QCD viscosity

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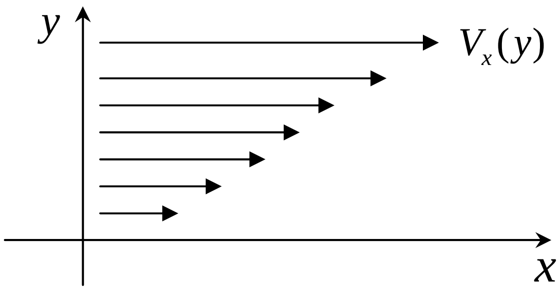
Viscosity

- Shear viscosity



Frictional force

$$T_{ij} = -\eta\left(\frac{\nabla_i V_j(x) + \nabla_j V_i(x)}{2} - \frac{1}{3}\delta_{ij}\nabla \cdot V(x)\right).$$

Question

- Whose shear viscosity is bigger? Liquid or gas water near 100 degree C at 1 atm?

Shear viscosity measures

how “perfect” a fluid is!

- Kovtun, Son, and Starinets ('05)

  Conjecture: Shear viscosity / entropy density

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

- Motivated by AdS/CFT

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \, \langle [T_{xy}(t, \mathbf{x}),\ T_{xy}(0, \mathbf{0})] \rangle$$

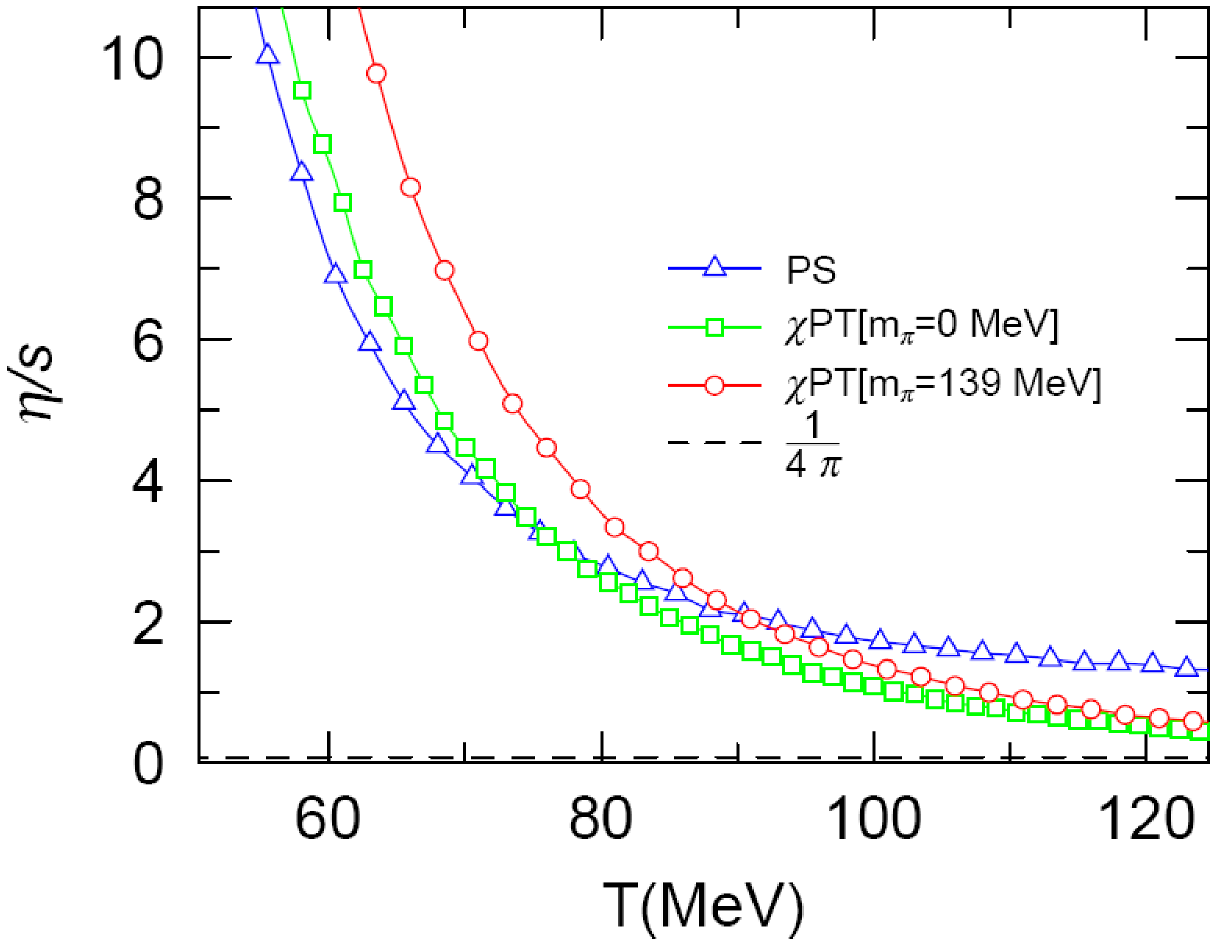
$$\eta = \frac{\sigma_{\rm abs}(0)}{16\pi G}$$

