Collider Phenomenology
on
Supersymmetry
Outline

1. Motivations.

2. A few supersymmetry breaking scenarios, associated phenomena and current experiment limits.

3. Connection with Cosmology (next lecture).
Motivations for Supersymmetry

- Provide an elegant solution to hierarchy problem
- Gauge coupling unification
- Dynamical electroweak symmetry breaking
- Provide a natural dark matter candidate
Gauge Hierarchy Problem

Two known scales in particle physics:

- Weak scale $M_W \sim 100$ GeV,
- Planck scale $M_{Pl} \sim 10^{19}$ GeV.

\[
\frac{M_{Pl}}{M_W} \sim 10^{17}
\]

A very large disturbing sensitivity to the Higgs potential.
Mass Protection

- Fermion mass protected by chiral symmetry.
- Gauge boson mass protected by gauge symmetry.
- Scalar boson mass ??

\[
\Delta M_H^2 = \frac{|\lambda_f|^2}{16\pi^2} \left[ -2\Lambda_{UV}^2 + 6m_f^2 \ln \left( \frac{\Lambda_{UV}}{m_f} \right) + \cdots \right]
\]

Two choices:
1. Make \( \Lambda_{UV} \) not too large, where some new physics appears.
2. Find some cancellation mechanism to remove \( \Lambda_{UV}^2 \) divergence
Suppose there exists a new scalar $S$

$$
\Delta M_H^2 = \frac{\lambda_S}{16\pi^2} \left[ \Lambda_{UV}^2 - 2m_S^2 \ln \left( \frac{\Lambda_{UV}}{m_S} \right) + ... \right]
$$

The leading term in $\Lambda_{UV}$ will cancel if

$$
\lambda_S = |\lambda_f|^2 \quad \text{and if there are 2 such scalars}
$$

Such systematic cancellation requires a new symmetry

$\Rightarrow$ Supersymmetry.

$$
Q|\text{boson}\rangle = |\text{fermion}\rangle \quad Q|\text{fermion}\rangle = |\text{boson}\rangle
$$

Cancellation OK if SUSY partner mass splitting $\sim M_W$
## Minimal Supersymmetric Standard model (MSSM)

<table>
<thead>
<tr>
<th>Standard Model</th>
<th>Supersymmetrize</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q = \begin{pmatrix} u_L \ d_L \end{pmatrix}$</td>
<td>$\tilde{u}_L \tilde{d}_L$</td>
</tr>
<tr>
<td>$L = \begin{pmatrix} \nu_L \ e_L \end{pmatrix}$</td>
<td>$\tilde{\nu}_L \tilde{\nu}_L$</td>
</tr>
<tr>
<td>$u^c, d^c, e^c$</td>
<td>$\tilde{u}^c, \tilde{d}^c, \tilde{e}^c$</td>
</tr>
<tr>
<td>$H_u = \begin{pmatrix} H^+_u \ H^0_u \end{pmatrix}$</td>
<td>$\tilde{H}^+_u \tilde{H}^0_u$</td>
</tr>
<tr>
<td>$H_d = \begin{pmatrix} H^0_d \ H^-_d \end{pmatrix}$</td>
<td>$\tilde{H}^0_d \tilde{H}^-_d$</td>
</tr>
<tr>
<td>$g$</td>
<td>$\tilde{g}$</td>
</tr>
<tr>
<td>$W^\pm, W^0$</td>
<td>$\tilde{W}^\pm, \tilde{W}^0$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\tilde{B}$</td>
</tr>
</tbody>
</table>
• Neutral higgsinos, winos, and bino mix to form mass eigenstates:

\[ \tilde{H}_u^0, \tilde{H}_d^0, \tilde{W}^0, \tilde{B} \implies \text{neutralinos } \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0 \]

• Charged higgsinos and winos mix to form mass eigenstates:

\[ \tilde{H}_u^+, \tilde{H}_d^-, \tilde{W}^\pm, \implies \text{charginos } \tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm \]

• The left-handed squark and right-handed squark mix to form mass eigenstates, especially the stop, sbottom, and stau

\[ \text{e.g. } \tilde{b}_L, \tilde{b}_R \implies \tilde{b}_1, \tilde{b}_2 \]
Soft supersymmetry breaking

\[ \mathcal{L} = \mathcal{L}_{\text{susy}} \]

