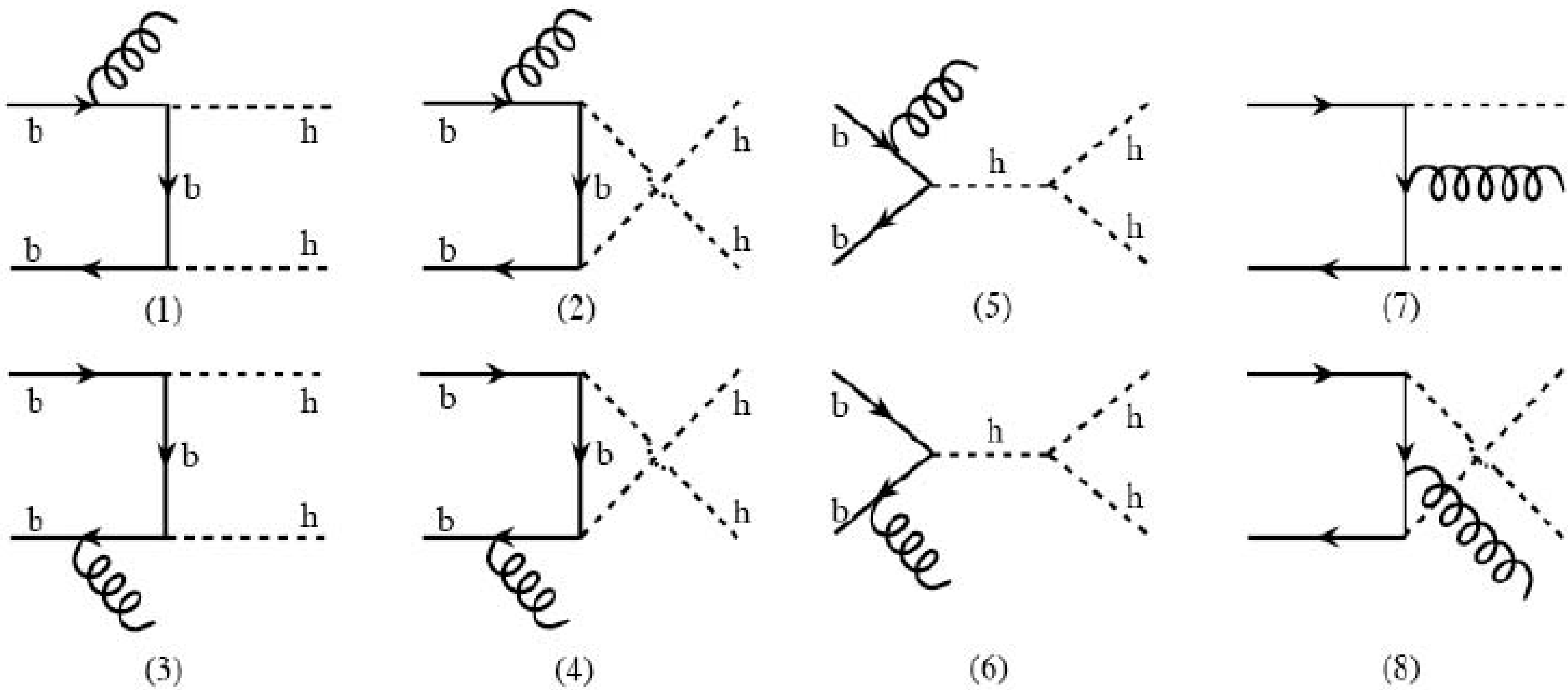
IR and UV divergences finite terms

$|M_D|^2$ includes all remaining finite terms.

IR divergences will be canceled by the IR divergences from real gluon emission diagrams

Real Gluon Emission

Corrections from real gluon emission



there is infrared and collinear singularities (m_b~0)

Soft Cutoff

We introduce a new cutoff parameter δ_s to separate the gluon phase space to soft and hard regions for numerical integration

- soft regions: $E_g \leq \delta_s \frac{\sqrt{\hat{s}}}{2}$

  Infrared and collinear singularities.

- hard regions: $E_g > \delta_s \frac{\sqrt{\hat{s}}}{2}$

  only collinear singularities.

$$\delta\hat{\sigma}_{\alpha_s} = \delta\hat{\sigma}_{v} + \delta\hat{\sigma}_{soft} + \delta\hat{\sigma}_{hard}$$

Corrections from Soft Gluons

- **soft region corrections:**

  We assume gluon momentum p_g is zero every where in the amplitude except in the denominators

  The amplitude is simplified to:

$$M_{soft} = g_s^2 T_{ji}^a \left(\frac{p_2^\mu}{p_2 \cdot p_g} - \frac{p_1^\mu}{p_1 \cdot p_g} \right) (\hat{M}_s^0 + \hat{M}_t^0 + \hat{M}_u^0)$$

infrared and collinear singularities

Three body phase space is simplified to:

$$d\Phi_3 |_{soft} = d\Phi_2 d\Phi_g |_{soft}$$

Set $p_g \to 0$ in δ function.

Phase Space of the Soft Gluon

gluon phase space

$$d\Phi_g|_{soft} = \frac{d^{N-1}p_g}{(2\pi)^{N-1}2E_g} = \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\frac{\pi^{\varepsilon}}{(2\pi)^3}$$

$$\int_0^{\frac{\sqrt{s}}{2}\delta_s} dE_g E_g^{1-2\varepsilon}\int_0^{\pi}\sin^{1-2\varepsilon}\theta_1 d\theta_1\int_0^{\pi}\sin^{-2\varepsilon}\theta_2 d\theta_2$$