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

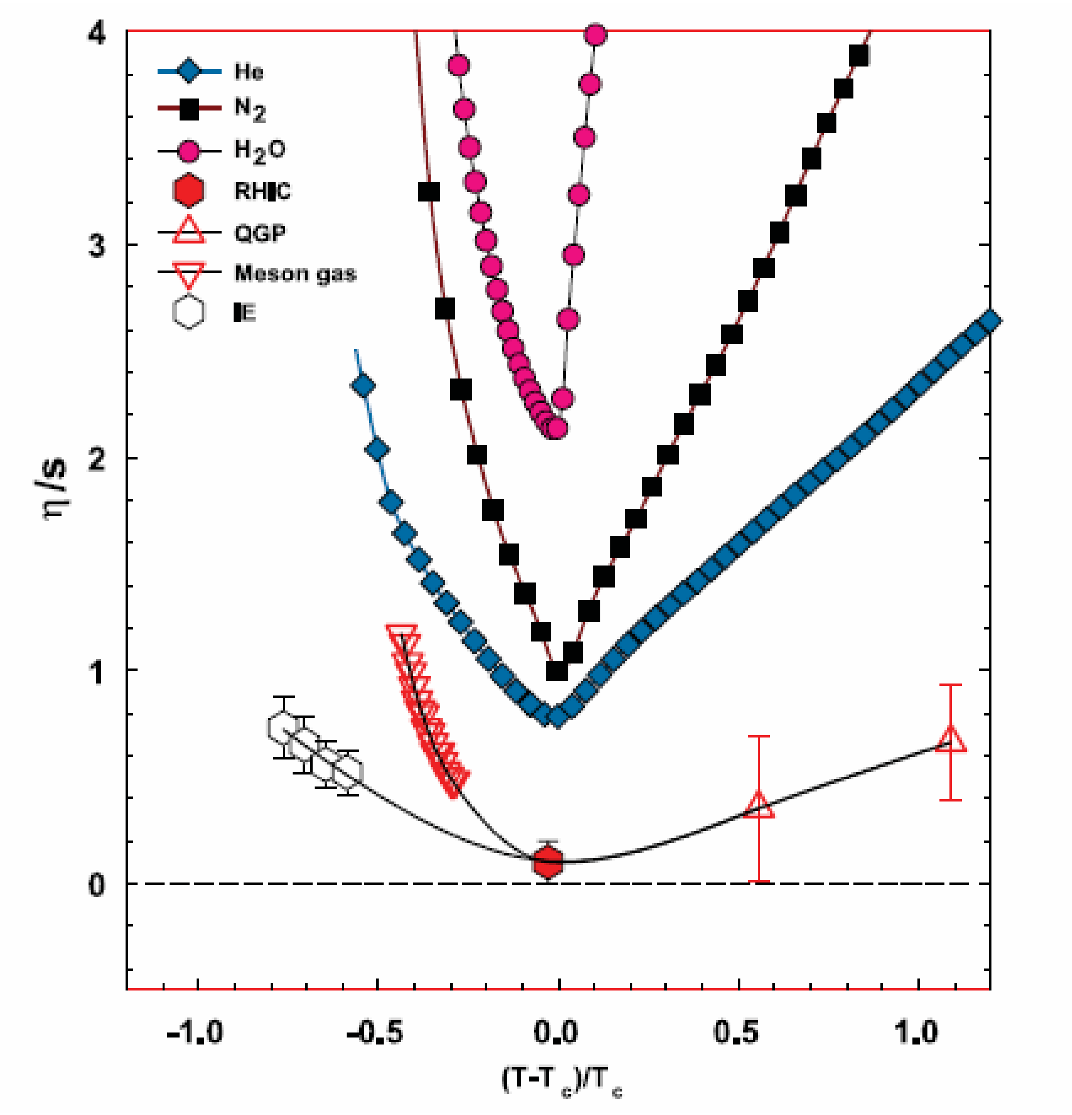
- QGP (quark gluon plasma) almost saturates the bound @ just above Tc --- a perfect fluid (Teaney)
- LQCD, gluon pasma (Karsch, Wyld; Nakamur, Sakai; Meyer)

- What happens below Tc?

