Probing Dark Energy via Supernova Observations : How Parametrization alters Conclusion

Je-An Gu （顧哲安）

National Center for Theoretical Sciences

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Content

- Introduction

  (Basic Knowledge; Motivation)

- Fits (fitting formulae) to $d_L - z$ Relation [& $w_{\rm DE}(z)$] (parametrization)

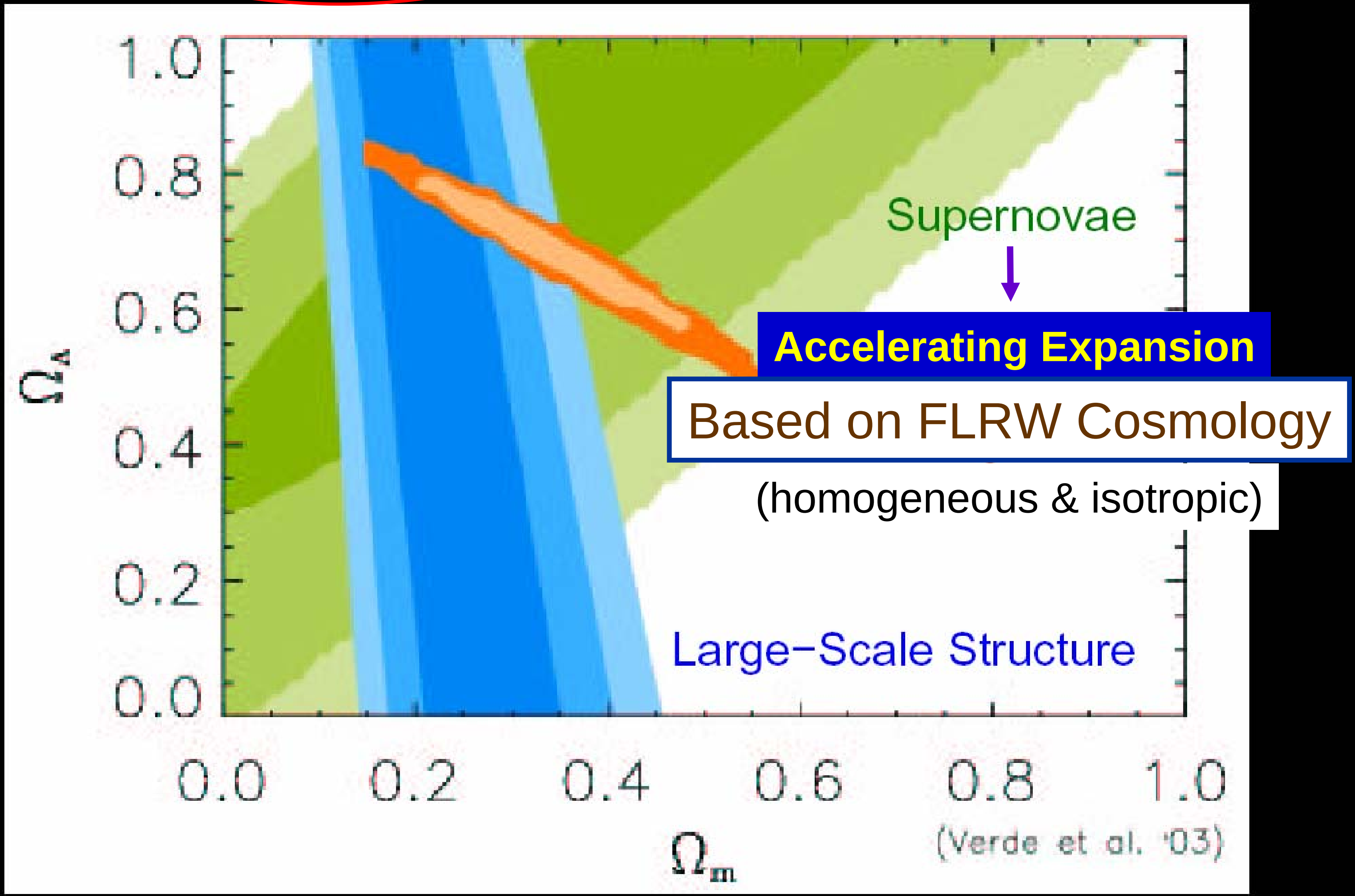
- Reconstruction of $w_{DE}(z)$ and Fit Testing

- Summary and Discussion

Introduction

(Motivation; Basic Knowledge, SNAP)

Concordance: $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$.



Supernova (SN)

Type Ia Supernova (SN Ia) :

– thermonulear explosion of carbon-oxide white dwarfs –

- Correlation between the peak luminosity and the decline rate

  $\Rightarrow$ absolute magnitude M

  $\Rightarrow$ luminosity distance d_L

  (distance precision: $\sigma_{mag} = 0.15$ mag $\rightarrow$ $\delta d_L/d_L \sim 7\%$)

- Spectral information $\rightarrow$ redshift z

SN Ia Data: $d_L(z)$ [i.e, $d_{L,i}(z_i)$]

[~ x(t) ~ position (time)]

Basic Knowledge

- Luminosity distance d_L

F: flux (energy/area×time)

L: luminosity (energy/time)

$$F = L \big/ 4\pi d_L^2$$

FLRW Cosmology $\quad ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$

$d_L(z) = a_0 r(z)(1+z) = r(z)(1+z)$ for $a_0 \equiv 1$ (flat RW),

$r(z)$: coordinate distance

(flat RW) $\quad r(z) = \int_0^z \frac{1}{H(z')} dz'$ $\qquad$ ($1+z = a_0/a$)

Basic Knowledge

- Magnitude m

$$F_2 / F_1 = 100^{(m_1 - m_2)/5}$$

$$(\text{ i.e. } F \propto 100^{-m/5})$$

- Absolute Magnitude $M = m$ (10pc)

$$100^{(m-M)/5} = F_{10\text{pc}} / F = (d_L / 10\text{pc})^2$$

- Distance Modulus $\mu \equiv m - M$

$$\mu \equiv m - M = 5\, log_{10}(d_L / M\text{pc}) + 25$$

SN Ia Data: $d_L(z) \rightarrow \mu(z)$

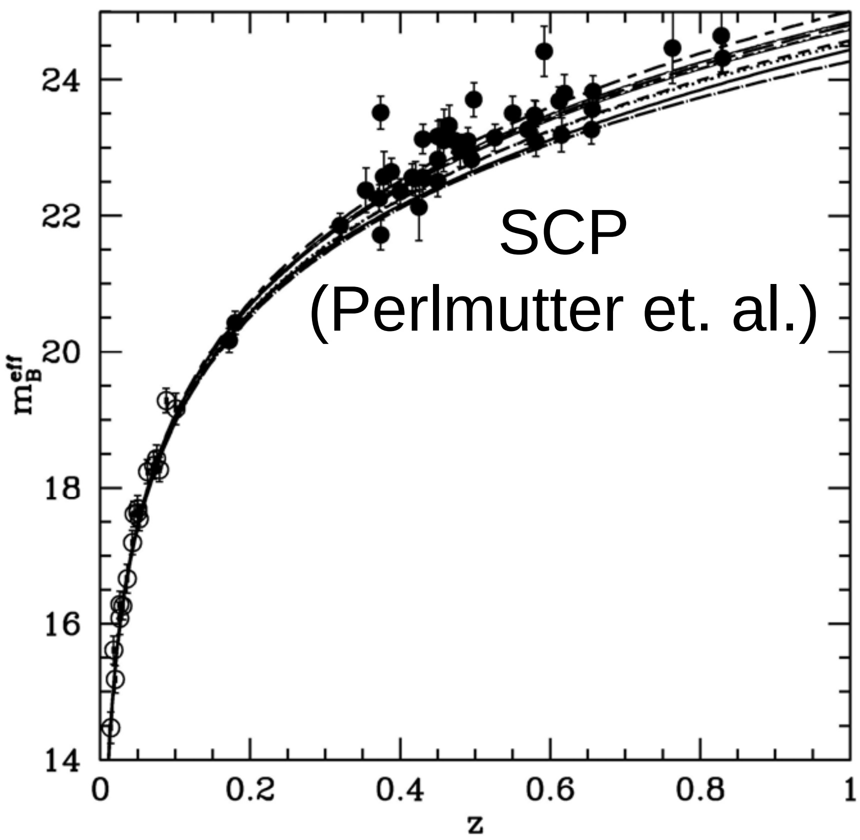
1998

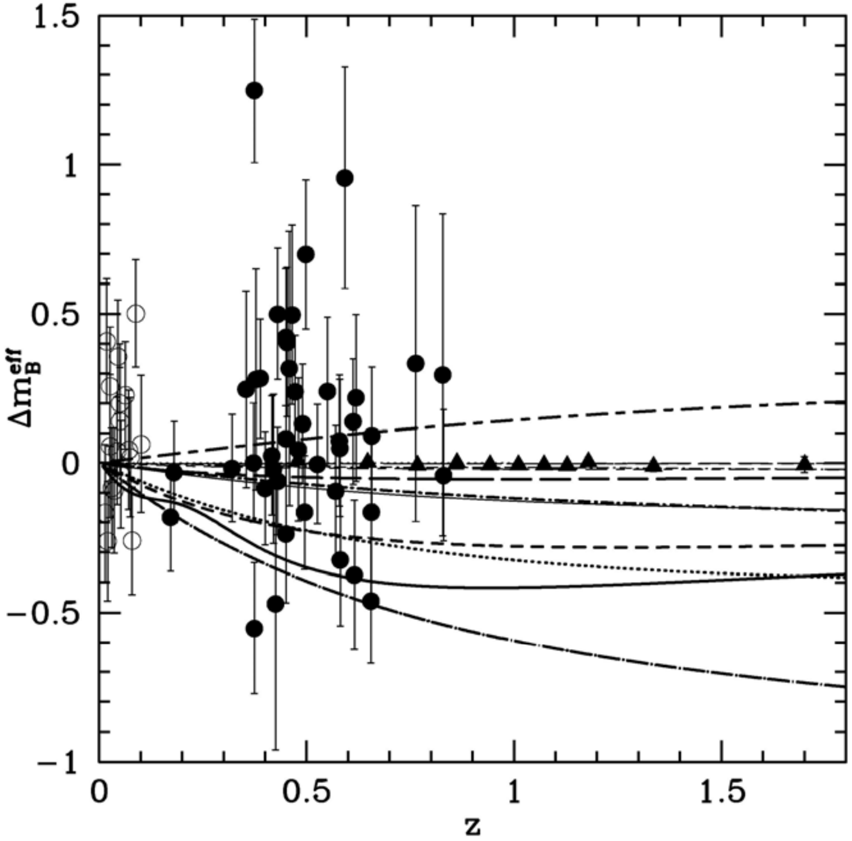
$$\Delta m(z) = m(z) - m_\Lambda(z)$$

↑

fiducial model

$$\Omega_\Lambda = 0.7,\ \Omega_m = 0.3$$





(can hardly distinguish different models)