Matrix element squared (integrated gluon phase space)

$$|M'_{soft}|^2 = \int d\Phi_g|_{soft}\,|M_{soft}|^2$$

$$= |M_0|^2 A\frac{64\pi\alpha_s}{3}\left[\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\ln(\delta_s^2) - \frac{1}{\varepsilon}\ln(\hat{s})\right.$$

$$\left. + \frac{1}{2}\ln^2(\hat{s}\delta_s^2) - \frac{\pi^2}{3}\right]$$

Cancellation of Infrared Divergences

Virtual diagrams plus soft contribution of real diagrams

Collinear singularity from soft region, will be absorbed into PDF

$$|M_v|^2 + |M'_{soft}|^2$$

$$= A\frac{64\pi\alpha_s}{3}\left(-\frac{1}{\varepsilon}\right)\left[\ln(\delta_s^2)+\frac{3}{2}\right]|M_0|^2$$

$$+ A\frac{64\pi\alpha_s}{3}\left[\frac{1}{2}\ln^2(s\delta_s^2)-\frac{\pi^2}{3}\right]|M_0|^2$$

$$- A\frac{64\pi\alpha_s}{3}|M_D|^2$$

Finite contributions from soft region

Finite virtual contributions

Collinear Cutoff

- **hard** region has collinear singularity

We introduce second new cutoff parameter δ_c to separate the hard region into hard/non-collinear and hard/collinear regions.

hard/collinear regions.

$$\frac{2p_1 \cdot p_g}{E_g\sqrt{\hat{s}}} < \delta_c \quad or \quad \frac{2p_2 \cdot p_g}{E_g\sqrt{\hat{s}}} < \delta_c \Rightarrow \begin{array}{c} -1 < \cos\theta_g < -1+\delta_c \\ 1-\delta_c < \cos\theta_g < 1 \end{array}$$

α_s corrections change to:

$$\delta\hat{\sigma}_{\alpha_s} = \delta\hat{\sigma}_v + \delta\hat{\sigma}_{soft} + \delta\hat{\sigma}_{hard/c} + \delta\hat{\sigma}_{hard/nc}$$

Hard/non-collinear corrections are finite and can be computed easily.

Hard Collinear Corrections

The initial b quark splits into a hard parton b' and a collinear hard gluon .

$$p_{b'} = zp_b \quad \text{and} \quad p_g = (1-z)p_b$$

Matrix element squared factorized to:

$$|\overline{M}_{hard/c}|^2 (b\bar{b} \to hhg)$$
$$\to (4\pi\alpha_s)\mu^{2\varepsilon} |M_0|^2 \frac{-2P_{b'b}(z,\varepsilon)}{z(p_1 - p_g)} + (1 \leftrightarrow 2)$$

Altarelli-Parisi splitting function:

$$P_{b'b}(z,\varepsilon) = C_F \left[\frac{1+z^2}{1-z} - \varepsilon(1-z) \right] = P_{bb}(z) + \varepsilon P'_{bb}(z)$$

Phase Space of the Hard Collinear Gluon

Define a new variable, $s_{bg} = 2p_1 \cdot p_g$

$$0 \le s_{bg} \le \frac{\hat{s}}{2}(1-z)\,\delta_c$$

The gluon phase space change to:

$$\frac{d^{N-1}p_g}{(2\pi)^{N-1}2E_g} = \frac{(4\pi)^{\varepsilon}}{16\pi^2}\frac{1}{\Gamma(1-\varepsilon)}\,dz\,ds_{bg}\,[(1-z)s_{bg}]^{-\varepsilon}$$

Together with matrix element squared, s_{bg} can be integrated out.

$$|\overline{M}_{hard/c}|^2 (b\bar{b} \to hhg)$$
$$\to (4\pi\alpha_s)\mu^{2\varepsilon}|M_0|^2 \frac{2P_{b'b}(z,\varepsilon)}{z\,s_{bg}} + (1 \leftrightarrow 2)$$

Hard Collinear Corrections

The cross section in hard –collinear region:

$$\sigma_{hard\ /c} = \int dx_1 dx_2 \bar{b}(x_2)\hat{\sigma}(b\bar{b} \to hh) \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(-\frac{1}{\varepsilon}\right)\delta_c^{-\varepsilon} \int_{x_1}^{1-\delta_s} P_{bb}(z,\varepsilon)\frac{dz}{z}\left[\frac{(1-z)^2}{2z}\right]^{-\varepsilon} b\left(\frac{x}{z}\right)$$

Absorb this into parton distribution function

At factorization scale μ_f, in $\overline{\mathrm{MS}}$ scheme

$$b(x) = b(x,\mu_f)\left\{1 + \frac{\alpha_s}{2\pi}(4\pi)^{\varepsilon}\Gamma(1+\varepsilon)\left(\frac{1}{\varepsilon}\right)\left[\ln(\delta_s^2) + \frac{3}{2}\right]\right\} + \frac{\alpha_s}{2\pi}(4\pi)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{1}{\varepsilon}\right)\int_{x_1}^{1-\delta_s} P_{bb}(z)\frac{dz}{z} b(x/z)$$

Cancellation of Collinear Divergences

Replace b(x) by b(x,μ_f) and drop terms high order than α_s

Extra terms in LO contributions.

$$\sigma_{LO} = \int dx_1 dx_2 b(x_1,\mu)\overline{b}(x_2,\mu)\hat{\sigma}_{LO}$$

$$+ \int dx_1 dx_2 b(x_1,\mu)\overline{b}(x_2,\mu)\hat{\sigma}_{LO} \frac{4\alpha_s}{3\pi}(4\pi)^{\varepsilon}\Gamma(1+\varepsilon)\left(\frac{1}{\varepsilon}\right)\left[\ln(\delta_s^2)+\frac{3}{2}\right]$$

$$+ \int dx_1 dx_2 \overline{b}(x_2,\mu)\hat{\sigma}_{LO} \frac{\alpha_s}{2\pi}(4\pi)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{1}{\varepsilon}\right) \int_{x_1}^{1-\delta_s} P_{bb}(z,\varepsilon)\frac{dz}{z} b(x_1/z,\mu)$$