η/s of QCD below T_c



JWC, Nakano



Lacey et al., PRL 98:092301,2007

QCD Phase Diagram

2 *M. Stephanov*

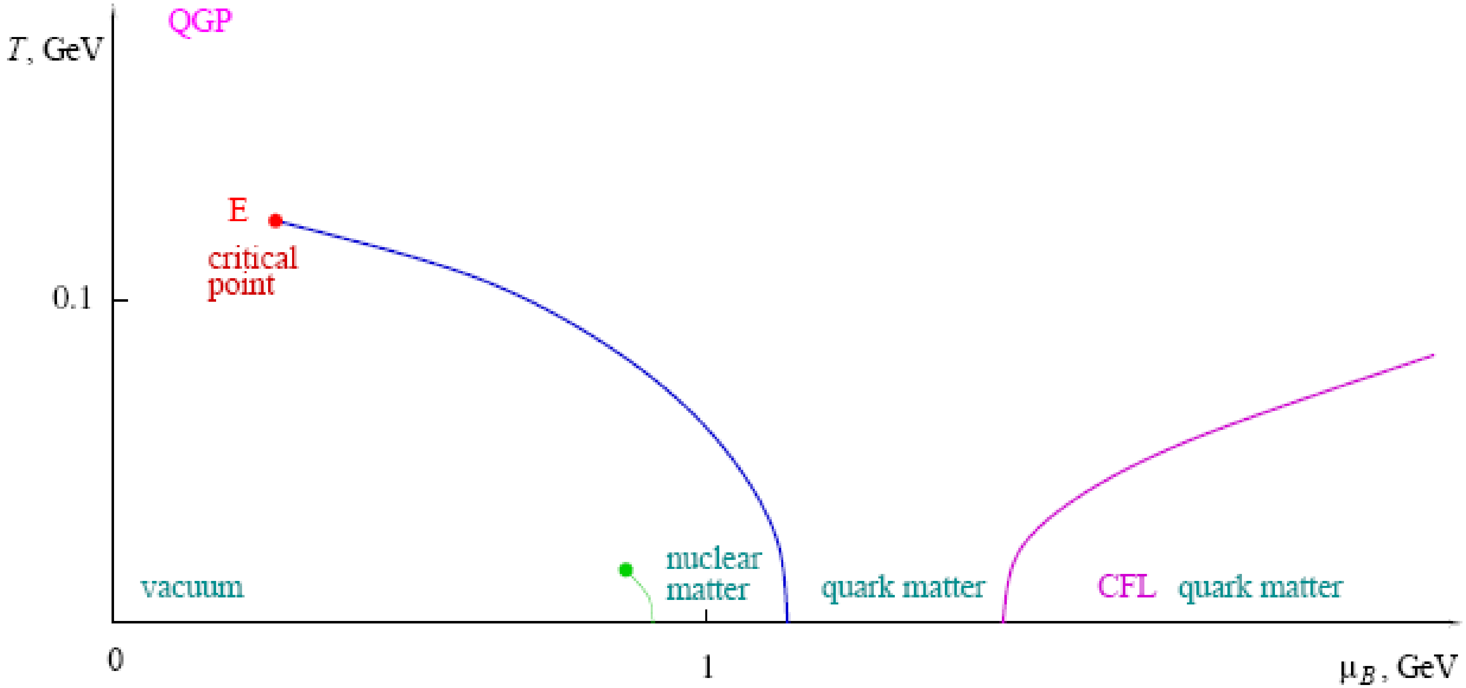
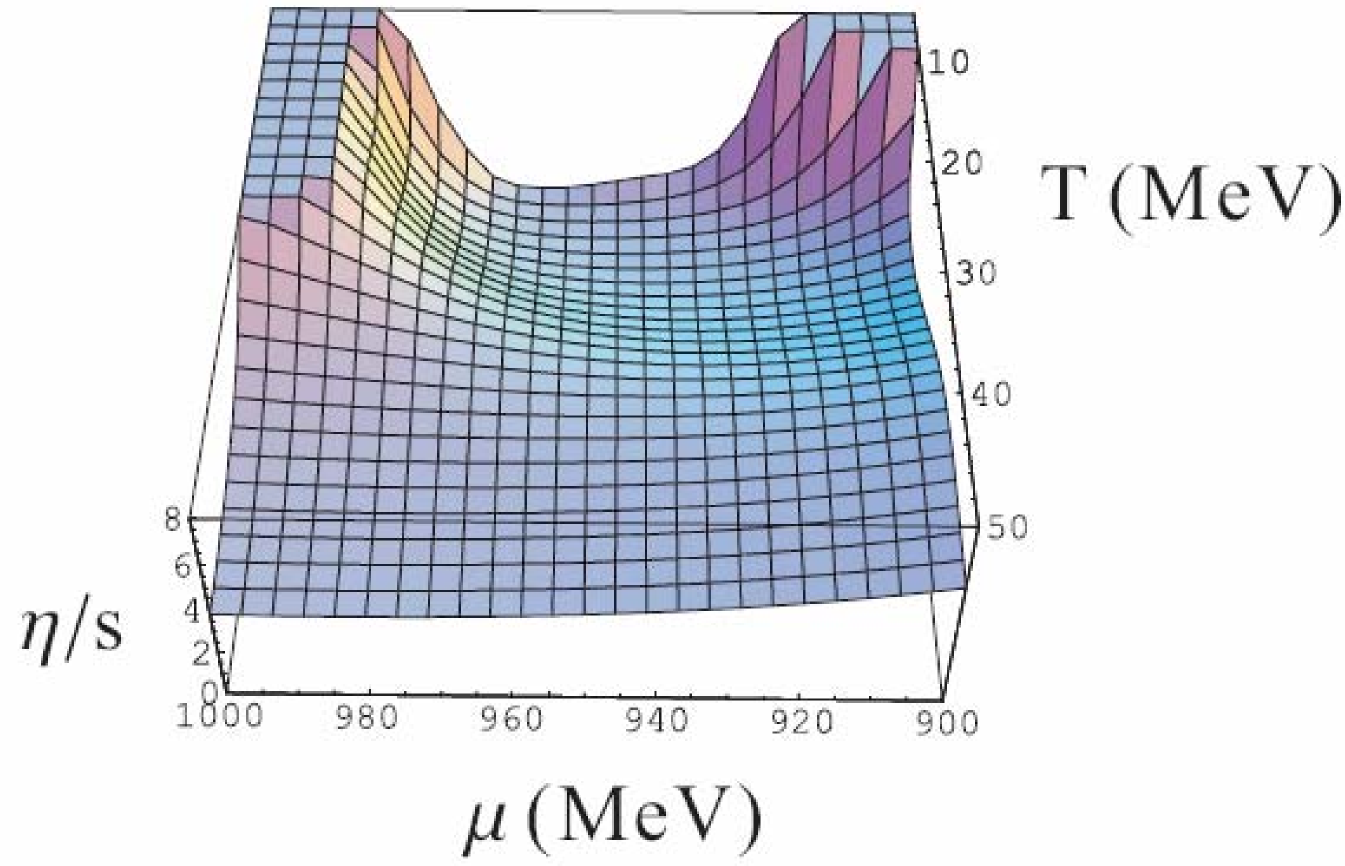
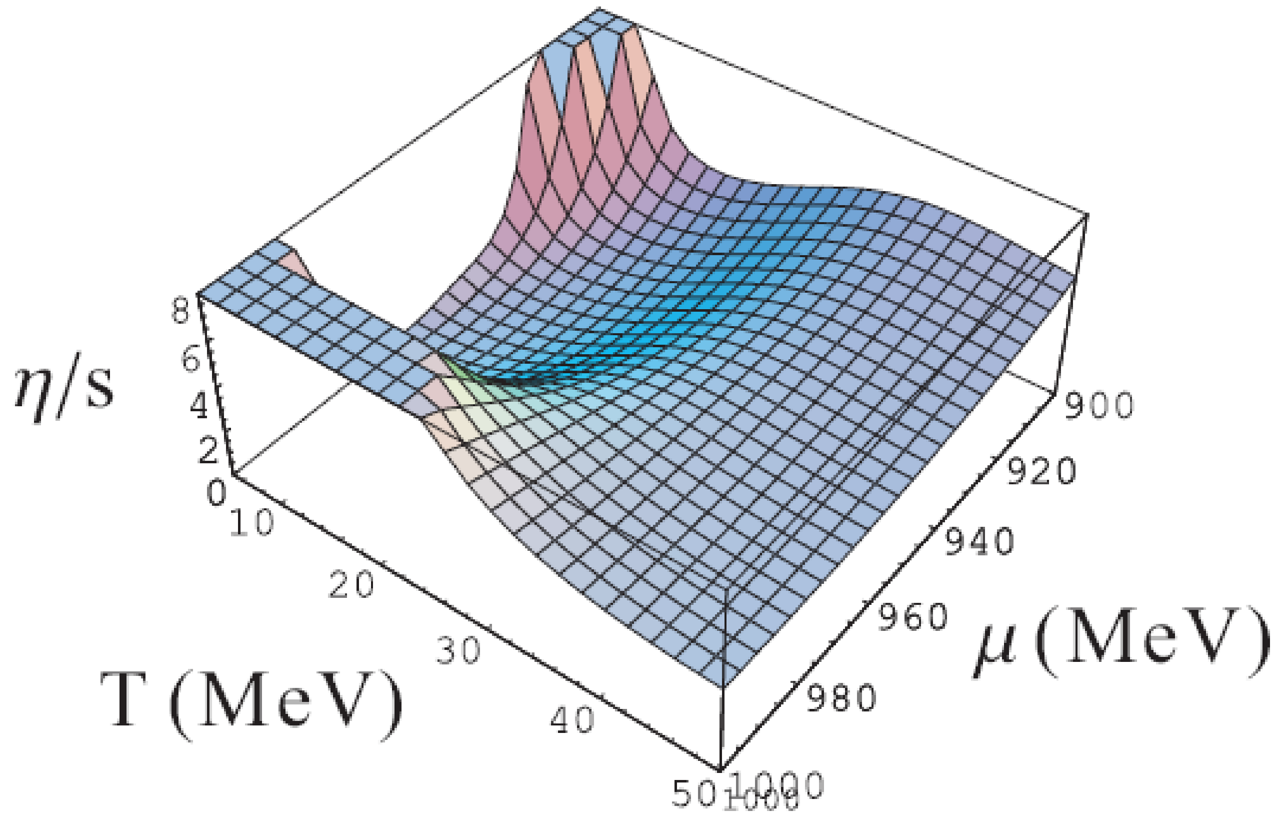