2004

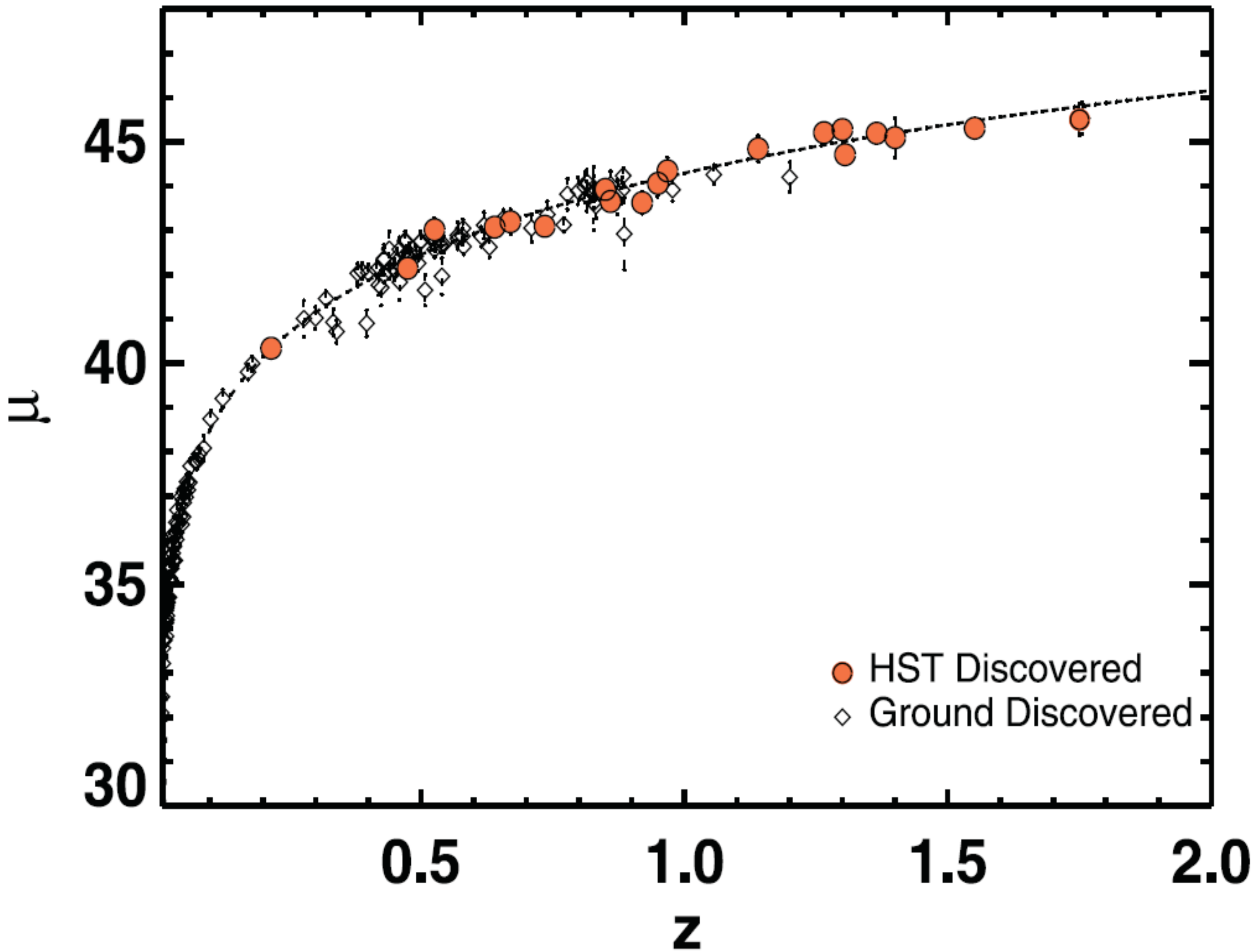


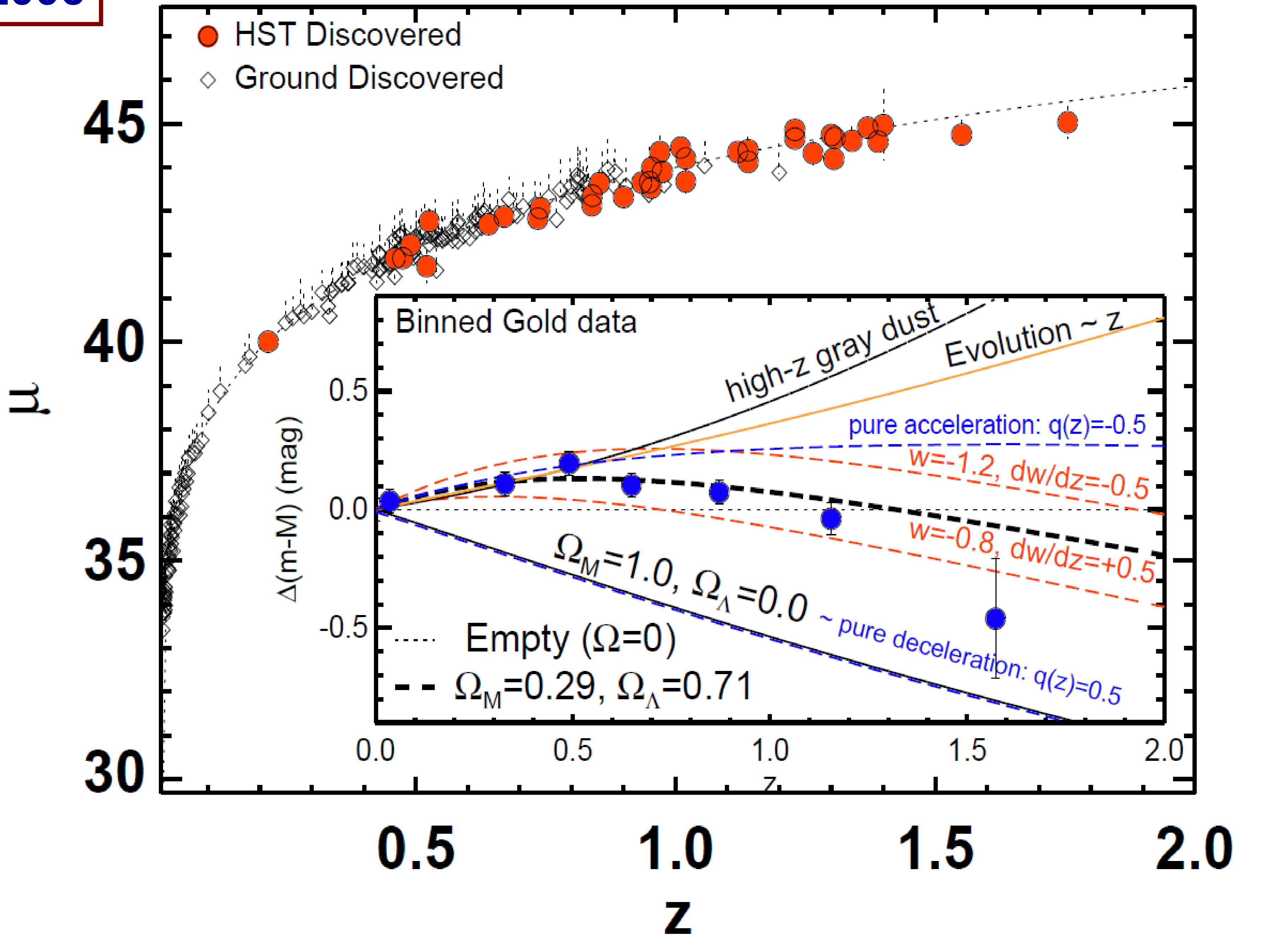
Fig.4 in astro-ph/0402512 [Riess et al., ApJ 607 (2004) 665]

Gold Sample (data set) [MLCS2k2 SN Ia Hubble diagram]

- Diamonds: ground based discoveries
- Filled symbols: HST-discovered SNe Ia
- Dashed line: best fit for a flat cosmology: Ω_M=0.29 Ω_Λ=0.71

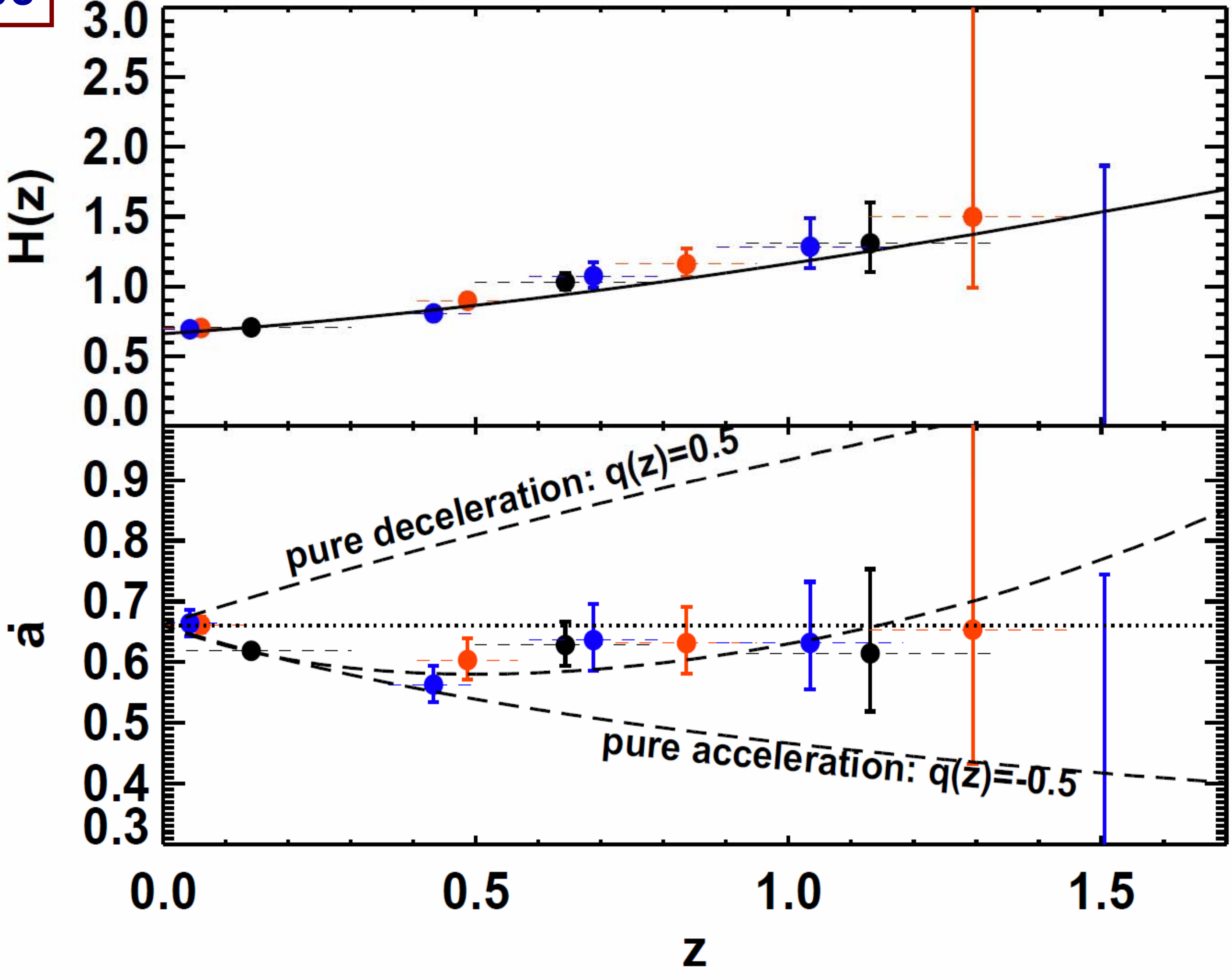
2006

Riess et al. astro-ph/0611572



2006

Riess et al. astro-ph/0611572



Supernova / Acceleration Probe (SNAP)

observe ~2000 SNe in 2 years

statistical uncertainty $\sigma_{\mathrm{mag}} = 0.15$ mag $\rightarrow$ 7% uncertainty in d_L