- Based on the underlying gauge symmetries and supersymmetry.
- So far supersymmetry is not broken. SM particles and SUSY partners are both massless.
- Gluon, \( W \), \( Z \) bosons are massless and so are the gluino, wino, bino before SUSY breaking.
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\[ \mathcal{L}_{\text{soft}} = -\frac{1}{2}(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B}) + c.c. \]

gaugino mass
give masses to the gluino, wino, bino. Thus, break SUSY.
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- Similarly,

\[ \mathcal{L}_{\text{soft}} = -\tilde{Q}^\dagger M_Q^2 \tilde{Q} - \tilde{L}^\dagger M_L^2 \tilde{L} - \ldots \]

give masses to the scalar quarks and scalar leptons.
Gauge Coupling Unification

\[ \frac{dg_i}{dt} = \frac{g_i}{16\pi^2} \left[ b_i g_i^2 + \frac{1}{16\pi^2} \left( \sum_{j=1}^{3} b_{ij} g_i^2 g_j^2 - \sum_{j=1}^{3} a_{ij} g_i^2 \lambda_j^2 \right) \right] \]

Ellis, Kelly, Nanopoulos; Amaldi, de Hoer, Furstenau; Langacker, Luo (1991).
Dynamical Electroweak Symmetry Breaking

(Chamseddine, Arnowitt and Nath; Gaume, Polchinski, and Wise; Ellis, Nanopoulos, Tamvakis (1982–1983))

The Higgs potential

\[ V_H = (M_{H_u}^2 + \mu^2)|H_u|^2 + (M_{H_d}^2 + \mu^2)|H_d|^2 + B(\epsilon_{ij} H_d^i H_u^j + h.c.) \]
\[ + \frac{1}{8}(g^2 + g'^2) \left[ |H_u|^2 - |H_d|^2 \right]^2 + \frac{1}{2}g^2|H_d^i H_u^i|^2 \]

EWSB occurs when one of the \((M_{H_u}^2 + \mu^2), (M_{H_d}^2 + \mu^2)\) becomes negative.

\(M_{H_u}^2\) can run to a negative value by the top Yukawa coupling.

\[ \frac{dM_{H_u}^2}{dt} = \frac{1}{8\pi^2} \left( -\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + 3\lambda_t^2(M_{QL}^2 + M_{t_R}^2 + M_{H_u}^2 + A_t^2) \right) \]
Evolution of sparticle masses

mass (GeV)

Q (GeV)

M_3
m_{\tilde{b}_R}
\bar{Q}_L
m_{\tilde{\tau}_R}
m_1
m_2
M_2
m_{\tilde{\tau}_L}
m_{\tilde{\tau}_R}
M_1

\sqrt{m_0^2 + \mu^2}
m_{1/2}
m_0
The lightest SUSY particle (LSP) is a **Dark Matter** candidate.

Imposing the *R*-parity conservation

- $R = 1$ for the SM particles
- $R = -1$ for SUSY particles

SUSY particles must be pair produced. E.g.

$$p\ p \rightarrow q\ \bar{q}\ \tilde{g}\ \tilde{g}$$

The LSP is stable:

$$\tilde{\chi}_1^0 \not\rightarrow \text{SM particles}$$

**LSP remains since freeze-out in the early universe**
Some problems of SUSY

- Too many soft parameters, more than 100.
- $\mu$ problem
- Proton decay operators
- Too many sources for FCNC and CP violation

No SUSY Particles (NSP) Found So Far
Various SUSY scenarios

and

Associated Phenomenology
Gravity mediated SUSY breaking

SUSY breaking theory origin is in the hidden sector, and the SUSY breaking effect is transmitted by a mediation sector.

Historically, the most popular one is the gravity. Gravitation, suppressed by $M_{\text{Pl}}$ couples the hidden sector to the visible. By dimension:

$$M_{\text{soft}} \sim \frac{\langle F \rangle}{M_{\text{Pl}}}$$

Naturalness requires $M_{\text{soft}} \sim O(0.1 - 1)$ TeV, implying

$$\sqrt{\langle F \rangle} \sim 10^{11-12} \text{ GeV}$$

Gravitino mass is $\sim F/M_{\text{Pl}}$. 
Supergravity Lagrangian

The hidden sector fields couple to the visible sector via gravity interactions. In the effective Lagrangian, it contains nonrenormalizable terms:

\[ \mathcal{L}_{\text{sugra}} = - \frac{F_X}{M_{P1}} \sum \frac{1}{2} f_a \lambda^a \lambda^a + c.c. - \frac{F_X F^*_X}{M_{P1}^2} k^i_j \phi_i \phi^*_j \]

\[ - \frac{F_X}{M_{P1}} \left( \frac{\alpha^{ijk}}{6} \phi_i \phi_j \phi_k + \frac{\beta^{ij}}{2} \phi_i \phi_j \right) + c.c. \]

where $F_X$ is the auxiliary field of a chiral superfield $X$ in the hidden sector.