Fig. 1. QCD phase diagram

The η/s Landscape

JWC, Li, Liu, Nakano





η/s
8
6
4
2
0
10
20
30
40
50
T (MeV)
900
920
940
960
980
1000
μ (MeV)

QCD Phase Diagram

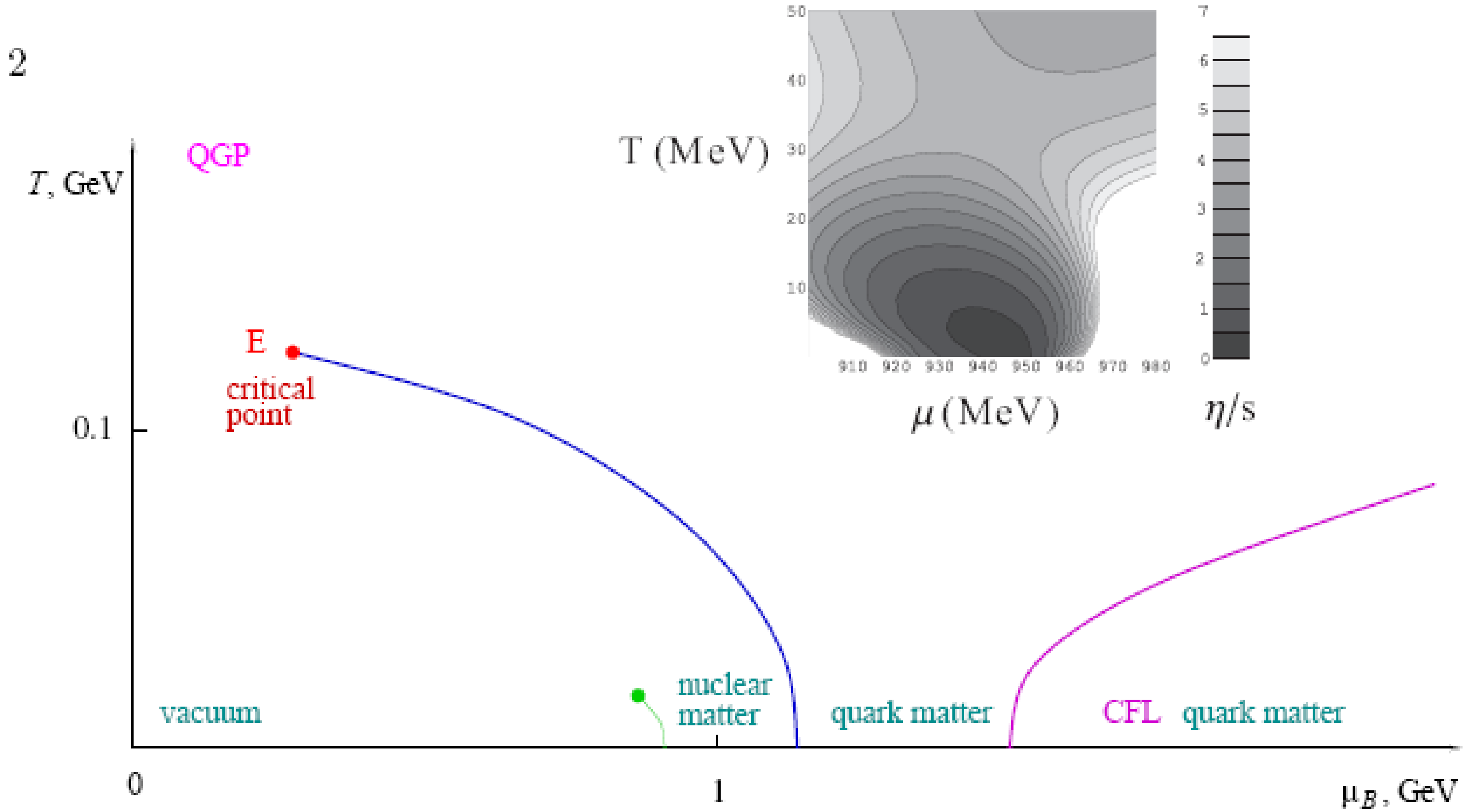
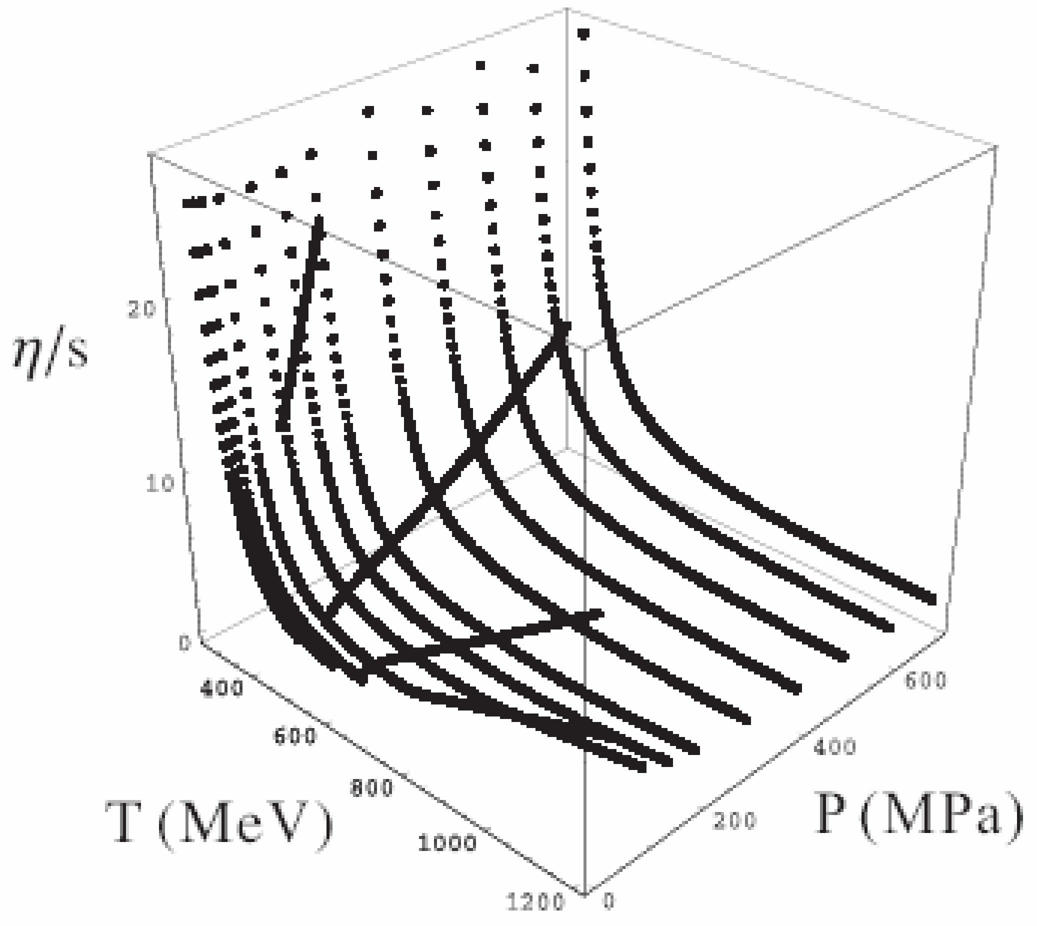