σ_{sys} = 0.02 mag at z =1.5

σ_z = 0.002 mag (negligible)

z	0~0.2	0.2~1.2	1.2~1.4	1.4~1.7
# of SN	50	1800	50	15

Observations

mapping out the evolution history

(e.g. SNe Ia , Baryon Acoustic Oscillation)

Phenomenology

Data Analysis

(invoking “fitting formula” / “parametrization”)

Model / Theory

Motivation

- Potential impact of improved future SNe data on our understanding of dark energy.
  [To obtain DE info, e.g. $w_{\mathrm{DE}}(z)$]

- Answering the questions: $\boldsymbol{w_{DE} = -1}$ **?** $\boldsymbol{w_{DE}' = 0}$ **?**

- Showing the importance of a "good" fit to the $d_L - z$ relation in order to draw conclusion on above issues.

Observations / Data

mapping out the evolution history

(e.g. SNe Ia , Baryon Acoustic Oscillation)

Fitting Formula / Parametrization

(model-independent ?)

$$\left(e.g. \quad d_L(z) = \sum_{i=0}^{N} c_i z^i ; \quad w_\phi = \sum_{i=o}^{N} w_i (1+z)^i \right)$$

Information about Physical Quantities

Characterizing our universe or DE models
to be reconstructed

(e.g. w_ϕ , ρ_ϕ , statefinders{r,s} , ……)

Fits (Fitting Formulae) to $d_L - z$ Relation

Two Fits

Fit 1

$$d_L(z) = \sum_{i=0}^{N} c_i z^i$$

polynomial fit of d_L

$\{ d_L(0) = 0 \Rightarrow c_0 = 0 \}$

Huterer & Turner, 1999, PRD

Fit 2

$$w_\phi(z) = \sum_{i=0}^{N} w_i (1+z)^i$$

Maor, Brustein, & Steinhardt, 2001, PRL

Astier, astro-ph/0008306

Weller & Albrecht, 2001, PRL

Fit 1

$$d_L(z) = \sum_{i=0}^{N} c_i z^i$$ (polynomial fit of d_L) $\{ d_L(0) = 0 \Rightarrow c_0 = 0 \}$

(flat RW)
$$\left\{ \begin{array}{ll} \dfrac{\dot{\rho}_\phi}{\rho_\phi} = -3H(1+w_\phi) & a^{-1} = 1+z \\ H = \dfrac{\dot{a}}{a} = \sqrt{\dfrac{8\pi G}{3}(\rho_m + \rho_\phi)} & r(z) = \displaystyle\int_0^z \frac{1}{H(z')} dz' \end{array} \right\} \quad (a_0 \equiv 1)$$

$1+w_\phi \leftarrow H$ & ρ_ϕ $\quad (H = 1/r')$

$\rho_\phi \leftarrow H$ (or r') & $\rho_m(z)$ (well known)

$[r(z) = d_L(z)/(1+z)] \leftarrow$ [data ➔ d_L or c_i] $\quad$ **($\Omega_i \equiv \rho_i / \rho_c$)**

$$\Rightarrow \quad 1 + w_\phi = \frac{1+z}{3} \frac{3\Omega_m H_0^2 (1+z)^2 + 2r''/r'^3}{\Omega_m H_0^2 (1+z)^3 - 1/r'^2}$$

Fit 2

$$w_\phi = \sum_{i=0}^{N} w_i (1+z)^i$$

$$d_L^{fit}(z) = \frac{1+z}{H_o} \int_0^z \frac{(1+z')^{-3/2} dz'}{\sqrt{\Omega_m + \Omega_\phi (1+z')^{3w_0} \exp\left\{3 \sum_{i=1}^{N} \left[(1+z')^i - 1\right] w_i / i \right\}}}$$

$$\left(\Omega_\phi = 1 - \Omega_m\right)$$

Other Fits / Parametrizations

- $w(a) = w_0 + w_a z/(1+z)$

  Linder 2003, PRL 90, 091301

- $$h(x) = \frac{H(x)}{H_0} = \sqrt{\Omega_m x^3 + A_0 + A_1 x + A_2 x}$$

  $$x = 1 + z\,, \quad A_0 + A_1 + A_2 = 1 - \Omega_m$$

  Alam, Sahni, Saini & Starobinsky, astro-ph/0311364

Reconstruction of $w_{DE}(z)$

Testing Parametrizations

(Weller & Albrecht 2001, PRD, astro-ph/0106079)

Procedures of Testing Parametrizations

Background cosmology
[pick a **DE Model** with $\boldsymbol{w_{DE}^{th}(z)}$]

↓

generate **Data** w.r.t. SNAP
(Monte Carlo simulation)

employ a **Fit** to the $d_L(z)$ or $w(z)$

↘ ↙

reconstruct $\boldsymbol{w_{DE}^{exp't}(z)}$

↓

compare $\boldsymbol{w_{DE}^{exp't}}$ and $\boldsymbol{w_{DE}^{th}}$

Fit 1

$$d_L^{\rm fit}(z) = \sum_{i=0}^{N} c_i z^i$$

background cosmology: ΛCDM { $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ }

$$\left(\Omega_i \equiv \rho_i / \rho_c \right)$$

$$1 + w_\phi = \frac{1+z}{3} \frac{3\Omega_m H_0^2 (1+z)^2 + 2r''/r'^3}{\Omega_m H_0^2 (1+z)^3 - 1/r'^2}$$

- Best Fit: Minimizing the χ^2 function: (function of c_i's)

$$\chi^2(\{c_i\}) = \sum_{k=0}^{N_z} \left(\frac{d_L^{\exp't}(z_k) - d_L^{fit}(z_k)}{\delta d_L(z_k)} \right)^2$$

- Error Evaluation: Gaussian error propagation: [from $d_L(z)$ to w(z)]

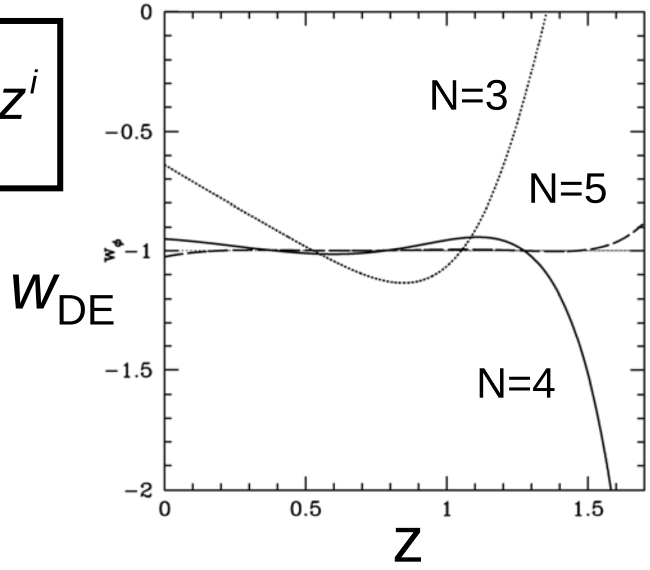
$d_L(z) \rightarrow w(z)$

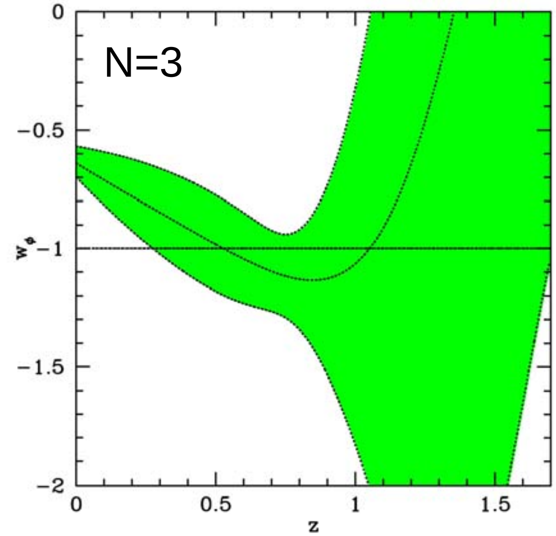
$$\delta w_\phi^2 = \sum_{ij} \frac{\partial w_\phi}{\partial c_i} \frac{\partial w_\phi}{\partial c_j} \sigma_{ij}$$

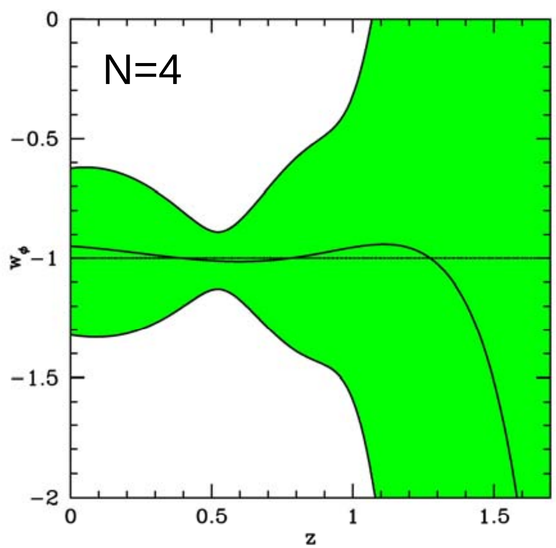
σ_{ij} : covariant matrix of the simulated sample of c_i

