

Indication of the Lowest-lying 1^{-+} Exotic Meson in QCD

Kwei-Chou Yang

Chung Yuan Christian University, Taiwan

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- ❖ Meson properties
- ❖ Experimental status
- ❖ Status for theoretical calculations
- ❖ 1^{-+} exotic mesons in language of the QCD field theory
- ❖ The 3-parton LCDAs of twist-3 of the lowest-lying 1^{-+} meson
- ❖ The mass of the lowest-lying 1^{-+} meson in QCD sum rules
- ❖ The comparison with previous studies
- ❖ Discussions and summary

Importance for predictions of QCD in the nonperturbative region

- ❖ Quantum Chromodynamics (QCD) is the presently accepted theory of strong interactions among quarks and gluons.
- ❖ Perturbative QCD has been developed in great detail and tested successfully.
- ❖ Nonperturbative QCD (related to the confinement dynamics) seems to be a much more difficult task.
- ❖ The approaches for nonperturbative QCD: Lattice QCD, QCD sum rules.
- ❖ Can we have any prediction going beyond conventional hadron? or the quark model?
- ❖ Pentaquark ($qqqqq$)?
- ❖ 1^{-+} states: Two evidences so far; candidates for the hybrid mesons ($q\bar{q}g$).

Meson properties

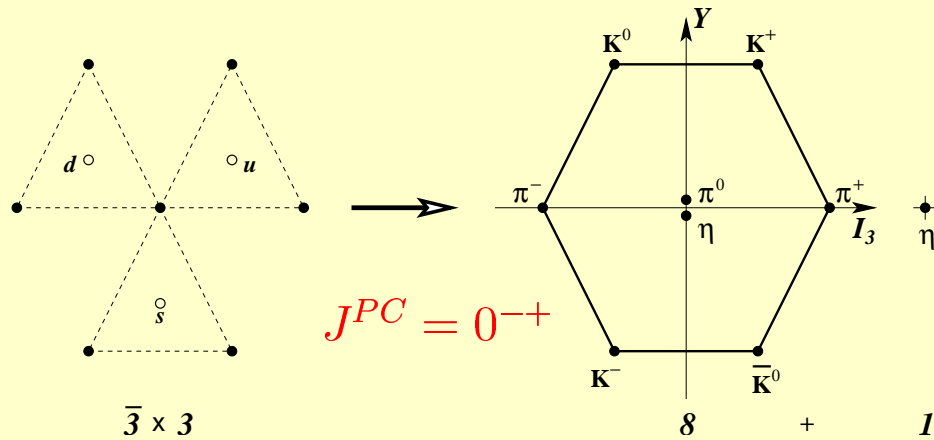
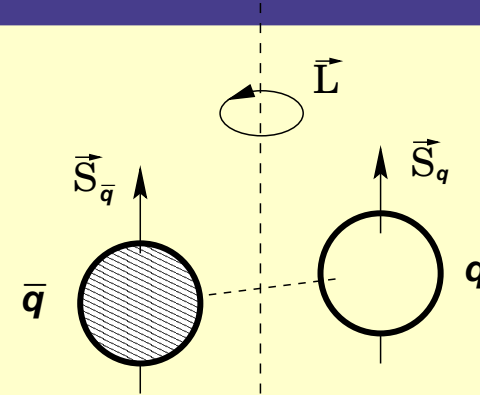
Normal Mesons

$$\vec{S} = \vec{S}_q + \vec{S}_{\bar{q}} \quad \text{Total Spin}$$

$$\vec{J} = \vec{S} + \vec{L} \quad \text{Total Angular Momentum}$$

$$P = (-1)^{L+1} \quad \text{Parity}$$

$$C = (-1)^{L+S} \quad \text{Charge-Parity}$$



Hadron03, Sep-2003

J. Kuhn, CMU - p.3/32

Normal Mesons

$L = 2$	$\rho_3 \omega_3 \phi_3 K_3$	3^{--}
	$\rho_2 \omega_2 \phi_2 K_2$	2^{--}
	$\rho_1 \omega_1 \phi_1 K_1$	1^{--}
	$\pi_2 \eta_2 \eta'_2 K_2$	2^{-+}
$L = 1$	$a_2 f_2 f'_2 K_2$	2^{++}
	$a_1 f_1 f'_1 K_1$	1^{++}
	$a_0 f_0 f'_0 K_0$	0^{++}
	$b_1 h_1 h'_1 K_1$	1^{+-}
$L = 0$	$\rho \omega \phi K^*$	1^{--}
	$\pi \eta \eta' K$	0^{-+}