To cancel the collinear singularity in soft region

To cancel the collinear singularity in hard collinear region

For simplification, we use $\mu_R = \mu_f = \mu$

α_s Corrections to $b\bar{b} \to hh$

$$\delta\sigma_{\alpha_s} = \sigma_v + \sigma_{soft} + \sigma_{hard/c} + \sigma_{hard/nc}$$

$$= \int dx_1 dx_2 b(x_1,\mu)\bar{b}(x_2,\mu)\hat{\sigma}_D$$

negative

soft

$$+ \int dx_1 dx_2 b(x_1,\mu)\bar{b}(x_2,\mu)\hat{\sigma}_{LO} \times \frac{4\alpha_s}{3\pi}\left\{\left[\frac{1}{2}\ln^2(\hat{s}\delta_s^2) - \frac{\pi^2}{3}\right] - \ln(\mu^2)\left[\ln(\delta_s^2) + \frac{3}{2}\right]\right\}$$

$$+ \frac{\alpha_s}{2\pi} C_F \int dx_1 dx_2 \bar{b}(x_2,\mu)\hat{\sigma}_{LO} \int_{x_1}^{1-\delta_s} \frac{dz}{z} b(x_1/z,\mu) \times \left\{\frac{1+z^2}{1-z}\ln\left[\frac{\hat{s}}{\mu^2}\frac{(1-z)^2}{z}\frac{\delta_c}{2}\right] + (1-z)\right\} + (b \leftrightarrow \bar{b})$$