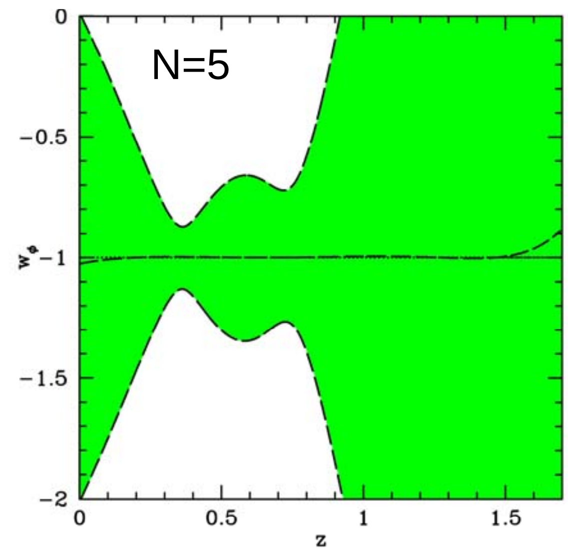
$$\delta d_L(z_k) = \sigma_{mag} d_L(z_k)(\ln 10)/5$$

$$d_L(z) = \sum_{i=0}^{N} c_i z^i$$









mean value: N=3 → wrong; N=4.5 → ok

error bar: N=4 → best reconstruction for w_ϕ

but only $-1.3 < w_\phi < -0.7$ at the 1σ level

Fit 2

$$w_\phi = \sum_{i=0}^{N} w_i (1+z)^i$$

$$d_L^{fit}(z) = \frac{1+z}{H_o} \int_0^z \frac{(1+z')^{-3/2} dz'}{\sqrt{\Omega_m + \Omega_\phi (1+z')^{3w_0} \exp\left\{ 3 \sum_{i=1}^{N} \left[(1+z')^i - 1 \right] w_i / i \right\}}} \quad \left(\Omega_\phi = 1 - \Omega_m \right)$$

- Best Fit: minimizing the χ^2 function:

$$\chi^2(\{w_i\}) = \sum_{k=0}^{N_z} \left(\frac{d_L^{exp't}(z_k) - d_L^{fit}(z_k)}{\delta d_L(z_k)} \right)^2$$

- Error Bar: Gaussian error propagation:

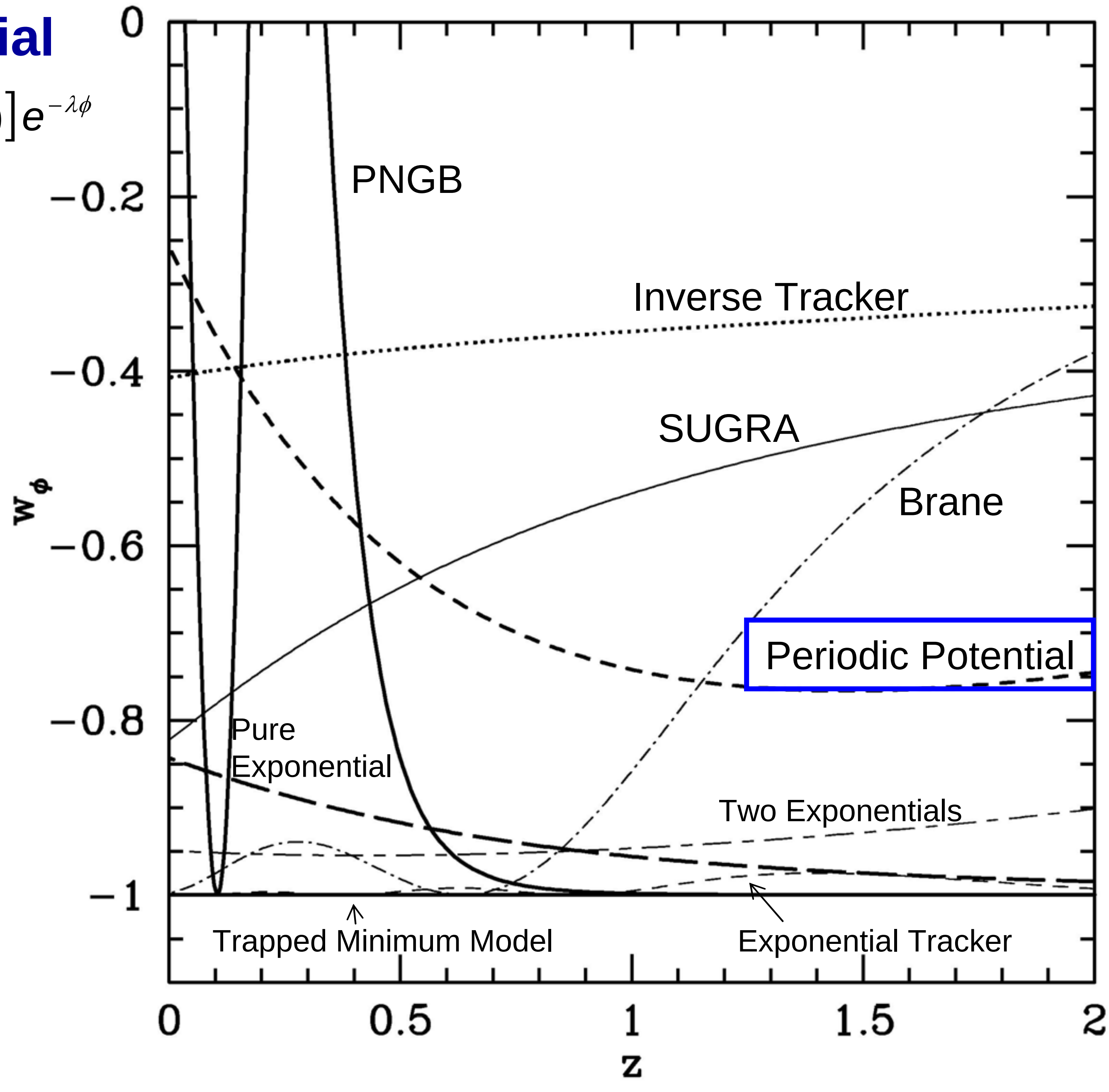
$$\delta w_\phi^2 = \sum_{i\,j} \frac{\partial w_\phi}{\partial w_i} \frac{\partial w_\phi}{\partial w_j} \sigma_{ij} = \sum_{i\,j} (1+z)^{i+j} \sigma_{ij}$$

$$\delta d_L(z_k) = \sigma_{mag} d_L(z_k) (ln 10)/5$$

Background Cosmology:

Periodic Potential

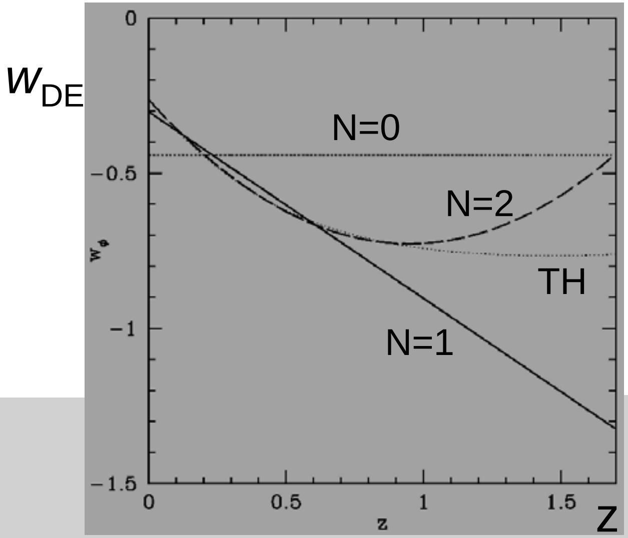
$$V(\phi) = V_0 [1 + \delta \sin(\beta\phi)] e^{-\lambda\phi}$$

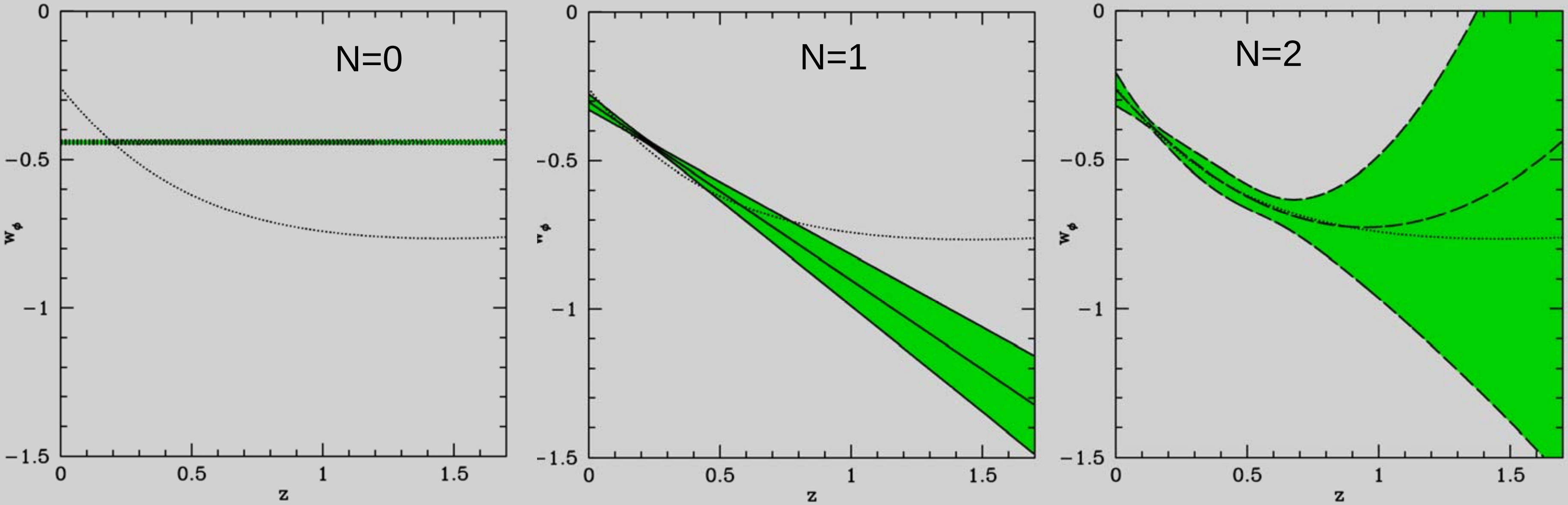


Background Cosmology:

Periodic Potential

$$V(\phi) = V_0 \left[1 + \delta \sin(\beta\phi)\right] e^{-\lambda\phi}$$





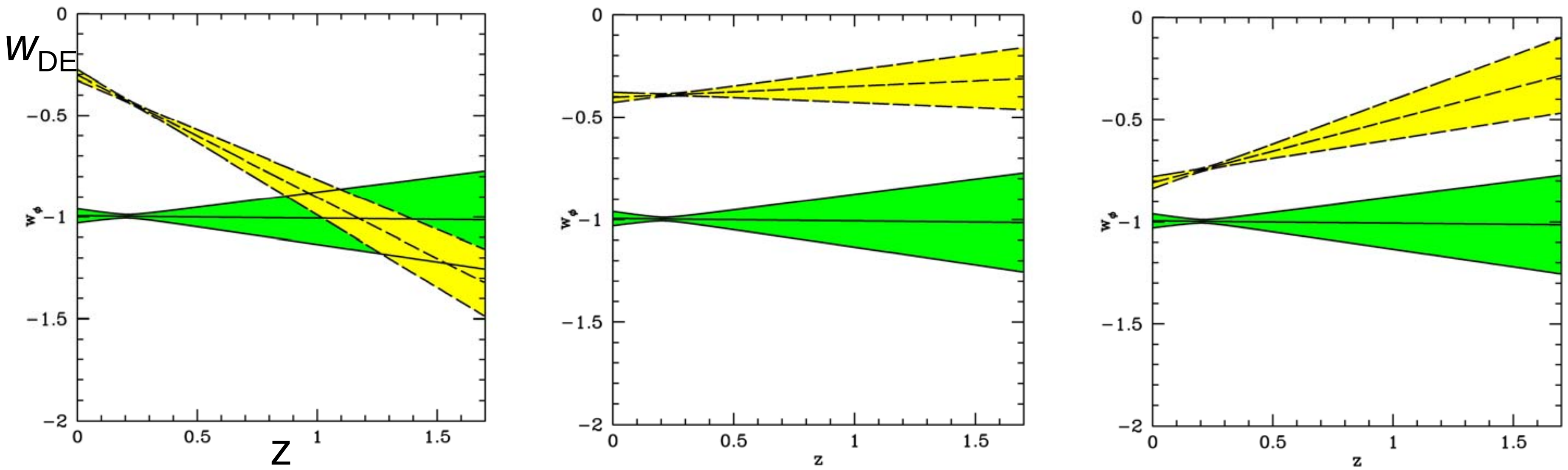
N=0 fit: w_ϕ = const cannot reproduce the evolving model

N=1 fit: poor for z > 0.6

N=2 fit: better

Error bars: {N=1} ~ {N=2} for z < 0.7 ; {N=2} increases rapidly for z > 0.7

Distinguishing Λ and other models via $\underline{N=1 \text{ fit}}$ $(\Omega_m=0.3)$



Periodic Potential

$$V(\phi) = V_0[1+\delta\sin(\beta\phi)]e^{-\lambda\phi}$$

Inverse Tracker

$$V(\phi) = M^{4+\alpha}/\phi^{\alpha}$$

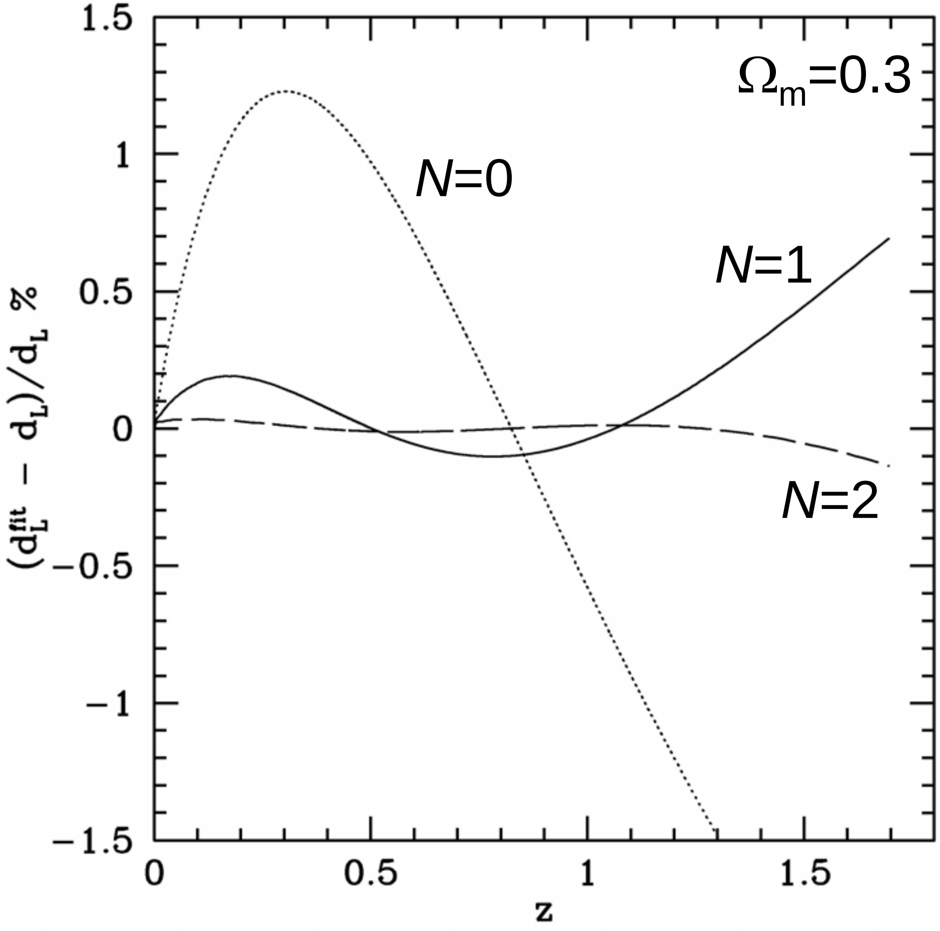
SUGRA

$$V(\phi) = M^{4+\alpha}\phi^{-\alpha}\exp[(\phi/M_{pl})^2/2]$$

Background Cosmology:

Periodic Potential

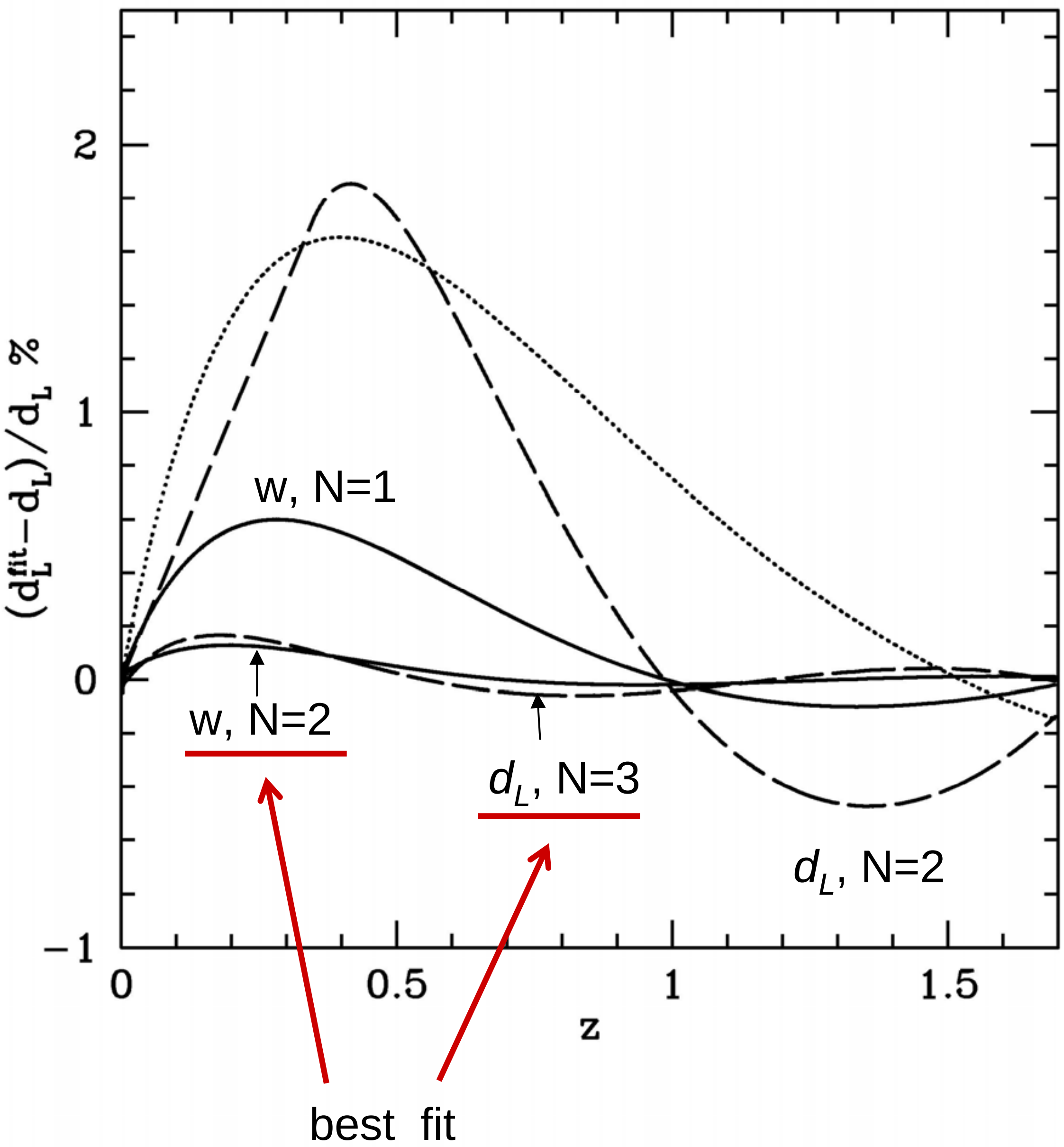
$$V(\phi) = V_0\left[1 + \delta \sin(\beta\phi)\right]e^{-\lambda\phi}$$



N=0, $\chi^2 = 31$ (poor)

N=1, $\chi^2 = 0.47$ (good enough)

N=2, $\chi^2 = 7.3\times10^{-3}$



Background Cosmology:

Periodic Potential

$$V(\phi) = V_0\left[1 + \delta \sin(\beta\phi)\right]e^{-\lambda\phi}$$

(Ω_m=0.3)

Can we reconstruct an evolving w_ϕ with SNe ?

$$w_\phi(z) = \sum_{i=0}^{N} w_i (1+z)^i = \sum_{i=0}^{N} \tilde{w}_i z^i$$

$$\tilde{w}_i = \sum_{k=0}^{N} \binom{k}{i} w_k$$

Gaussian error propagation : $\delta w_i^2 = \sum_{kl} \binom{k}{i}\binom{l}{i} \sigma_{kl}$