$$\vec{S} = \vec{S}_q + \vec{S}_{\bar{q}}$$

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

Non- $q\bar{q}$ sequence (“exotic”):

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$$

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Searching for exotic mesons

For a quark-antiquark system, we have

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S} \quad (\text{neutral mesons}).$$

Therefore, if $S = 0$, it should have $C = -P$.

	S=0	S=1
L=0	0^{-+}	1^{--}
L=1	1^{+-}	0^{++} 1^{++}
L=2	2^{-+}	1^{--} 2^{--} 3^{--}

\implies states with $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}$ are exotic mesons if they exist.

Summary of $J^{PC} = 1^{-+}$ States

$\pi_1(1400)$ Decay: $\eta\pi$

$\pi^- p$ production: weak signal

first exotic state resonance

Strong signal in $\bar{p}n$ annihilation

Decay and mass disagree with predictions \Rightarrow possible $q\bar{q}q\bar{q}$ state?

$\pi_1(1600)$ Decays: $\rho\pi$, $\eta'\pi$ and FTM preferred $f_1\pi$, $b_1\pi$

Only seen in $\pi^- p$ production (E852 and VES)

$\eta'\pi$ decay is unexpectedly strong

Mass $M = 1.6 - 1.7$, Width $\Gamma = 0.16 - 0.4$

$\pi_1(2000)$ Decays: $f_1\pi$, $b_1\pi$

Only seen in $\pi^- p$ production (E852)

Mass and decays agree with predictions for a hybrid meson

3 states with same quantum numbers \Rightarrow mixing?

Hadron03, Sep-2003

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Experimental status

- ❖ We now have two evidence for two 1^{-+} exotics, $\pi_1(1400)$ and $\pi_1(1600)$.
- ❖ A negatively charged exotic state, $\pi_1(1400)$, with $J^{PC} = 1^{-+}$ was observed in $\pi^- p \rightarrow \eta \pi^- p$ [E852, 1997, 1999] and in $\bar{p} n \rightarrow \pi^0 \pi^- \eta$ [Crystal Barrel, 1998].
- ❖ The corresponding neutral state was reported by the Crystal Barrel (1999) and E852 (2003,2006) collaborations in the reactions of $\bar{p} p \rightarrow \pi^0 \pi^0 \eta$ and $\pi^- p \rightarrow \eta \pi^0 n$, respectively, where the decay channel $\eta \pi$ is isovector and hence cannot be confused with a glueball. The charge conjugate is a good quantum number in the neutral $\eta \pi^0$ system.
- ❖ The world average of the mass for $\pi_1(1400)$ is 1376 ± 19 MeV.
- ❖ The lowest-lying 1^{-+} state is $\pi_1(1400)$.

Status for theoretical studies

A hybrid ($q\bar{q}g$) picture for the (lowest-lying) 1^{-+} meson:

- ❖ Flux-tube model: the quark and anti-quark are connected by a SHO-like gluon, F.E. Close and P.R. Page, NPB443, 233 (1995).

$$m \approx 1.9 \text{ GeV}$$

$\pi_1(1400)$ was suggested as a $q\bar{q}q\bar{q}$ bound state, while $\pi_1(1600)$ might be a hybrid state. F.E. Close and H.J. Lipkin, PLB196, 245 (1987).

- ❖ Lattice Calculation: $m \gtrsim 1.7 \text{ GeV} \sim 2.1 \text{ GeV}$.

C.W. Bernard *et al.*, PRD56, 7039 (1997); PRD 68, 074505 (2003); ...