When $F_X$ develops a VEV, SUSY is broken in the hidden sector and thus communicates to the visible sector. It develops

- gaugino masses: $M_a = f_a \langle F_X \rangle / M_{P1}$
- sfermion masses: $(M^2)_i^j = k^i_j \langle F_X \rangle^2 / M_{P1}^2$
- Trilinear terms: $A^{ijk} = \alpha^{ijk} \langle F_X \rangle / M_{P1}$
- B term: $B_{ij} = \beta^{ij} \langle F_X \rangle / M_{P1}$

It is not obvious they are flavor-blind.
Minimal Supergravity (mSUGRA)

Arnowitt, Chamseddine, Nath

One solution to flavor problem is to adopt universal boundary conditions. The soft parameters are defined by 5 parameters only:

- $M_0$: universal scalar mass
- $M_{1/2}$: universal gaugino mass
- $A_0$: universal $A$ term
- $\tan \beta \equiv v_u/v_d$: ratio of VEV of the Higgs fields
- $\text{sign}(\mu)$: sign of the $\mu$ parameter

The LSP is usually the lightest neutralino (the combination of bino, neutral wino and higgsinos). It is the dark matter candidate.

Only 5 parameters $\Rightarrow$ very predictive. But somehow quite restrictive now by the WMAP data. Relaxing the universality can relieve the suitable parameter space.
SPS4: mSUGRA: $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$, $\tan \beta = 50$, $\mu > 0$.
SPS5: mSUGRA: $m_0 = 150$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -1000$, $\tan \beta = 5$, $\mu > 0$. 
Collider Phenomenology of SUGRA

The neutral LSP will not leave any tracks in the detector, ie, it gives to missing energies.

**Smoking gun signatures:**

1. Multi-leptons plus $E_T$

   $$pp \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \ell^+ \ell^- \nu \bar{\nu} \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

   $$pp \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow \ell^\pm \ell^- \nu \bar{\nu} \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

2. Multi-lepton multi-jets plus $E_T$

   $$pp \rightarrow \tilde{g} \tilde{g}, \tilde{q} \tilde{q}^*, \tilde{g} \tilde{g}$$

   Gluino is majorana that can decay into leptons of either charges $\Rightarrow$ same-sign dilepton + $E_T$ signal.

3. Gluino can decay into $b\bar{b}_1^* \rightarrow b\bar{b} \tilde{\chi}_1^0 \Rightarrow 4b$ + $E_T$ signal.
Tri-lepton + $E_T$ signal

Multi-jet + $E_T$
CDF search for gluino decays $\tilde{g} \rightarrow \tilde{b}\tilde{b}$

In the gluino pair production

$$p\bar{p} \rightarrow \tilde{g}\tilde{g} \rightarrow (\tilde{b}^* b/\tilde{b}\bar{b}) (\tilde{b}^* b/\tilde{b}\bar{b}) \rightarrow 4b + 2\tilde{\chi}_1^0$$

The signal is defined by a $E_T > 80$ GeV plus $b$ tags.

<table>
<thead>
<tr>
<th>Process</th>
<th>Exclusive Single B-Tag</th>
<th>Inclusive Double B-Tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWK</td>
<td>$5.66 \pm 0.76(stat) \pm 1.72(sys)$</td>
<td>$0.61 \pm 0.21(stat) \pm 0.19(sys)$</td>
</tr>
<tr>
<td>TOP</td>
<td>$6.18 \pm 0.12(stat) \pm 1.42(sys)$</td>
<td>$1.84 \pm 0.06(stat) \pm 0.46(sys)$</td>
</tr>
<tr>
<td>QCD</td>
<td>$4.57 \pm 1.64(stat) \pm 0.57(sys)$</td>
<td>$0.18 \pm 0.08(stat) \pm 0.05(sys)$</td>
</tr>
<tr>
<td>Total Predicted</td>
<td>$16.41 \pm 1.81(stat) \pm 3.15(sys)$</td>
<td>$2.63 \pm 0.23(stat) \pm 0.66(sys)$</td>
</tr>
<tr>
<td>Observed</td>
<td>21</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 24: Number of expected and observed events in signal region.