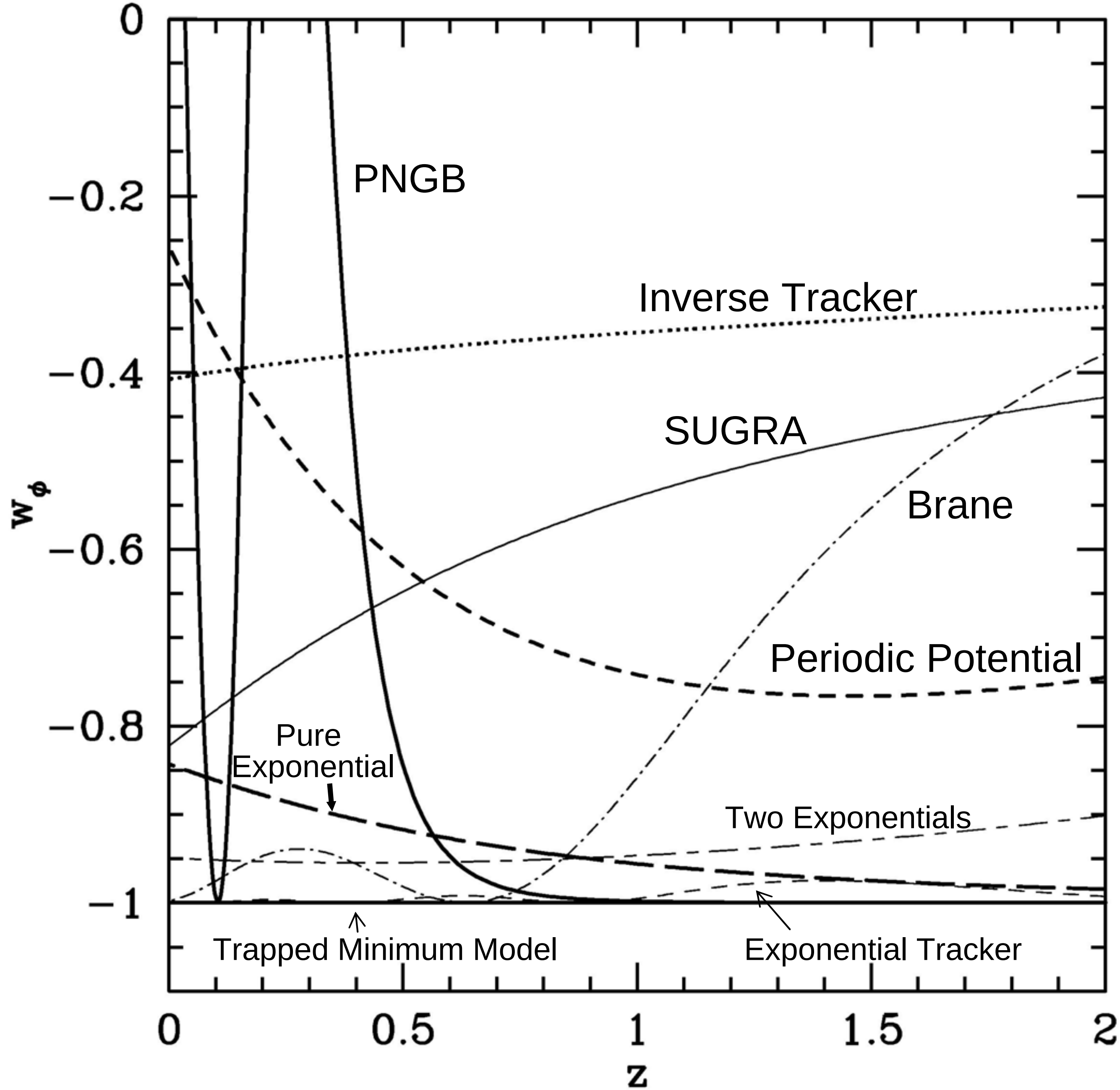
Use $N = 1$ fit : $\tilde{w}_0 = w_0 + w_1$, $\tilde{w}_1 = w_1$

The evolution coefficients with error bars for the linear fit

$$w_\phi = \tilde{w}_0 + \tilde{w}_1 z$$

"+" : evolution ; "–" : no evolution; "0" : marginal evolution

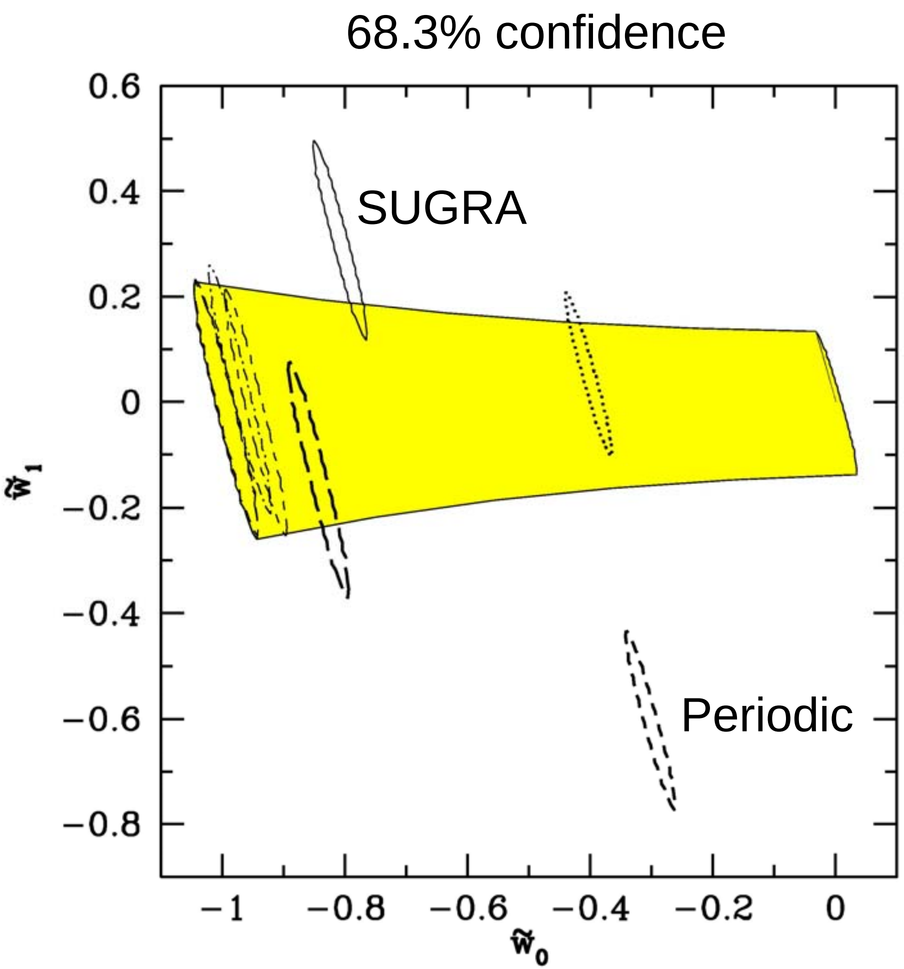
	$\tilde{w}_0$	$\delta\tilde{w}_0$	$\tilde{w}_1$	$\delta\tilde{w}_1$	Theoretical evolution	Evolution reconstructed
Λ	−1.00	0.035	−0.011	0.16	−	−
trapped minimum	−0.99	0.035	−0.0057	0.16	−	−
Brane	−0.97	0.034	0.028	0.16	+	−
two exponentials	−0.95	0.034	−0.016	0.16	−	−
periodic	−0.30	0.027	−0.60	0.11	+	+
pure exponential	−0.84	0.033	−0.14	0.15	0	0
PNGB	−0.00	0.025	−0.94	0.10	+	+
SUGRA	−0.81	0.029	0.31	0.13	+	+
exponential tracker	−1.00	0.035	−0.011	0.16	−	−
inverse tracker	−0.40	0.025	0.054	0.10	−	−

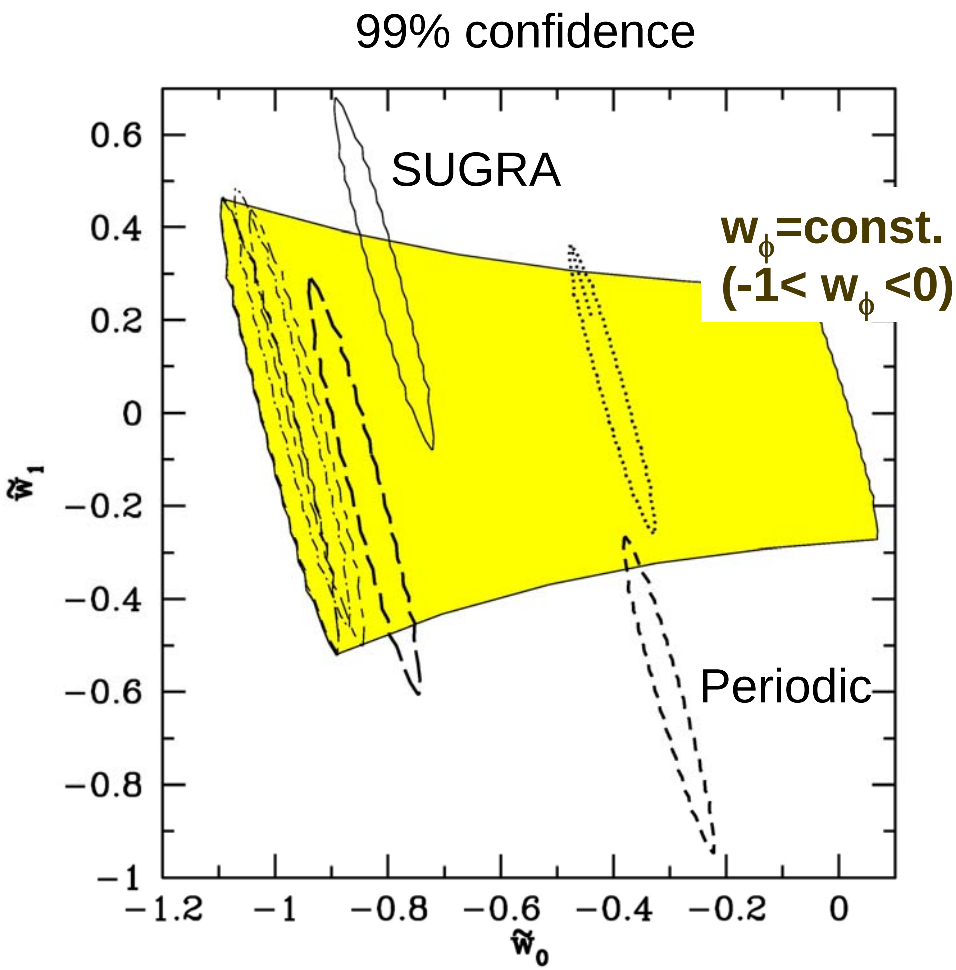


PNGB
Inverse Tracker
SUGRA
Brane
Periodic Potential
Pure Exponential
Two Exponentials
Trapped Minimum Model
Exponential Tracker
w_ϕ
z
0
−0.2
−0.4
−0.6
−0.8
−1
0
0.5
1
1.5
2

$$w_\phi = \widetilde{w}_0 + \widetilde{w}_1 z$$

Distinguish w_ϕ = const and evolving w_ϕ





Conclusion

- Fit 2 $\left[w_\phi = \sum_{i=0}^{N} w_i (1+z)^i \right]$ better
- SNAP data + {w, N=1} fit
  - distinguishing Λ and other models? **Good**
  - telling us whether w_ϕ evolves? **Not so good**