- ❖ QCD sum rules

Without radiative corrections

J. Govaerts *et al.*, NPB248,1, 1984: $m_{\pi_1} = 1.3 \text{ GeV}$.

I. I. Balitsky1, *et al.*, Z.Phys.C33,265, 1986. Nonstrange: $\sim 1.5 \text{ GeV}$; strange $\sim 1.6 \text{ GeV}$.

With radiative corrections:

H.Y.Jin & J.G. Körner, PRD64, 074002, 2001: $m_{\pi_1} \geq 1.55 \text{ GeV}$.

K.G. Chetyrkin & S. Narison, PLB485, 145 (2000), $m_{\pi_1} \approx (1.6 \sim 1.7) \text{ GeV}$.

Status for theoretical studies

Decays of a hybrid in the flux-tube model are:

$$b_1\pi : \pi f_1 : \pi\rho : \pi\eta : \pi\eta' = 170 : 60 : 5 - 20 : 0 - 10 : 0 - 10$$

compared with the data (VES) for $\pi_1(1600)$:

$$b_1\pi : \eta'\pi : \rho\pi = 1 : 1.0 \pm 0.3 : 1.5 \pm 0.5$$

1^{-+} exotic mesons in language of the QCD field theory

$$|1^{-+}\rangle = \psi_{q\bar{q}}|\bar{q}q\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \dots,$$

where ψ_i are distribution amplitudes.

Note that a nonlocal $\psi_{q\bar{q}}$ does not vanish and is antisymmetric under interchange of momentum fractions of \bar{q} and q in SU(3) limit.

Projecting a lowest-lying 1^{-+} meson (π_1) along the light-cone ($z^2 = 0$), the leading-twist light-cone distribution amplitudes (LCDAs) $\phi_{\parallel, \perp}$ are defined as

$$\begin{aligned} \langle 0|\bar{q}_1(z) \not{z} q_2(0)|\pi_1(P, \lambda)\rangle &= m_{\pi_1}^2 \epsilon^{(\lambda)} \cdot z \int_0^1 du e^{-i\bar{u}pz} \phi_{\parallel}(u), \\ \langle 0|\bar{q}_1(z) \sigma_{\perp\nu} z^\nu q_2(0)|\pi_1(P, \lambda)\rangle &= im_{\pi_1} \epsilon_{\perp}^{(\lambda)} p \cdot z \int_0^1 du e^{-i\bar{u}pz} \phi_{\perp}(u), \end{aligned}$$

where $p_\mu = P_\mu - z_\mu m_{\pi_1}^2 / (2pz)$, and the nonlocal quark-antiquark pair, connected by the Wilson line which is not shown, is at light-like separation.

Considering the G-parity, $\phi_{\parallel, \perp}$ are antisymmetric under interchange $u \leftrightarrow \bar{u}$ in SU(3) limit, i.e., the amplitudes vanish in the $z \rightarrow 0$ limit (but *not* in the $z^2 \rightarrow 0$ limit).

Remarks on G-parity properties of LCDAs

$$\text{G-parity: } \hat{G} = \hat{C}i\sigma_2 = \hat{C} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\langle 1^{-+} | G^\dagger = \langle 1^{-+} | (-1),$$

$$G\bar{d}(x)\gamma_\mu u(0)G^\dagger = -C\bar{u}(x)\gamma_\mu d(0)C^\dagger = +\bar{d}(0)\gamma_\mu u(x)$$

Remarks on G-parity properties of LCDAs

$$\text{G-parity: } \hat{G} = \hat{C}i\sigma_2 = \hat{C} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\langle 0^{++} | G = \langle 0^{++} | + 1, \quad \langle \eta^{(\prime)}(0^{-+}) | G = \langle \eta^{(\prime)}(0^{-+}) | + 1, \quad \langle \omega(1^{--}) | G = \langle \omega(1^{--}) | - 1,$$

$$G^\dagger \bar{u}(x) g_s G_{\mu\nu}(vx) \sigma_{\alpha\beta} u(0) G = +\bar{d}(x) g_s G_{\mu\nu}(vx) \sigma_{\alpha\beta} d(0)$$