Fig. 1. QCD phase diagram

η/s of Water



η/s on the Lattice

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}),\ T_{xy}(0, \mathbf{0})] \rangle$$

- Re and Im time two-point functions have the same spectral function

$$\eta(T) = \pi \left. \frac{d\rho}{d\omega} \right|_{\omega=0}$$

Extracting the Spectral Function

$$C(x_0) = L_0^5 \int d^3\mathbf{x} \ \langle \overline{T}_{12}(0)\overline{T}_{12}(x_0, \mathbf{x})\rangle$$

$$C(x_0) = L_0^5 \int_0^\infty \rho(\omega) \frac{\cosh \omega(\frac{1}{2}L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega.$$

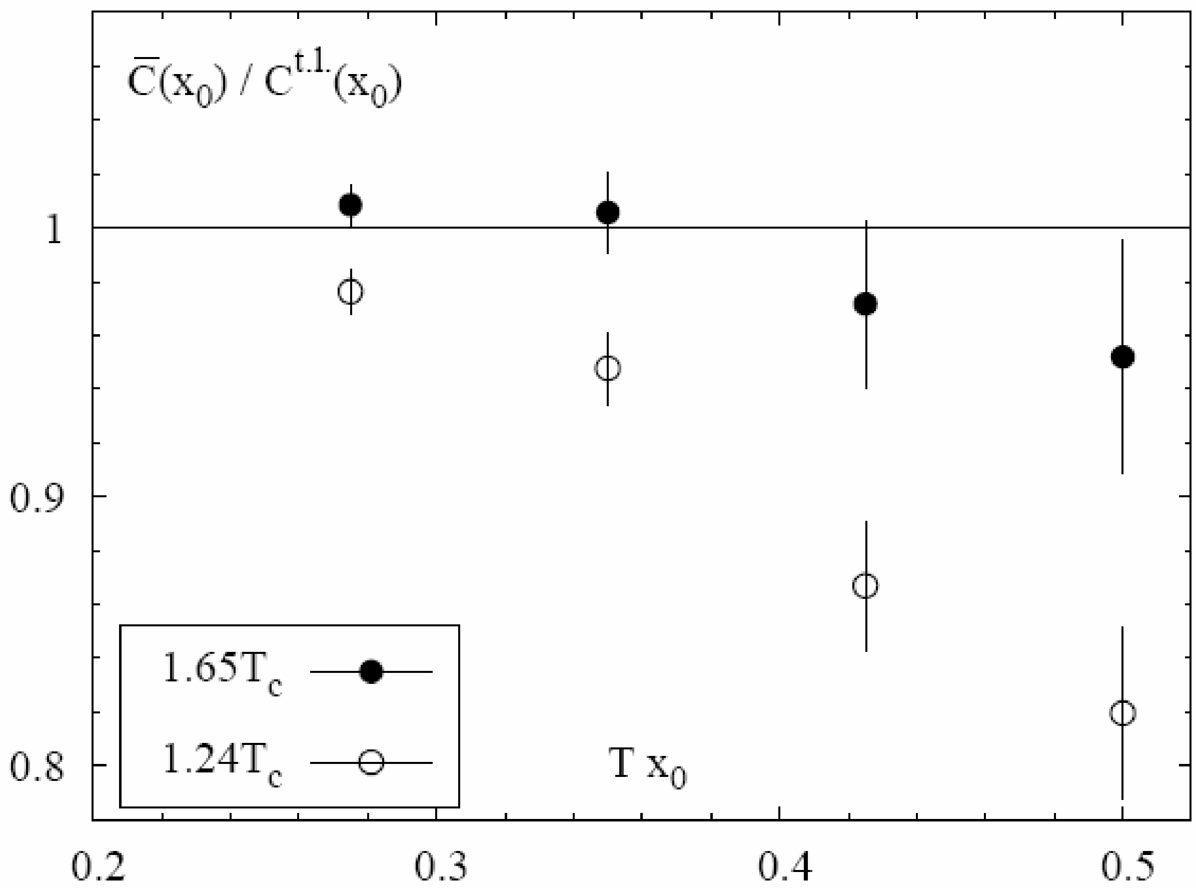
- $\rho(\omega)$ not unique w/ finite lattice sites

Tree Level Spectral Function

$$\rho^{\mathrm{t.l.}}(\omega) \;=\; \frac{A_{\mathrm{t.l.}}\,\omega^4}{\tanh\frac{1}{4}\omega L_0} + BL_0^{-4}\,\omega\delta(\omega),$$

$$A_{\mathrm{t.l.}} \;=\; \frac{1}{10}\frac{d_A}{(4\pi)^2}, \qquad B = \left(\frac{2\pi}{15}\right)^2 d_A.$$

- $d_A = 8$ the number of gluons



Harvey B. Meyer ‘07

"Unsatisfactory attempt" I
(Meyer)