Another Example

- Alam, Sahni, and Starobinsky, 2004, JCAP, astro-ph/0403687

  "***The case for Dynamical dark energy revisited***"

  (*Hey! Surprise! Constant* w_{DE} *is disfavored!!*)

- Jönsson, Goobar, Amanullah, and Bergström, 2004, JCAP, astro-ph/0404468

  "***No evidence for dark energy metamorphosis?***"

  (*You must be kidding me ……*)

Alam, Sahni, and Starobinsky, 2004

$$\rho_{DE}(x) / \rho_{0c} = A_0 + A_1 x + A_2 x^2 \qquad x \equiv 1 + z$$

(~ a truncated Taylor series of the dark energy density)

- $$w_{DE}(x) = -1 + \frac{A_1 x + 2A_2 x^2}{3(A_0 + A_1 x + A_2 x^2)}$$

- Flatness : $\Omega_m + \Omega_{DE} = 1$ ➔ $A_0 + A_1 + A_2 = 1 - \Omega_m$

  Prior : $\Omega_m = 0.3$

($\Omega_i \equiv \rho_i / \rho_c$)

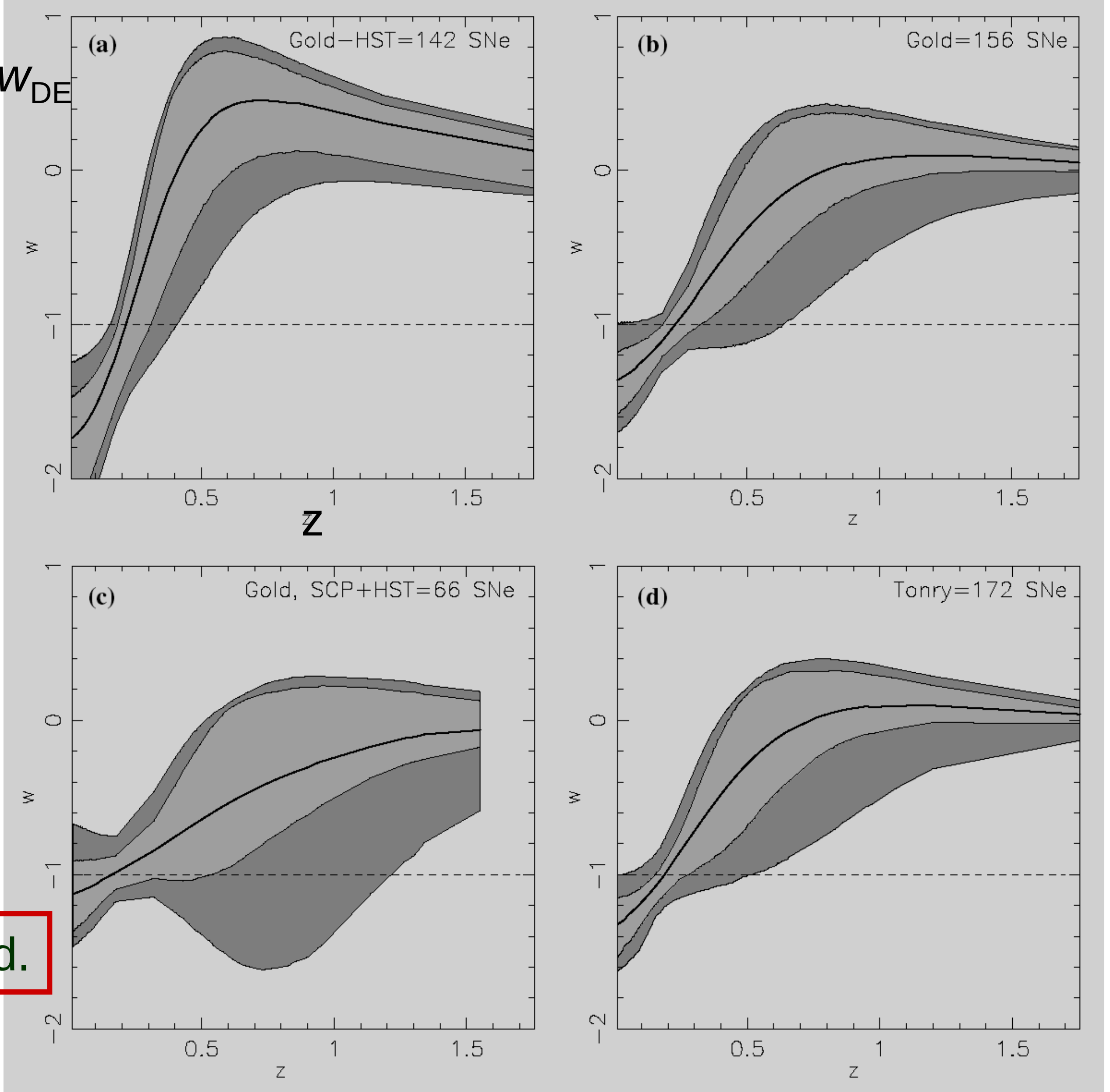
Alam, Sahni, and Starobinsky, 2004

- Parametrization :

$$\frac{\rho_{DE}(x)}{\rho_{0c}} = A_0 + A_1 x + A_2 x^2$$

- Prior :

  flatness ; $\Omega_m = 0.3$



⇓

Constant w_{DE} is disfavored.

Testing the parametrization: $\rho_{DE}(x)/\rho_{0c} = A_0 + A_1 x + A_2 x^2$

Parametrization vs. Reality

$$\begin{cases} \text{Reality}: \; f_R(x) \sim \sin x \\ \text{Parametrization}: \; f_P(x) = a_0 + a_1 x \end{cases}$$

In this case, it is not surprising to have something wrong.

$$\begin{cases} \text{Reality}: \; f_R(x) = \text{constant} \\ \text{Parametrization}: \; f_P(x) = a_0 + a_1 x \end{cases}$$

In this case, $f_p(x)$ can exactly realize $f_R(x)$.
Naively, this parametrization should be good to obtain info.

Jönsson, Goobar, Amanullah, & Bergström, 2004

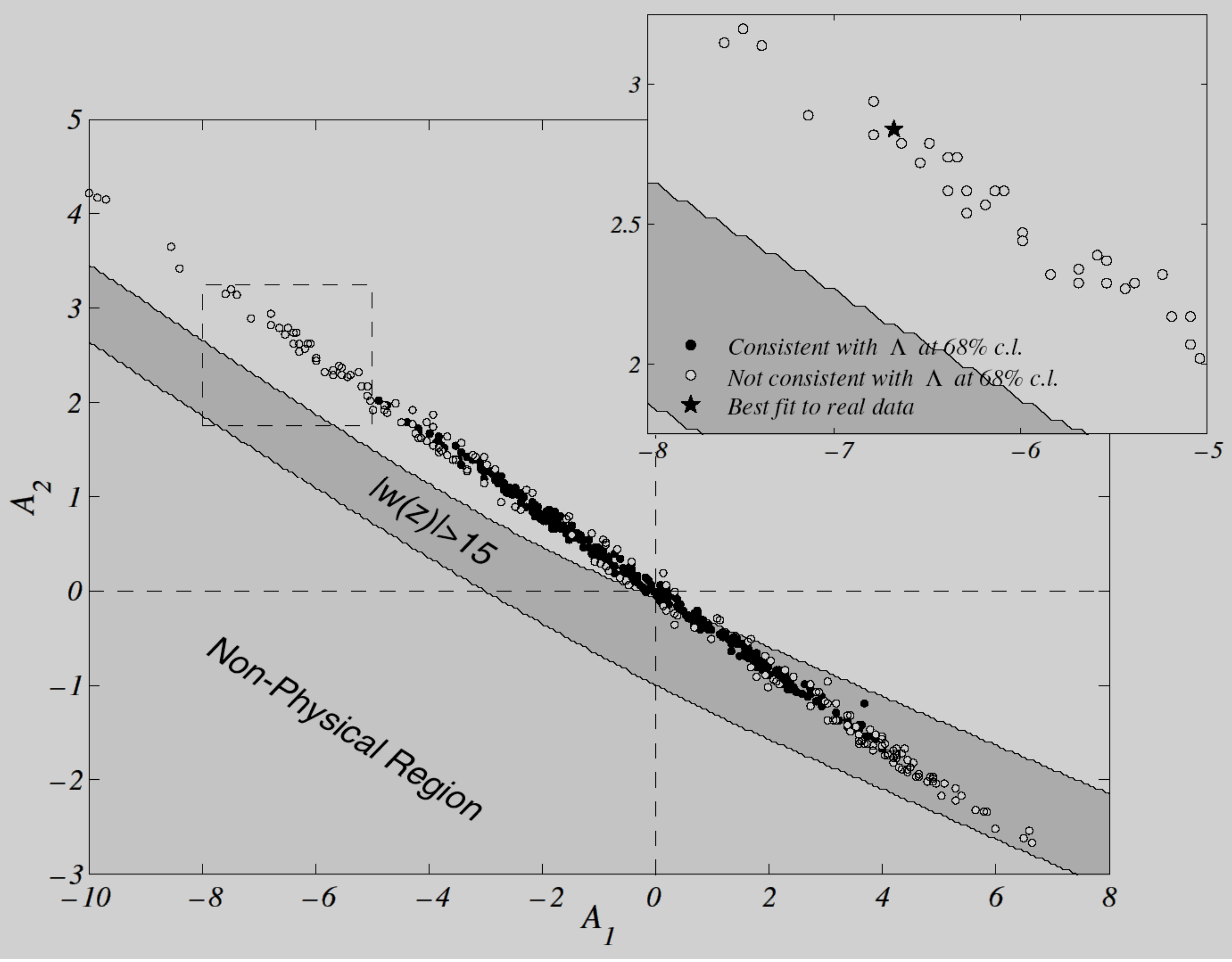
Testing the parametrization: $\rho_{DE}(x)/\rho_{0c} = A_0 + A_1 x + A_2 x^2$

(which can exactly realize ρ_Λ)