$$G^\dagger \bar{u}(x) \gamma_\mu \gamma_5 g_s \tilde{G}_{\alpha\beta}(vx) u(0) G = -\bar{d}(x) \gamma_\mu \gamma_5 g_s \tilde{G}_{\alpha\beta}(vx) d(0)$$

$$G^\dagger \bar{u}(x) \gamma_\mu g_s G_{\alpha\beta}(vx) u(0) G = +\bar{d}(x) \gamma_\mu g_s G_{\alpha\beta}(vx) d(0)$$

$$\langle 0^{++}(q) | \bar{u}(x) g_s G_{\mu\nu}(vx) \sigma_{\alpha\beta} u(0) | 0 \rangle$$

$$= f_{3,0^{++}} [q_\beta (q_\mu g_{\nu\alpha} - q_\nu g_{\mu\alpha}) - q_\alpha (q_\mu g_{\nu\beta} - q_\nu g_{\mu\beta})] \int \mathcal{D}\alpha \phi_{3,0^{++}} e^{iqx(\alpha_u + v\alpha_g)},$$

$$\langle 0^{++}(q) | \bar{u}(x) \gamma_\mu \gamma_5 g_s \tilde{G}_{\alpha\beta}(vx) u(0) | 0 \rangle = -f_{0^{++}} (q_\alpha g_{\beta\mu} - q_\beta g_{\alpha\mu})$$

$$\times \int \mathcal{D}\alpha \tilde{\phi}_\perp e^{iqx(\alpha_u + v\alpha_g)} + f_{0^{++}} \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha (\tilde{\phi}_\parallel + \tilde{\phi}_\perp) e^{iqx(\alpha_u + v\alpha_g)}.$$

$$\langle 0^{++}(q) | \bar{u}(x) \gamma_\mu g_s G_{\alpha\beta}(vx) u(0) | 0 \rangle$$

$$= if_{0^{++}} \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \phi_\perp(\alpha_i) e^{iqx(\alpha_u + v\alpha_g)}$$

$$+ if_{0^{++}} \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \phi_\parallel(\alpha_i) e^{iqx(\alpha_u + v\alpha_g)},$$

$\phi_{3,0^{++}}, \phi_\perp, \phi_\parallel$ are symmetric, **but $\tilde{\phi}_\perp, \tilde{\phi}_\parallel$ is antisymmetric**, under $\alpha_u \leftrightarrow \alpha_{\bar{u}}$

Estimate the electroproduction rate of the exotic meson

$$\phi_{\parallel}(u) = f_{\pi_1} 30u(1-u)(1-2u)$$

with

$$f_{\pi_1} = 50 \text{ MeV}.$$

Estimating the cross section for $\gamma^* p \rightarrow \pi_1^0 p$, it is found

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} \approx \left(\frac{5f_H}{3f_\rho} \right)^2 \approx 0.15.$$

Exotic hybrid meson can be therefore electroproduced in an experimentally feasible way in actual experiments at JLAB, HERMES or Compass.

See I.V. Anikin, P.Pire, L. Szymanowski, O.V. Teryaev, S. Wallon, hep-ph/0401130.

The 3-parton LCDAs of twist-3 of the lowest-lying 1^{-+} meson

$\psi_{q\bar{q}g}$ can be non-vanishing under interchange of momentum fractions of quarks.