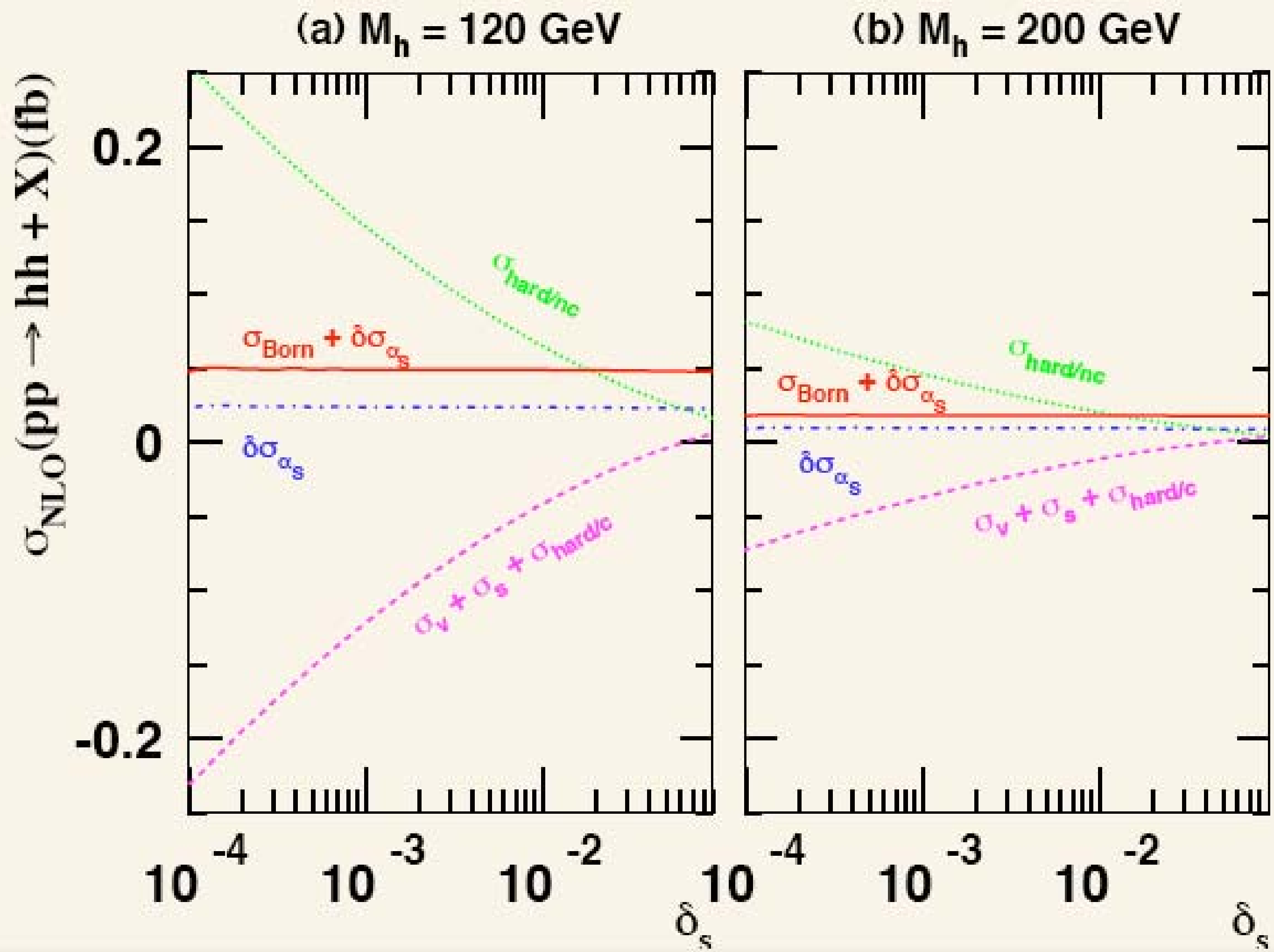
collinear

$$+ \int dx_1 dx_2 b(x_1,\mu)\bar{b}(x_2,\mu)\hat{\sigma}_{hard/nc}$$

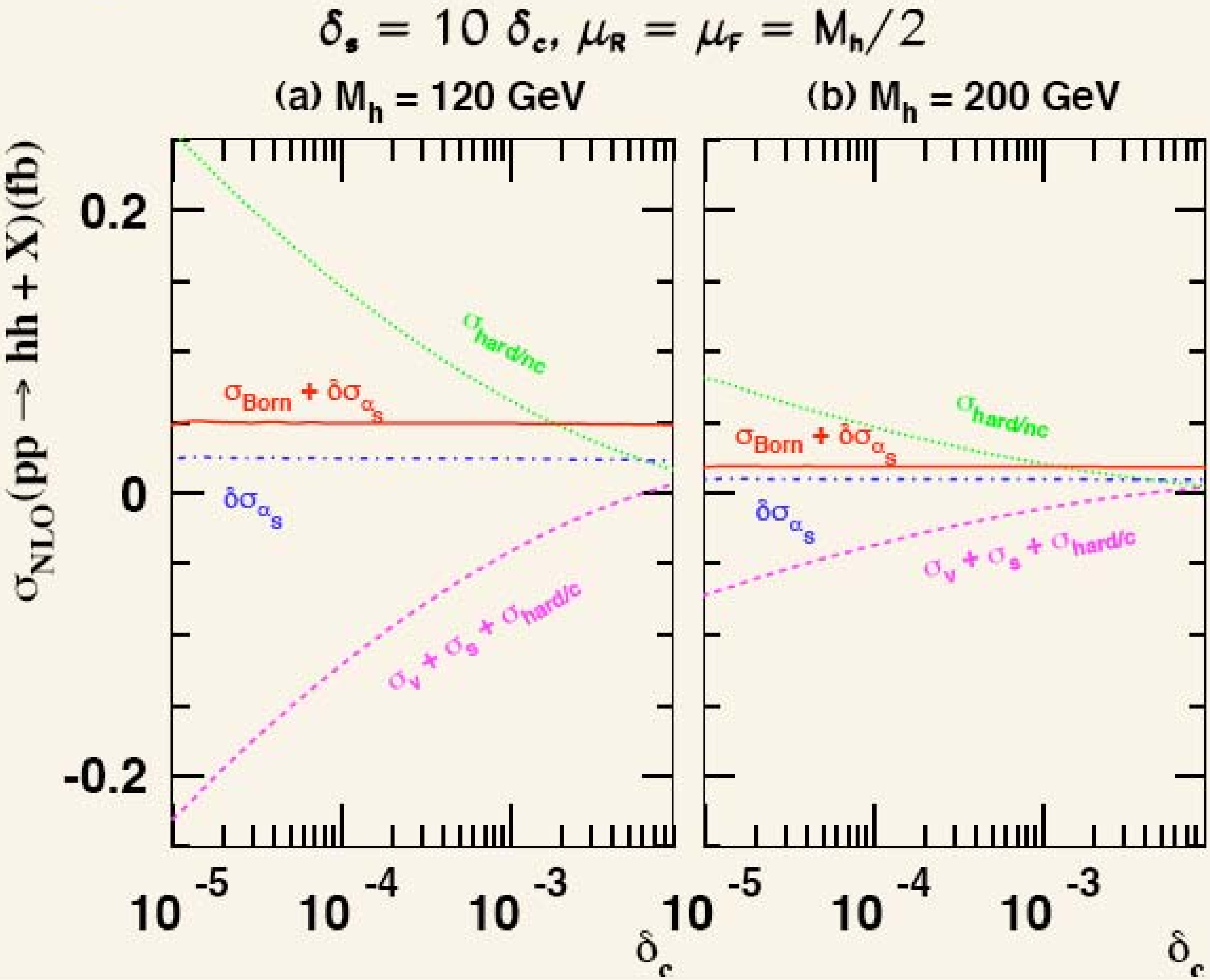
$$+ (1 \leftrightarrow 2)$$

Independence on the Soft Cutoff

$\delta_c = \delta_s/10,\ \mu_R = \mu_F = M_h/2$

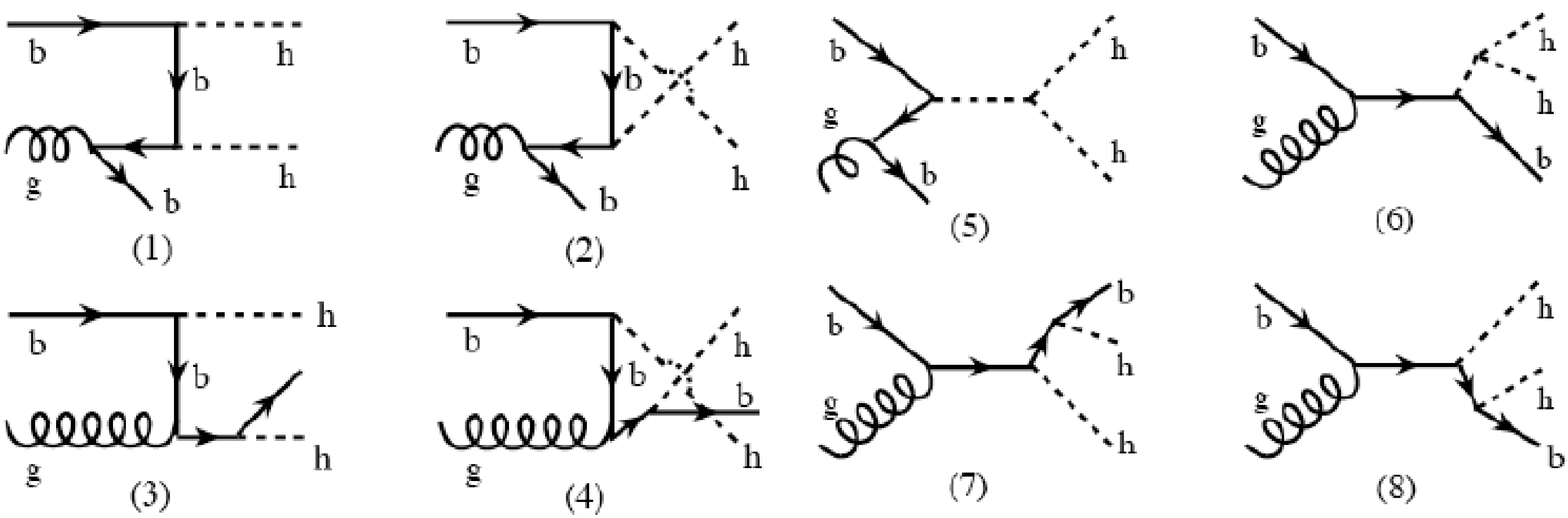


Independence on the Collinear Cutoff



Corrections from $bg \to bhh$

1/Λ corrections from lowest-order b g → b hh



Initial gluon splits into a collinear b $\bar{b}$ pair

diagram (1) , (2) and (5) have collinear singularities

Collinear Cutoff for $bg \to bhh$

only collinear singularity exists

Gluon splits into a pair of collinear b and $\bar{b}$
this singularity is absorbed into
gluon distribution function