- Breit-Wigner ansatz (same as Karsch+Wyld & Nakamura+Sakai)

$$\rho(\omega)/\omega = \frac{F}{1+b^2(\omega-\omega_0)^2} + \frac{F}{1+b^2(\omega+\omega_0)^2}$$

$$\eta/s\big|_{T=1.65T_c} \;=\; 0.33(3)$$

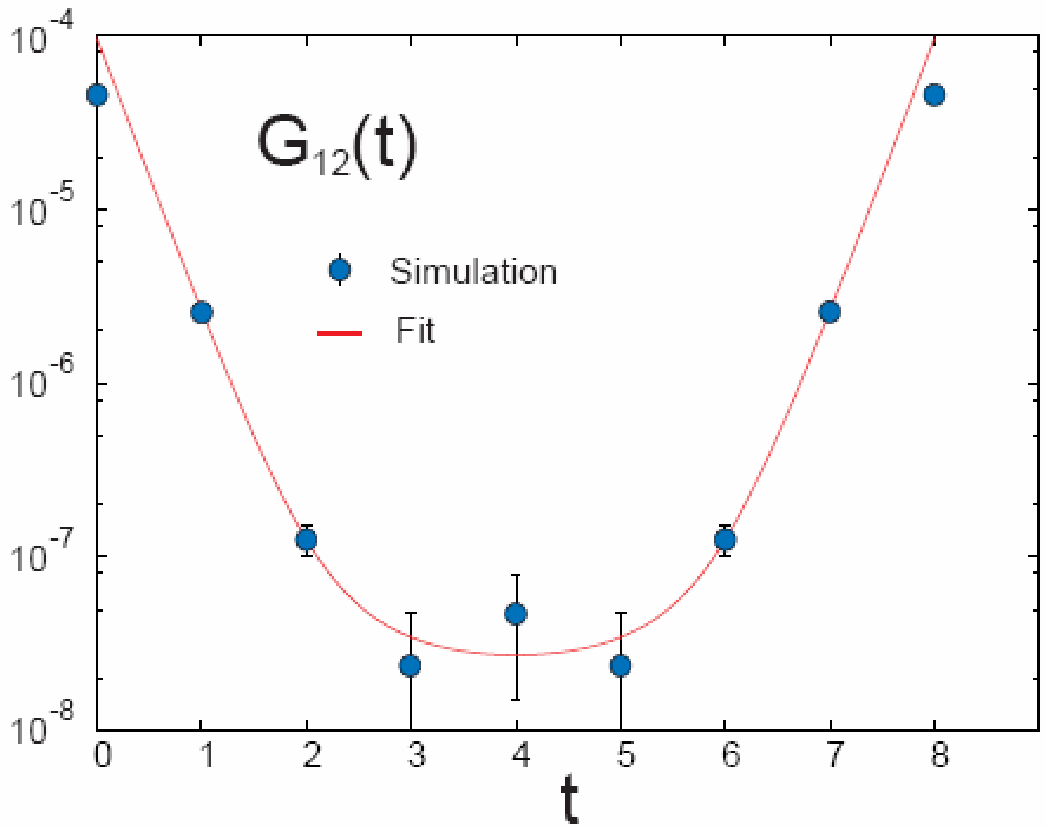
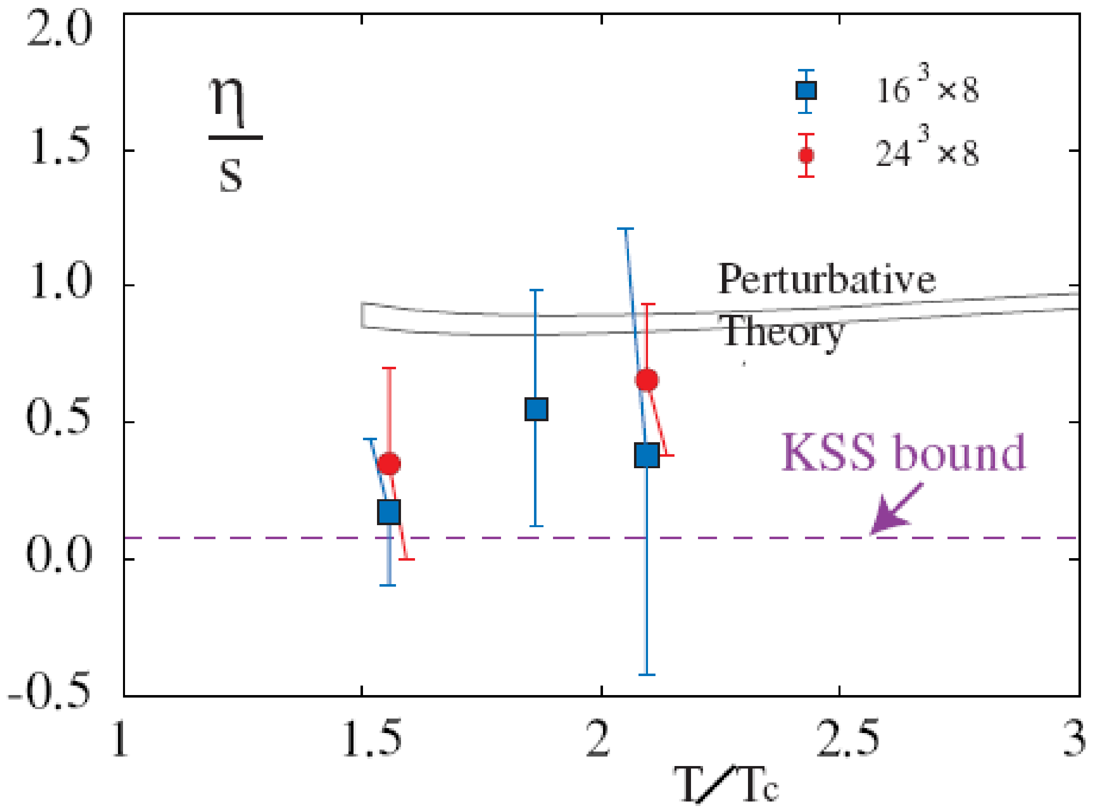


FIG. 1: Numerical data points and fitting results of Matsubara Green's function $G_{12}(t)$ on a $24^3 \times 8$ lattice

Nakamura & Sakai, PRL 94, 072305 (2005)



Nakamura & Sakai, PRL 94, 072305 (2005)

“Unsatisfactory attempt” II

(Meyer)