The three-parton distribution amplitudes of twist-3 are defined by

$$\begin{aligned} \langle \pi_1(P, \lambda) | \bar{q}_1(-x) \gamma_\alpha g_s G_{\mu\nu}(vx) q_2(x) | 0 \rangle &= -ip_\alpha [p_\mu \epsilon_{\perp\nu}^{*(\lambda)} - p_\nu \epsilon_{\perp\mu}^{*(\lambda)}] f_{3,\pi_1}^V \mathcal{V}(v, -px) + \dots, \\ \langle \pi_1(P, \lambda) | \bar{q}_1(-x) \gamma_\alpha \gamma_5 g_s \tilde{G}_{\mu\nu}(vx) q_2(x) | 0 \rangle &= -p_\alpha [p_\mu \epsilon_{\perp\nu}^{*(\lambda)} - p_\nu \epsilon_{\perp\mu}^{*(\lambda)}] f_{3,\pi_1}^A \mathcal{A}(v, -px) + \dots, \\ \langle \pi_1(P, \lambda) | \bar{q}_1(-x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q_2(x) | 0 \rangle \\ &= \frac{\epsilon^{*(\lambda)} x}{2(px)} [p_\alpha p_\mu g_{\beta\nu}^\perp - p_\beta p_\mu g_{\alpha\nu}^\perp - p_\alpha p_\nu g_{\beta\mu}^\perp + p_\beta p_\nu g_{\alpha\mu}^\perp] f_{3,\pi_1}^\perp m_{\pi_1} \mathcal{T}(v, -px) + \dots, \end{aligned}$$

where the shorthand notation is used:

$$\mathcal{F}(v, -px) \equiv \int \mathcal{D}\underline{\alpha} e^{ipx(\alpha_{q_2} - \alpha_{q_1} + v\alpha_g)} \mathcal{F}(\alpha_{q_1}, \alpha_{q_2}, \alpha_g),$$

Due to G-parity, V, T [A] is symmetric [antisymmetric] under the interchange $\alpha_{q_2} \leftrightarrow \alpha_{q_1}$ in the SU(3) limit.

(cf. the cases of vector mesons, P. Ball *et al.*, NPB529, 323 (1998)) and axial-vector mesons, KCY, to appear in NPB.)

The mass of the lowest-lying 1^{-+} meson in QCD sum rules

Adopting the local gauge-invariant current

$$J(x) = z^\beta z^\mu \bar{d}(x) \sigma_{\alpha\beta} g_s G_\mu^\alpha(x) u(x),$$

The residue of J coupled to the 1^{-+} state is defined as

$$\langle 0 | J(0) | 1^{-+}(p, \lambda) \rangle = f_{3,1^{-+}}^\perp m_{1^{-+}} (\varepsilon^{(\lambda)} \cdot z)(p \cdot z).$$

$f_{3,1^{-+}}^\perp$ determines the normalization of \mathcal{T} , the twist-3 3-parton LCDA, and is also the coefficient with conformal spin $7/2$ in the conformal partial wave expansion for the LCDA.

We consider the two-point correlation function

$$i \int d^4x e^{iqx} \langle 0 | T J(x) J^\dagger(0) | 0 \rangle = \Pi(q^2) (q \cdot z)^4.$$

It should be noted that $J(x)$ can couple not only to 1^{-+} states but also to 0^{++} states as

$$\langle 0 | J(0) | 0^{++}(p, \lambda) \rangle = -2f_{3,S}(p \cdot z)^2,$$

Considering the two-point correlation function

$$i \int d^4x e^{iqx} \langle 0 | T J(x) J^\dagger(0) | 0 \rangle = \Pi(q^2) (q \cdot z)^4,$$

we approximate the correlation function as

$$\frac{4(f_{3,a_0})^2}{m_{a_0}^2 - q^2} + \frac{(f_{3,\pi_1}^\perp)^2}{m_{\pi_1}^2 - q^2} = \frac{1}{\pi} \int_0^{s_0} ds \frac{\text{Im}\Pi^{\text{OPE}}}{s - q^2},$$

where Π^{OPE} (see hep-ph/0703186, KCY) is the OPE result at the quark-gluon level, and s_0 is the threshold of higher resonances.