We only need one cutoff to separate final b phase space into collinear and non-collinear regions.

collinear regions $\frac{-(p_g - p_b)^2}{E_g \sqrt{\hat{s}}} < \delta_c$

Corrections from bg →bhh is separated to:

$$\delta \hat{\sigma}_{bg} = \delta \hat{\sigma}_c + \delta \hat{\sigma}_{nc}$$

Cancellation of the Collinear Singularity

Cross section in collinear region is simplified to

$$\delta\sigma_{bg/c} = \int dx_1 dx_2 b(x_2)\hat{\sigma}(b\bar{b} \to hh)$$
$$\frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(-\frac{1}{\varepsilon}\right)\delta_c^{-\varepsilon}$$
$$\int_{x_1}^{1} P_{bg}(z,\varepsilon)\frac{dz}{z}\left[\frac{(1-z)^2}{2z}\right]^{-\varepsilon} G(x_1/z)$$

Absorb this divergence into parton distribution function

$$G(x) = G(x,\mu) + \frac{\alpha_S}{2\pi}(4\pi)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{1}{\varepsilon}\right)$$
$$\int_{x_1}^{1} P_{bg}(z)\frac{dz}{z}G(x/z)$$

Contributions from $bg \to bhh$

$$P_{bg}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right] - \varepsilon z(1-z)$$
$$= P_{bg}(z) + \varepsilon P'_{bg}(z)$$

Corrections from bg → bhh

$$\sigma_{bg} = \int dx_1 dx_2 b(x_2) G(x_1) \hat{\sigma}_{LO}(bg \to bhh)$$
$$= \int dx_1 dx_2 b(x_2) G(x_1, \mu) \hat{\sigma}_{LO}(bg \to bhh)$$
$$+ \int dx_1 dx_2 b(x_2) \hat{\sigma}(b\bar{b} \to hh)$$
$$\times \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{1}{\varepsilon}\right) \int_{x_1}^{1} P_{bg}(z) \frac{dz}{z} G(x_1/z, \mu)$$

Collinear cancellation

Cross Section of $bg \to bhh$

$$\delta\sigma_{bg} = \frac{\alpha_s}{2\pi}\int dx_1 dx_2 b(x_2)\int_{x_1}^{1}\frac{dz}{z}G(x_1/z,\mu)\hat{\sigma}_{LO}(b\bar{b} \to hh)$$

$$\times\left\{\frac{z^2+(1-z)^2}{2}\ln\left[\frac{\hat{s}}{\mu^2}\frac{(1-z)^2}{z}\frac{\delta_c}{2}\right]+z(1-z)\right\}$$

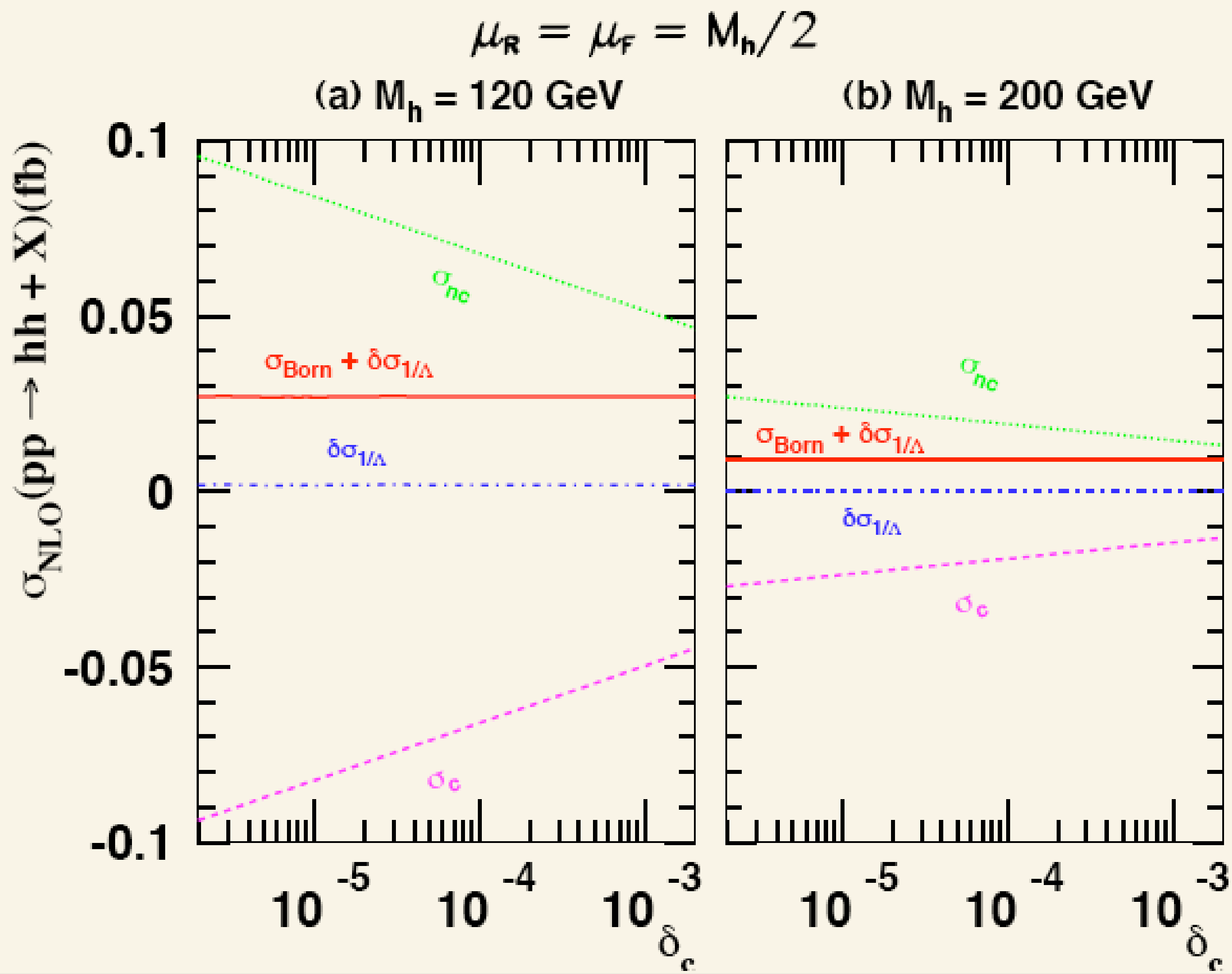
$$+\int dx_1 dx_2 G(x_1,\mu)b(x_2,\mu)\hat{\sigma}_{nc}(b\mathbf{g} \to bhh)$$