- Hard-thermal-loop motivated ansatz

$$\rho(\omega)/\omega = \frac{\eta/\pi}{1+b^2\omega^2} + \theta(\omega-\omega_1)\frac{A\omega^3}{\tanh\omega/4T}$$

$$b = 0 \quad \omega_1/T = 7.5(2) \quad A/A_{\rm t.l.} = 0.996(8)$$

$$\eta/s = 0.25(3) \qquad \chi^2_{\rm min} = 4.0$$

An Upper Bound on η/s
(Meyer)

$$C(\tfrac{1}{2}L_0) \geq L_0^5 \left[\int_0^{\sqrt{2}T} \rho_{BW}(\omega) + \int_\Lambda^\infty \rho_{\text{t.l.}}(\omega)\right] \frac{d\omega}{\sinh \omega L_0/2}$$

$$\Lambda = \max(\tfrac{1}{2}[M_2 + M_{2^*}] \approx 2.6\text{GeV}, 5T$$

$M_{2(*)}$ masses of the two lightest tensor glueballs

$$\eta/s < \begin{cases} 0.96 & (T = 1.65T_c) \\ 1.08 & (T = 1.24T_c) \end{cases}$$

An Sophisticated Approach

(Meyer)

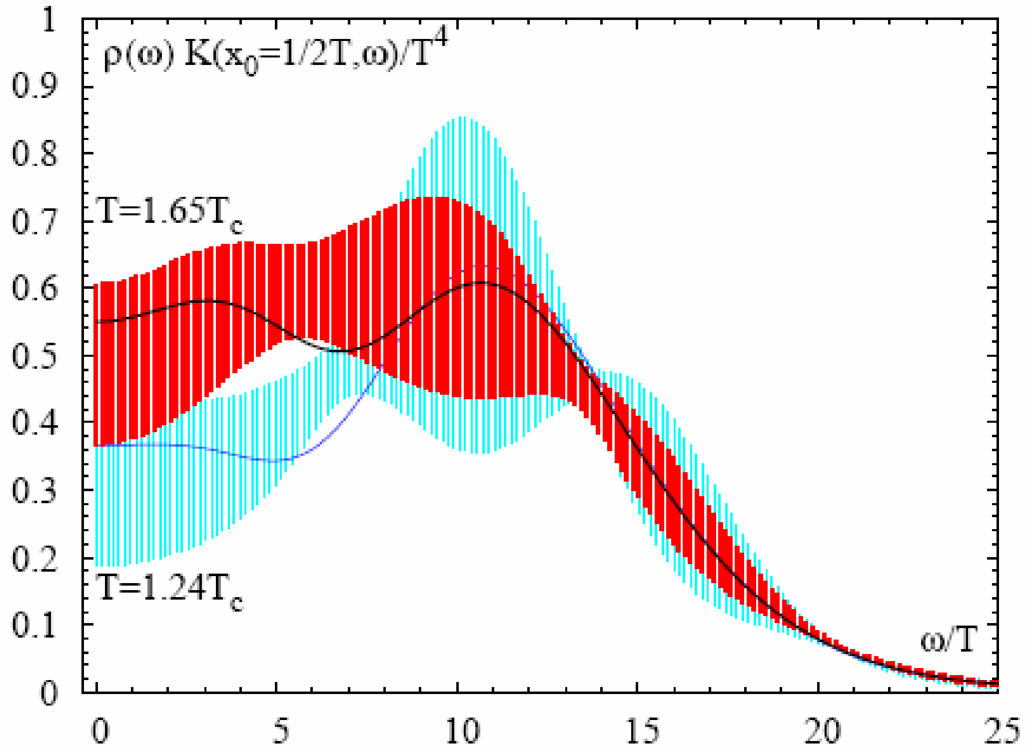


FIG. 3: The result for $\rho(\omega)$. The meaning of the error bands and the curves is described in the text. The area under them equals $\overline{C}(L_0/2) = 8.05(31)$ and $9.35(42)$ for $1.24T_c$ and $1.65T_c$ respectively.

An Sophisticated Approach

(Meyer)

$$\eta/s = \begin{cases} 0.134(33) & (T = 1.65T_c) \\ 0.102(56) & (T = 1.24T_c) \end{cases}$$

Outlook

- η/s on the lattice: modeling spectral function, below Tc, including quarks or in cold atoms with large scattering length