To subtract the contribution arising from the lowest-lying scalar meson, we further evaluate the following non-diagonal correlation function

$$i \int d^4x e^{iqx} \langle 0 | T J(x) \bar{u}(0) d(0) | 0 \rangle = \bar{\Pi}(q^2) (q \cdot z)^2,$$

where the scalar current can couple to the lowest-lying scalar meson $a_0(980)$:

$$\langle 0 | \bar{u}(0) d(0) | a_0(980) \rangle = m_{a_0} f_{a_0}.$$

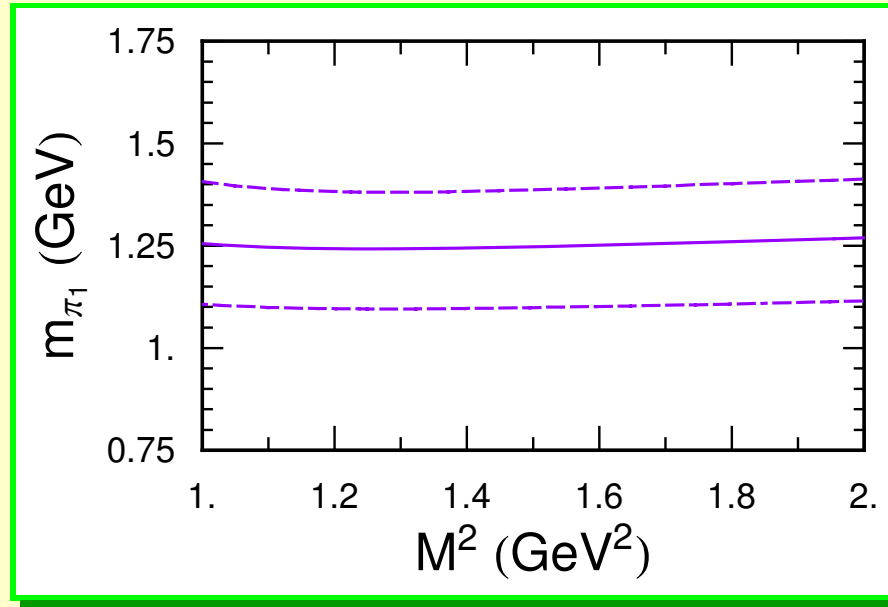
- ❖ To obtain the mass sum rule for the lowest-lying 1^{-+} exotic state, we substitute f_{3,a_0} with the sum rule given by the latter correlation function, and use $f_{a_0} = (0.380 \pm 0.015)$ GeV and $\bar{s}_0 = (3.0 \pm 0.2)$ GeV² (see H.Y.Cheng, C.K.Chua, KCY, PRD73, 014017).
- ❖ We consider the renormalization-group (RG) improved QCD sum rules. The scale dependence of the operator J is given by $J(Q) = J(\mu)L^{\Gamma_{T_2^+}/b}$ with $L \equiv \alpha_s(Q)/\alpha_s(\mu)$, $\Gamma_{T_2^+} = 7C_F/3 + N_c$, and $b = (11N_c - 2n_f)/3$, where N_c and n_f are numbers of colors and flavors, respectively. (see KCY, to appear in NPB.)
- ❖ The procedure for performing the RG-improvement on the “mass” sum rule is very important. If the anomalous dimension of the current J was neglected, the stable sum rule could not be obtained within the Borel window and the resulting mass was reduced by 300 MeV.

Results

Numerically, we get the mass for the lowest-lying 1^{-+} exotic meson:

$$m_{\pi_1} = (1.26 \pm 0.15) \text{ GeV},$$

corresponding to $s_0 = 2.5 \pm 0.7 \text{ GeV}^2$, where s_0 is determined by the maximum stability of the mass sum rule within the Borel window $1 \text{ GeV}^2 < M^2 < 2 \text{ GeV}^2$.



The solid curve is obtained by using the central values of input parameters. The region between two dashed lines is variation of the mass within the allowed range of input parameters.