$$+\ (1 \leftrightarrow 2)$$

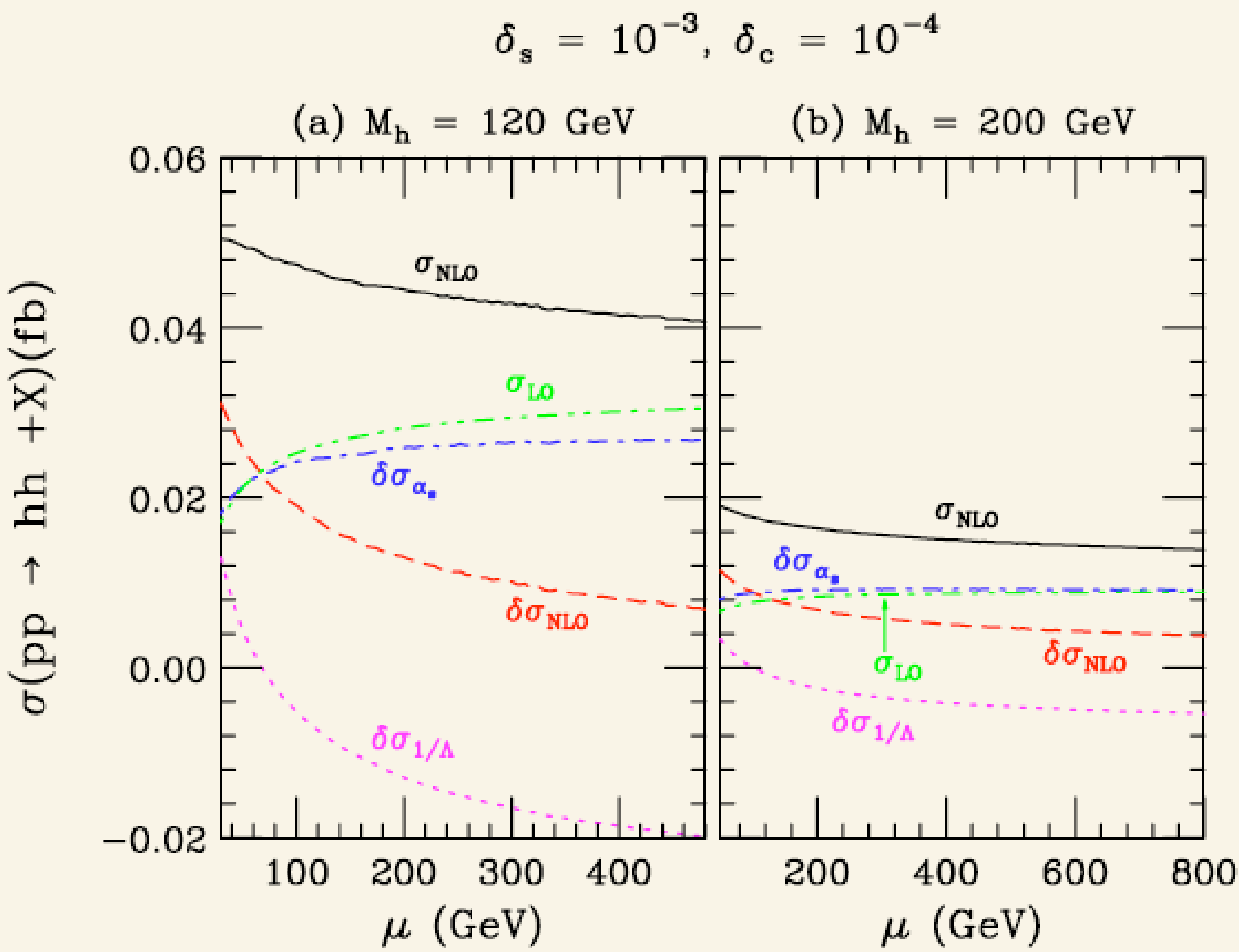
$\bar{b}g \to \bar{b}hh$ Corrections have same results.