Thus, no evidence for gluino pair production with sequential decay into sbottom-bottom is observed.
Gluino-sbottom 95% exclusion

CDF Run II Preliminary, 156 pb$^{-1}$

- PROSPINO, NLO
- $Q^2$ Scale Uncertainty
- $m(\tilde{q}) = 500 \text{ GeV}/c^2$
- $\Delta m(\tilde{g}b_1) = 60 \text{ GeV}/c^2$
- $m(\tilde{g}) = 60 \text{ GeV}/c^2$

Cross Section [pb]

100 10 1

Mass of the Gluino [GeV/$c^2$]

180 200 220 240 260 280 300

CDF Run II Preliminary

- Excl. Single B-Tag
- Incl. Double B-Tag

Sbottom mass [GeV/$c^2$]

100 120 140 160 180 200

Gluino mass [GeV/$c^2$]

180 200 220 240 260 280

Gluino $\rightarrow \tilde{b}\tilde{b}$, 95% C.L. Cross Section Limit

BR(\tilde{g} \rightarrow b\tilde{b})=100%

CDF Run I excluded

CDF Run II Preliminary

(s, incl. double tag)

(excl. single tag)

\tilde{g} \rightarrow t\bar{t}, kinematically forbidden
CDF Tri-lepton search for chargino-neutralino production

Tri-lepton signal comes from the associated chargino-neutralino production

\[ p\bar{p} \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 \rightarrow \ell^+ \ell^- \ell \nu \tilde{\chi}_1^0 \tilde{\chi}_1^0 \]

With \( \mathcal{L} = 745 \text{ pb}^{-1} \) the \( \mu + \ell \ell \) final state only found one event, consistent with the SM background of 0.64 event.
DØ Tri-lepton search for chargino-neutralino production (325 pb⁻¹)

Heavy squarks: no negative interference in the production. 3ℓ-max: leptonic BR is enhanced maximally for $m_{\tilde{\ell}} \gtrsim m_{\tilde{\chi}_2^0}$. Large $m_0$: gives small leptonic BR.
LEP 2 limits

At LEP2 $e^-e^+$ collider, it can produce many sparticle pairs:

$$e^-e^+ \rightarrow \tilde{\ell}^+\tilde{\ell}^-, \tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^0\tilde{\chi}_1^0, \tilde{\chi}_2^0\tilde{\chi}_2^0, \tilde{q}\tilde{q}^*$$

There are limits on slepton masses, chargino masses, as they can readily produced at LEP if they exist. The exclusion is almost up to half of the $\sqrt{s}$ if the mass difference from the LSP is large enough.

Another constraint comes from SM Higgs mass bound:

$$m_h > 114.4 \text{ GeV}$$

Most of the SUSY parameter space yields a SM-like Higgs boson, therefore the Higgs mass bound is applicable.
LEP2 slepton and squark mass limits

\[ \sqrt{s} = 183-208 \text{ GeV} \]

ADLO

Excluded at 95% CL

(\mu = 200 \text{ GeV/c}^2, \tan\beta = 1.5)

Observed

Expected

CDF

D0

UA1 & UA2

LEP

ADLO

LEP
LEP2 chargino mass limits

$\sqrt{s} > 206.5$ GeV

$\tan \beta = 2$, $\mu = -200$ GeV

Excluded at 95% C.L.

ADLO preliminary
Gaugino - $M_{\tilde{\nu}} > 500$ GeV

Expected limit
Gauge mediated SUSY breaking

Dine, Nelson, Shirman; Dimopoulos, Dine, Raby, Thomas

This is a very simple idea to use the gauge interactions to communicate the SUSY breaking from the hidden sector to the visible sector. It is flavor-blind. It could just be the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry.

Typical soft masses are of order

$$M_{\text{soft}} \sim \frac{\alpha}{4\pi} \frac{\langle F_X \rangle}{M_{\text{mess}}}$$

where $F_X$ is the auxillary field of a chiral superfield in the hidden sector, $M_{\text{mess}}$ is the mass scale of the messenger sector.

Both $\langle F_X \rangle$ and $M_{\text{mess}}$ can be as low as 10 TeV.

The gravition mass $M_{3/2} \sim \langle F_X \rangle / M_{\text{Pl}} \ll M_{\text{soft}}$ can be as low as sub-eV.
A minimal GMSB model

Suppose the messenger sector contains $q, q^c, \ell, \ell^c$ that transform under $SU(3) \times SU(2)_L \times U(1)_Y$ as

$$
q \sim (3, 1, -1/3), \quad q^c \sim (\bar{3}, 1, 1/3), \quad \ell \sim (1, 2, 1/2), \quad \ell^c \sim (1, 2, -1/2)
$$

which contain the fermionic and scalar parts. They couple to a gauge singlet chiral superfield of the hidden sector

$$
W_{\text{mess}} = y_2 S \ell \ell^c + y_3 S qq^c
$$

Suppose $F_S$ develops a VEV by some dynamical SUSY breaking mechanisms, and we assume the scalar part of $S$ develops a VEV too: $\langle S \rangle$.