- mapping the phase diagram by η/s
  tricritical point

QCD Phase Diagram

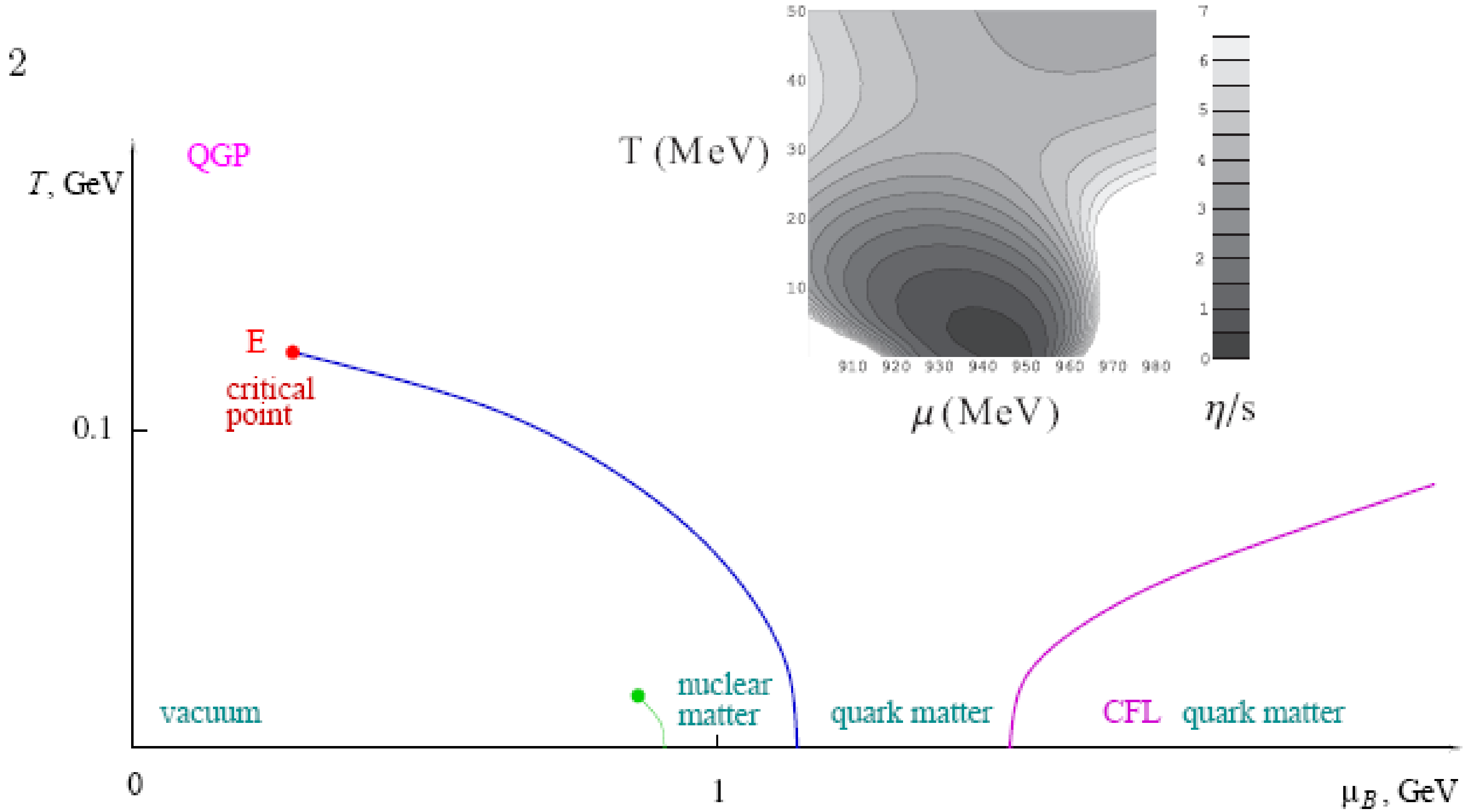
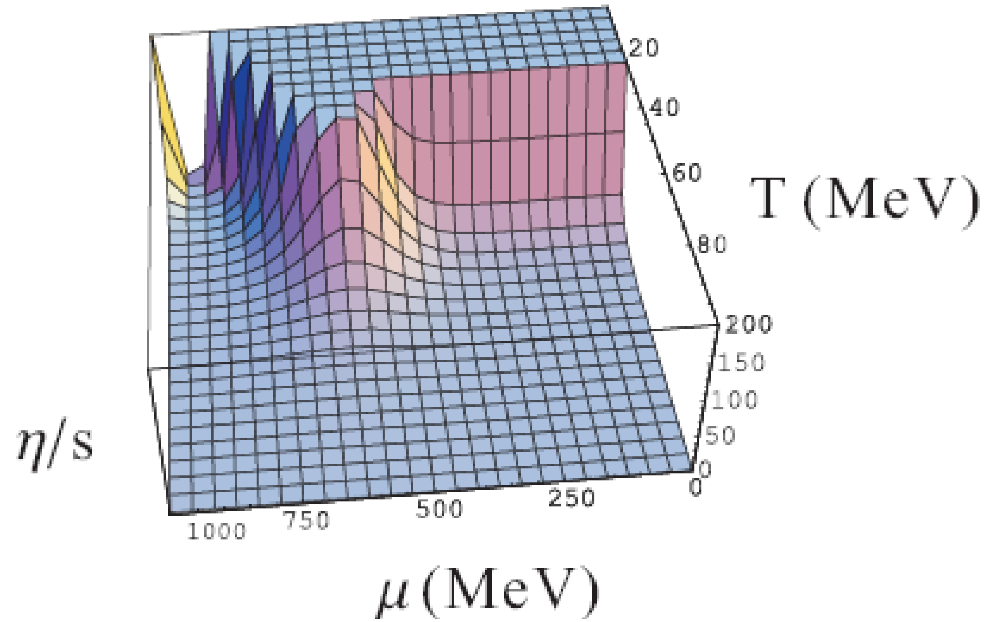
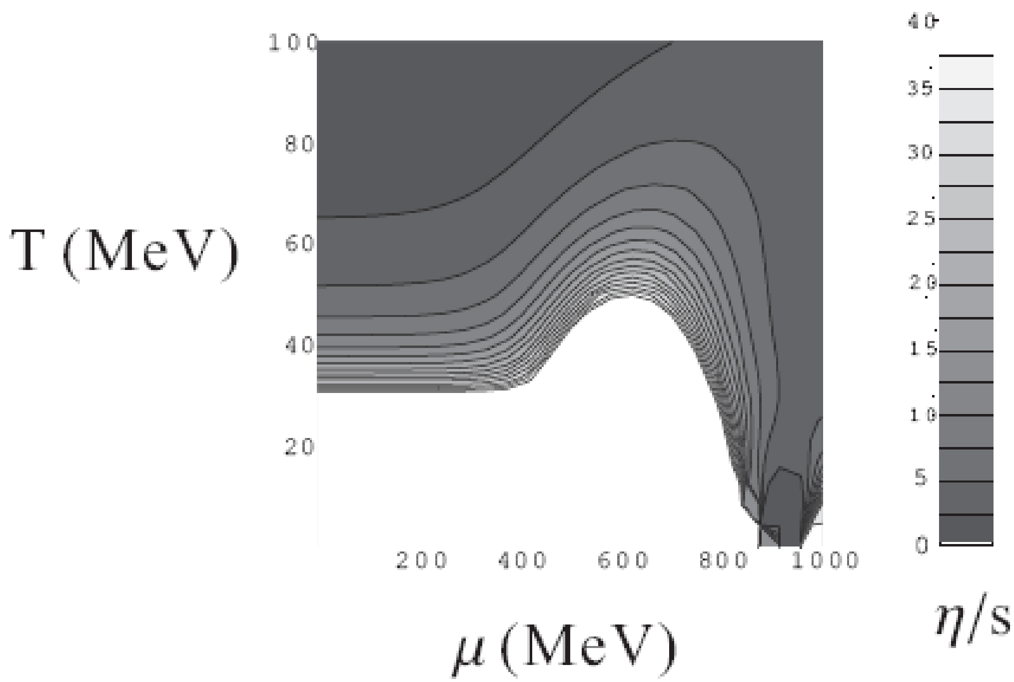


Fig. 1. QCD phase diagram



T (MeV)
20
40
60
80
η/s
200
150
100
50
0
0
250
500
750
1000
μ (MeV)



T (MeV)
100
80
60
40
20
200
400
600
800
1000
μ (MeV)
40
35
30
25
20
15
10
5
0
η/s