$$\delta\sigma_{1/\Lambda} = \delta\sigma_{bg} + \delta\sigma_{\bar{b}g}$$

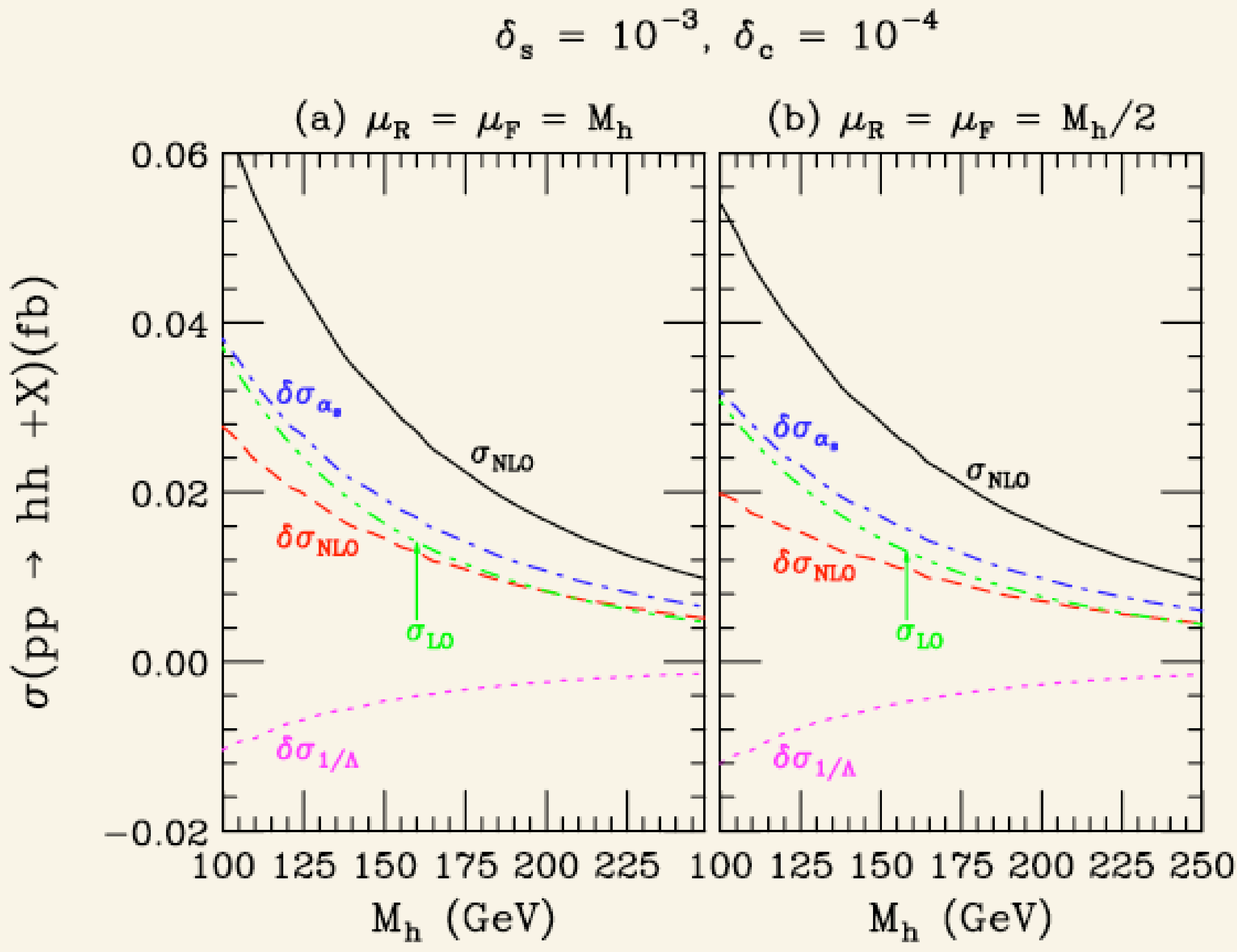
Independence on the Collinear Cutoff



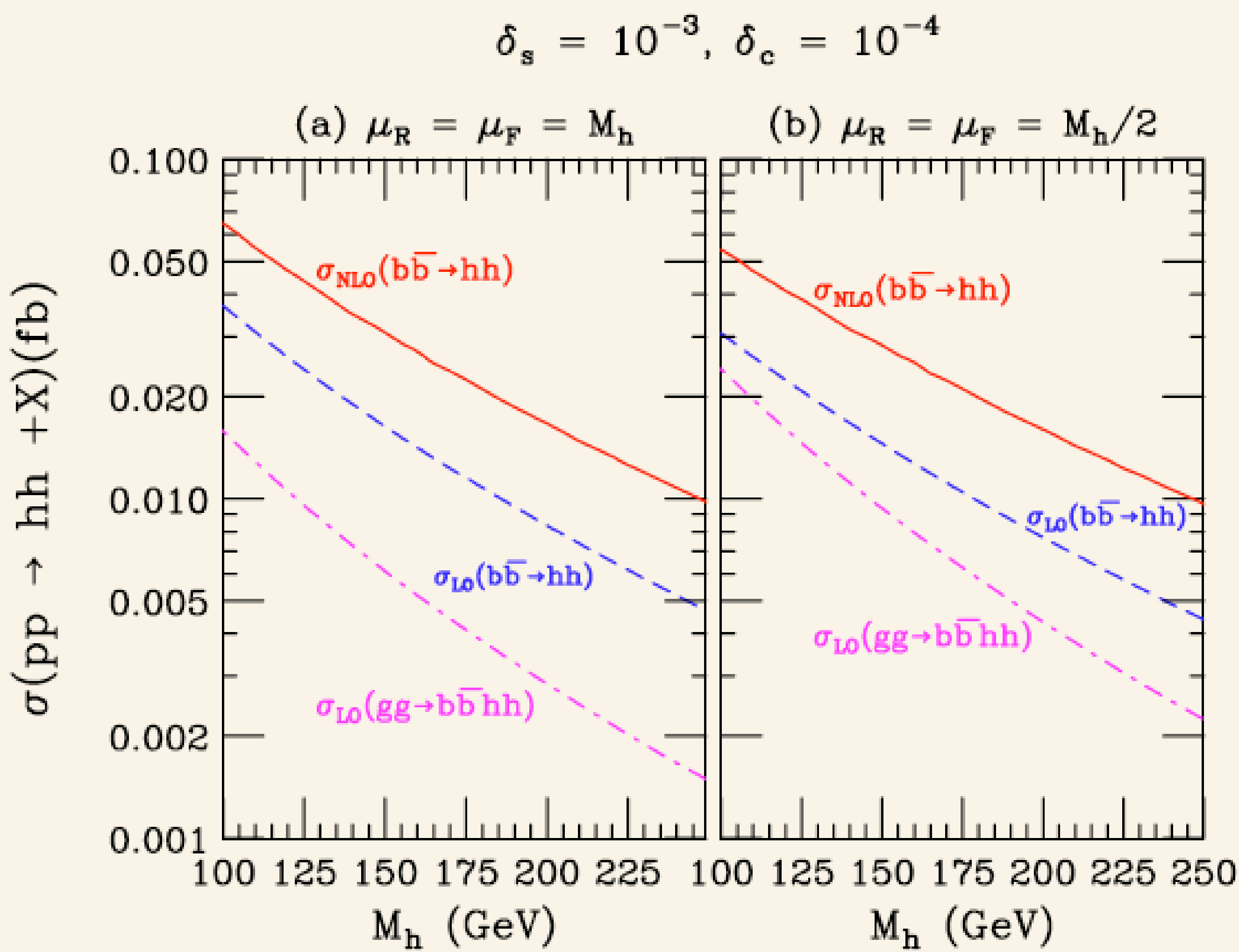
Dependence on μ



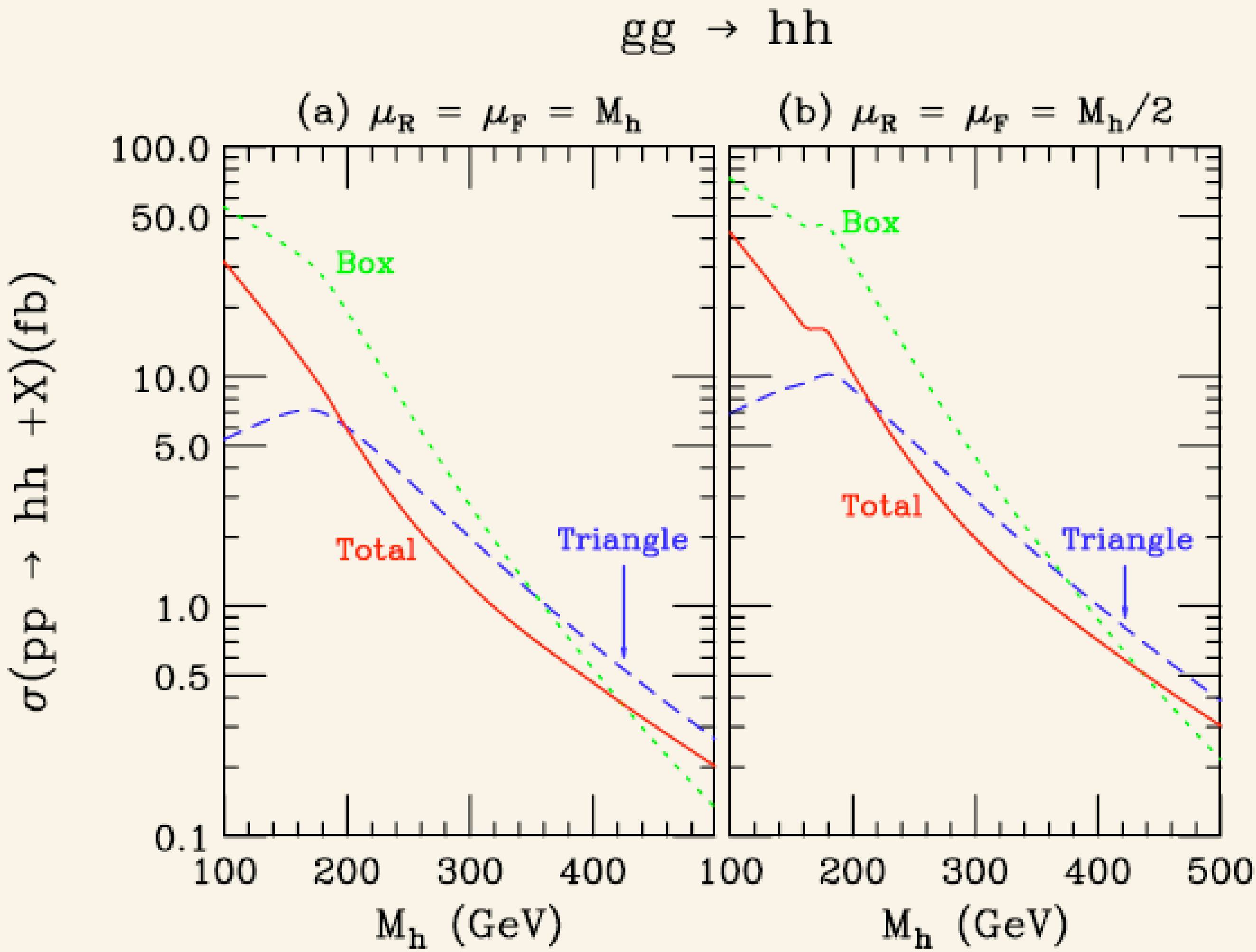
Cross Section versus Higgs Mass



Associated Higgs Pair Production



Higgs Pair Production via Gluon Fusion



Conclusions

- We have presented the NLO corrections to Higgs pair production via bottom quark fusion in the Standard Model.
- Our NLO results are not sensitive to the difference between renormalization and factorization scales and we use the same renormalization and factorization scales.
- The rate of Higgs pair production in the Standard Model is very small, although the NLO corrections significantly increase this rate.
- However, the rate for Higgs pair production will be enhanced in models with large couplings of the Higgs bosons to b quarks.
- Our results are of interest in attempts to measure the trilinear Higgs coupling in such models.

$b\bar{b} \to H$ at NNLO

Harlander and Kilgore (2003)