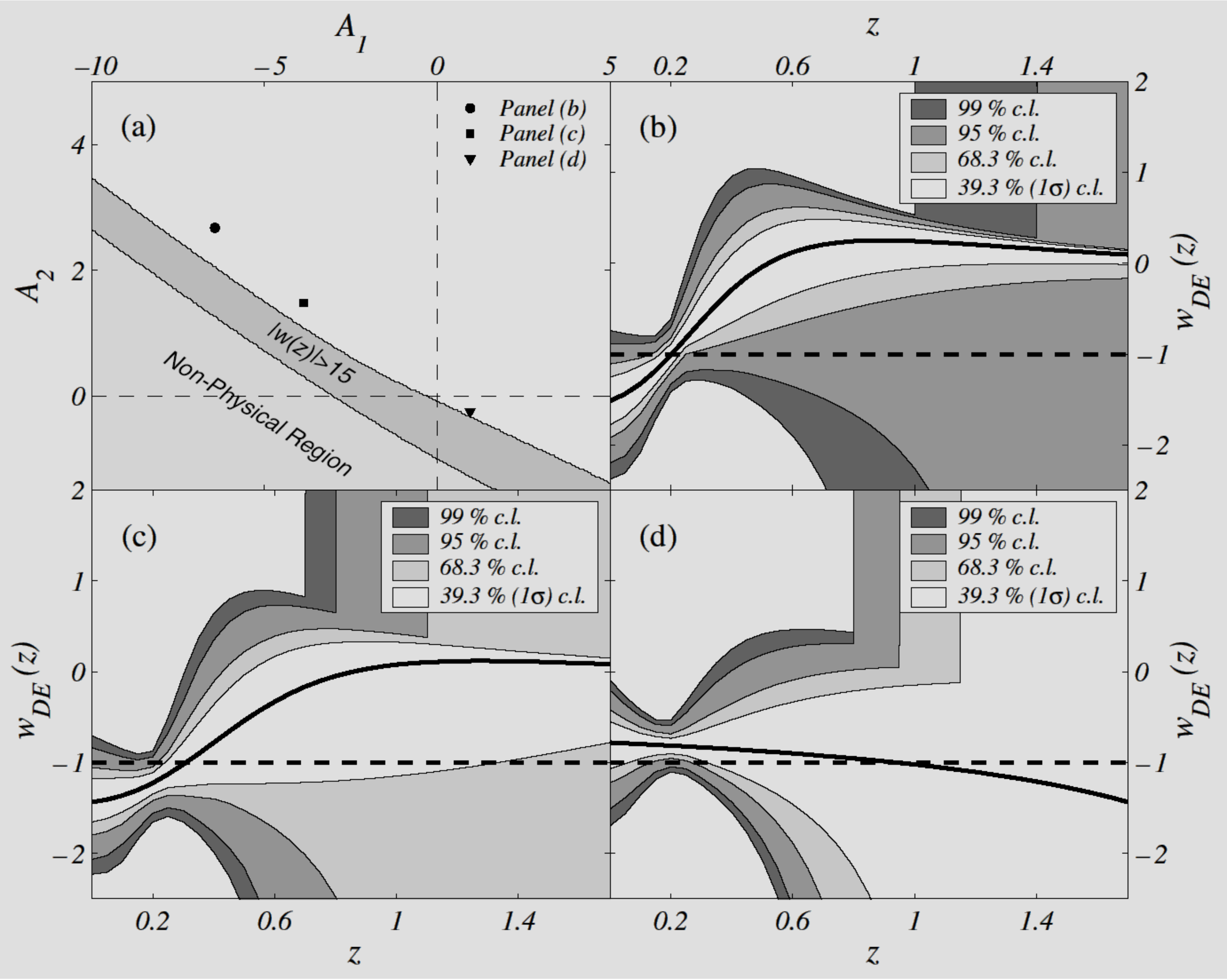
Fiducial model : flat ΛCDM (exactly realized by the above)

- ➔ Simulated SN data
- ➔ Fitting data via $\rho_{DE}(x)/\rho_{0c} = A_0 + A_1 x + A_2 x^2$
- ➔ Obtaining information about parameters *As* and physical quantities, e.g., $w_{DE}(z)$.
- ➔ Comparing with the flat-ΛCDM fiducial model

Jönsson, Goobar, Amanullah, & Bergström, 2004



Jönsson, Goobar, Amanullah, & Bergström, 2004



Jönsson, Goobar, Amanullah, & Bergström, 2004

Data

analyzed
by invoking

- Parametrization :

$$\frac{\rho_{DE}(x)}{\rho_{0c}} = A_0 + A_1 x + A_2 x^2$$

**Physical Information
Conclusion**

very unstable

Very sensitive to
the fine details of
the data

Summary and Discussion

Observations / Data

mapping out the evolution history

(e.g. SNe Ia , Baryon Acoustic Oscillation)

analyzed by invoking

Fitting Formula / Parametrization

(model-independent ?)

$$\left(e.g. \quad d_L(z) = \sum_{i=0}^{N} c_i z^i ; \quad w_\phi = \sum_{i=o}^{N} w_i (1+z)^i \right)$$

Information about Physical Quantities

Characterizing our universe or DE models to be reconstructed

(e.g. w_ϕ , ρ_ϕ , statefinders{r,s} , ……)

Summary & Discussion

- Different parametrizations may give different conclusions.

- A suitable parametrization for answering the question whether w = constant is yet to be worked out.

  [Linder 2003 (PRL 90, 091301): $w(a) = w_0 + w_a z/(1+z)$]

- For answering different questions or for obtaining different physical information, we may need different parametrizations.

Happy Women's Day !!

International Women's Day (March 8)

http://www.un.org/ecosocdev/geninfo/women/womday97.htm

Pure exponential: $V(\phi) = V_0 e^{-\lambda\phi}$

$$V_0 = 10^{-120} M_{pl}^{\ 4},\ \lambda = M_{pl}^{\ -1},\ \phi(0) = 0.135 M_{pl},\ \dot{\phi}(0) = 0$$

Pseudo-Nambu-Goldstone-boson (PNGB): $V(\phi) = M^4 [\cos(\phi / f) + 1]$

$$M^4 = 1.001 \times 10^{-120} M_{pl}^{\ 4},\ f = 0.1 M_{pl},\ \phi(0) = 1.184 \times 10^{-4} M_{pl},\ \dot{\phi}(0) = 0$$

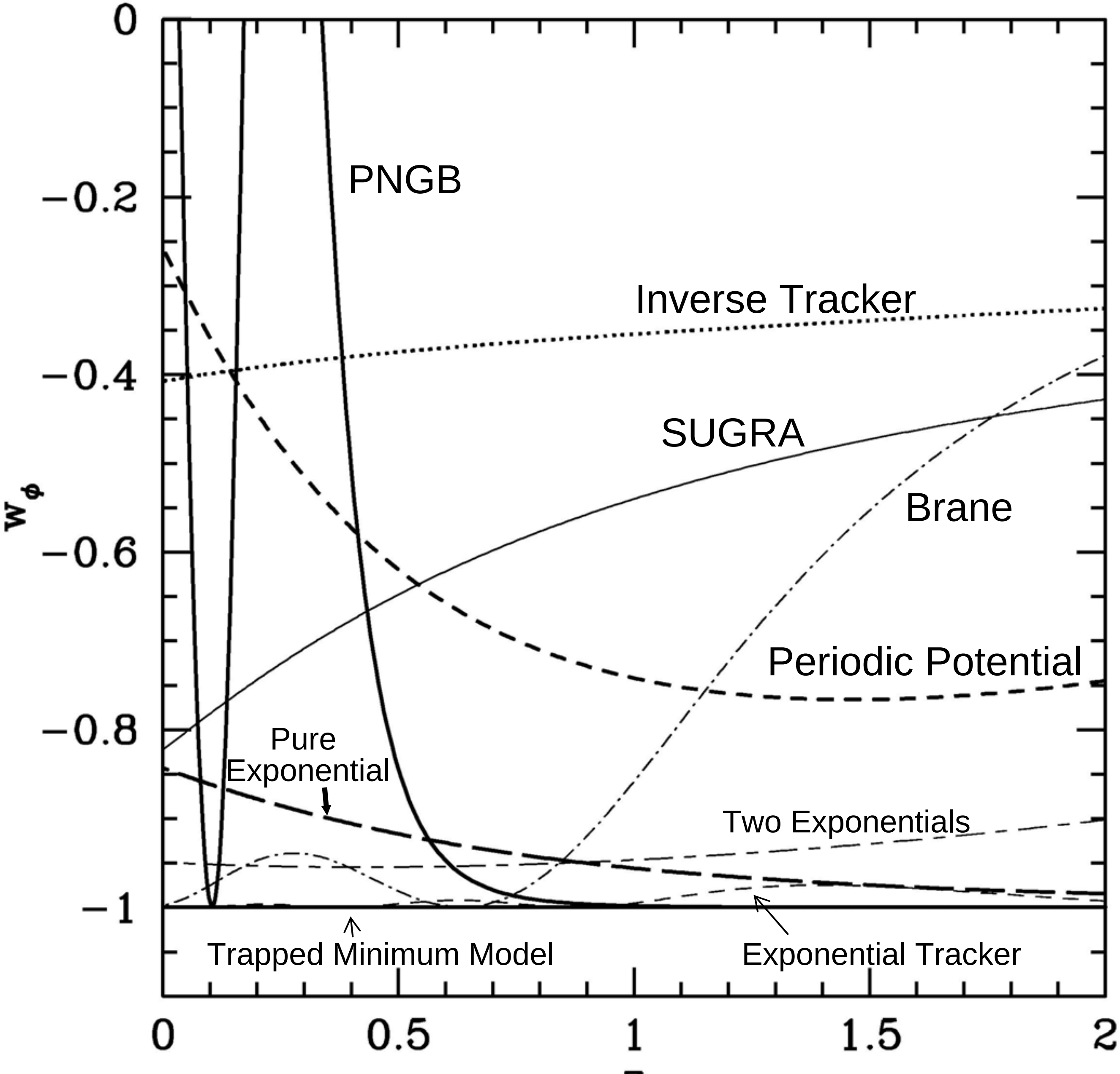
Cosmological tracker solutions: $V(\phi) = M^{4+\alpha} / \phi^{\alpha}$ $\quad M = 9.09 \times 10^{-31} M_{pl}$

$$V(\phi) = M^4 e^{M/\phi} \qquad M = 2.11 \times 10^{-12} M_{pl},\ \ \alpha = 6$$

Supergravity (SUGRA) potential: $\dfrac{M^{4+\alpha}}{\phi^{\alpha}} \exp\left[\dfrac{1}{2}\left(\dfrac{\phi}{M_{pl}}\right)^2\right]$

$$M = 1.611 \times 10^{-8} M_{pl}$$

$$\alpha = 11$$



PNGB
Inverse Tracker
SUGRA
Brane
Periodic Potential
Pure Exponential
Two Exponentials
Trapped Minimum Model
Exponential Tracker
w_ϕ
z
0
−0.2
−0.4
−0.6
−0.8
−1
0
0.5
1
1.5
2

Dark Energy Phenomenology :

How Parametrization alters Conclusion

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