The result for the lowest-lying strange 1^{-+} meson

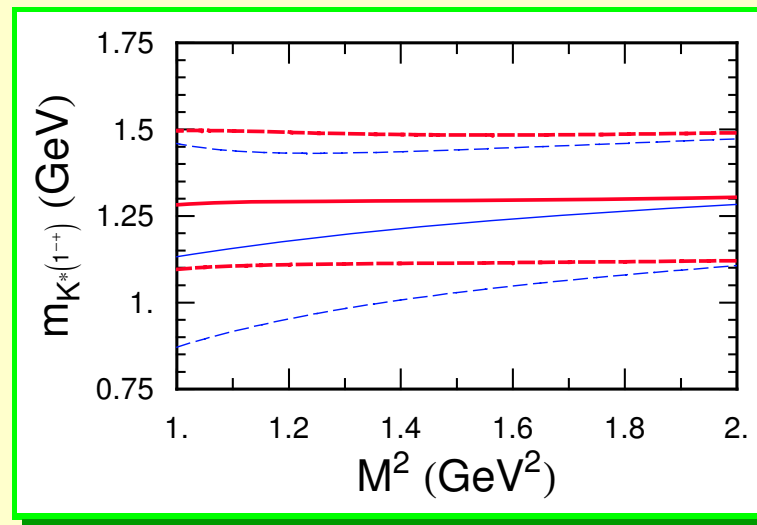
Because it is still questionable for the existence of the $\kappa(800)$, we consider two possible scenarios.

In scenario 1, the $\kappa(800)$ is treated as the lowest-lying strange scalar meson with mass being 0.8 ± 0.1 GeV and $f_\kappa = 0.37 \pm 0.02$ GeV, $\bar{s}_0 = 2.9 \pm 0.2$ GeV²,

In scenario 2, the $K_0^*(1430)$ is considered as the lowest-lying strange scalar meson with $f_{K_0^*(1430)} = 0.37 \pm 0.02$ GeV, $\bar{s}_0 = 3.6 \pm 0.3$ GeV².

The result in scenario 2 is

$$m_{K^*(1^{-+})} = 1.31 \pm 0.19 \text{ GeV}, \quad \text{corresponding to } s_0 = 2.3 \pm 0.9 \text{ GeV}^2.$$



The red (blue) curves are for scenario 2 (1), where $s_0 = 2.3 \pm 0.9$ (2.7 ± 0.7) GeV².

- ❖ Using an arbitrary set of allowed inputs, within the Borel window, the mass is stable only for scenario 2

- ❖ The result hints that the κ may not be a real particle or suitable in the sum rule study due to its large width.

Another possible choice for the mass sum rules

$$\begin{aligned} i \int d^4x e^{iqx} \langle 0 | J_\mu(x) J_\nu^\dagger(0) | 0 \rangle \\ = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_v(q^2) + q_\mu q_\nu \Pi_s(q^2), \end{aligned}$$

with

$$J_\mu(x) = i \bar{d}(x) \gamma_\alpha g_s G_\mu^\alpha(x) u(x),$$

and

$$\langle 0 | J_\mu(0) | \pi_1(p, \lambda) \rangle = f_{4,\pi_1} m_{\pi_1}^2 \varepsilon_\mu^{(\lambda)}.$$

The resulting sum rule is

$$e^{-m_{\pi_1}^2/M^2} m_{\pi_1}^4 (f_{4,\pi_1})^2 = \frac{1}{\pi} \int_0^{s_0} ds e^{-s/M^2} \text{Im} \Pi_v^{\text{OPE}}(s),$$

The f_{4,π_1} determines the normalization of twist-4 LCDAs of the lowest-lying 1^{-+} meson.

The mass can then be obtained by applying $(M^4 \partial / \partial M^2 \ln)$ to both sides of the above equation.

Results for previous studies

❖ Without radiative corrections

J. Govaerts *et al.*, NPB248,1, 1984: $m_{\pi_1} = 1.3 \text{ GeV}$.

I. I. Balitsky¹, D. I. Dyakonov¹ and A. V. Yung, Z.phys.C33,265, 1986.

Nonstrange: $\sim 1.5 \text{ GeV}$; strange $\sim 1.6 \text{ GeV}$.

❖ With radiative corrections:

H.Y.Jin & J.G. Körner, PRD64, 074002, 2001.

Conclusion: (1) $\pi_1(1400)$ is excluded from being a pure hybrid state. (2)

Conservatively, $m_{\pi_1} \geq 1.55 \text{ GeV}$. No stable results can be obtained. (3) The result depends on the excited threshold s_0 and Borel window.

K.G. Chetyrkin & S. Narison, PLB485, 145 (2000).

Conclusion: $m_{\pi_1} \approx (1.6 \sim 1.7) \text{ GeV}$, $\leq 1.9 \text{ GeV}$ by Finite Energy Sum Rule.

Comments on previous works

Three remarks:

- (1) The mass sum rule strongly depend on α_s which is scale-dependent.
- (2) The mass sum rule result is strongly s_0 -dependent, if RG-effects are not included.
- (3) Radiative corrections give $\sim 30\%$ corrections to the mass.

RG-corrections are important:

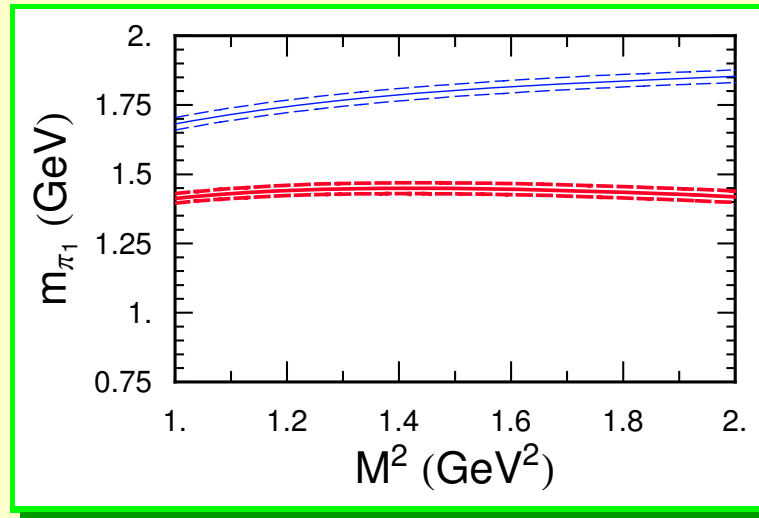
$$J_\mu(Q) = J_\mu(\mu) \left(\frac{\alpha_s(Q)}{\alpha_s(\mu)} \right)^{32/(9b)}$$

$$\alpha_s(Q) \simeq \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}, \quad \beta_0 = 11 - 2n_f/3.$$

We obtain

$$\begin{aligned} m_{\pi_1} &= (1.43 \pm 0.04) \text{ GeV}, \\ m_{K^*(1-+)} &= (1.44 \pm 0.04) \text{ GeV}, \end{aligned}$$

corresponding to the same $s_0 = (4.9 \pm 0.1) \text{ GeV}^2$.



The red curves are obtained from RG-improved sum rule, while blue curves, using the same s_0 as the heavy curves and the scale $\mu = 2 \text{ GeV}$ for α_s , do not contain RG corrections. s_0 is determined by the maximum stability of the mass sum rule within the Borel window $1 \text{ GeV}^2 < M^2 < 2 \text{ GeV}^2$.

Discussions and Summary

- ❖ We calculate the mass of the lowest-lying 1^{-+} exotic meson from QCD sum rules. The obtained non-strange meson masses from two different sum rules consist with the observation for $\pi_1(1400)$.
- ❖ Our results obtained from two different sum rule analyses agree with each other.
- ❖ The two strange mesons, $K^*(1410)$ and $K^*(1680)$, are currently assigned to be 2^3S_1 and 1^3D_1 states, respectively. However, because the $K^*(1410)$ is too light compared with the remaining 2^3S_1 nonet states, therefore it could be replaced by $K^*(1680)$ as the 2^3S_1 state. If so, our result hints that the $K^*(1410)$ is very likely to belong to the lowest-lying 1^{-+} nonet.
- ❖ The other possible choice of the twist-3 current is to consider $z^\beta z^\alpha \bar{d}(x) g_s \gamma_\beta G_{\mu\alpha}(x) u(x)$. One can also consider the 4-quark operator relevant to the $|q\bar{q}q\bar{q}\rangle$ Fock state; however the resulting sum rule will be clouded by the factorization of condensates.