The effect of SUSY breaking is then transmitted to the messenger sector:

$$
\ell, \ell^c : \quad M_{\text{ferm}}^2 = |y_2 \langle S \rangle|^2, \quad M_{\text{scal}}^2 = |y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle|$
$$
$$
q, q^c : \quad M_{\text{ferm}}^2 = |y_3 \langle S \rangle|^2, \quad M_{\text{scal}}^2 = |y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle|
$$
The SUSY breaking is then transmitted to MSSM gauginos via 1 loop diagram:

\[ M_a(M_{\text{mess}}) = \frac{\alpha_a}{4\pi} \Lambda \text{ where } \Lambda = \frac{\langle F_S \rangle}{\langle S \rangle} \]
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MSSM scalars receive mass from 2 loop diagrams:

\[ M_i^2(M_{\text{mess}}) = 2\Lambda^2 \left[ \left( \frac{\alpha_3}{4\pi} \right)^2 C_3^i + \left( \frac{\alpha_2}{4\pi} \right)^2 C_2^i + \frac{3}{5} \left( \frac{\alpha_1}{4\pi} \right)^2 C_1^i \right] \]

where \( C_{3,2,1} = 0 \) for gauge singlets, and otherwise \( C_{3,2,1} = 4/3, 3/4, (Y/2)^2 \) for fundamental representation of \( SU(3), SU(2)_L, U(1)_Y \), respectively.
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\[
M_a(M_{\text{mess}}) = N \frac{\alpha_a}{4\pi} \Lambda
\]

\[
M_i^2(M_{\text{mess}}) = 2N \Lambda^2 \left[ \left( \frac{\alpha_3}{4\pi} \right)^2 C_3^i + \left( \frac{\alpha_2}{4\pi} \right)^2 C_2^i + \frac{3}{5} \left( \frac{\alpha_1}{4\pi} \right)^2 C_1^i \right]
\]
Phenomenology

- Trilinear terms are further suppressed by $\alpha_a/4\pi$ relative to gaugino mass.
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$$M_a, M_i \sim \frac{\alpha_a}{4\pi} \Lambda$$
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$$M_a, M_i \sim \frac{\alpha_a}{4\pi} \Lambda$$

- Note that the gaugino mass increases as $N$, but scalar mass increases as $\sqrt{N}$. For $N = 1$ bino is the NLSP while for $N \geq 2$ the stau is the NLSP.
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- Note that the gaugino mass increases as $N$, but scalar mass increases as $\sqrt{N}$. For $N = 1$ bino is the NLSP while for $N \geq 2$ the stau is the NLSP.
- Collider signature depends on which is the NLSP:
  \[ \tilde{\chi}_1^0 \rightarrow \tilde{G} + (\gamma, Z, h), \quad \tilde{\tau} \rightarrow \tilde{G} + \tau \]

  One is the multi-photon while another is multi-tau-lepton in the final state.

  Typical decay length if the NLSP is
  \[ L \simeq (10 \text{ km}) \langle \beta \gamma \rangle \left( \frac{\sqrt{F}}{10^7 \text{ GeV}} \right)^4 \left( \frac{100 \text{ GeV}}{M_{\text{NLSP}}} \right)^5 \]
SPS7: GMSB with $\tilde{\tau}$ NLSP: $\Lambda = 40$ TeV, $M_{\text{mess}} = 80$ TeV, $N = 3$, $\tan \beta = 15$, $\mu > 0$.

SPS8: GMSB with $\tilde{\chi}_1^0$ NLSP: $\Lambda = 100$ TeV, $M_{\text{mess}} = 200$ TeV, $N = 1$, $\tan \beta = 15$, $\mu > 0$. 
Combined CDF and DØ limit on GMSB diphoton plus $E_T$ events

![Graph showing the relationship between Chargino Mass and Neutralino Mass](image)

- **CDF**: 202 pb$^{-1}$
- **DØ**: 263 pb$^{-1}$

- **GMSB** $\gamma\gamma+E_T$

- **M=2Λ, N=1, tanβ=15, µ>0**

- **σ × BR ($\gamma+X)^2$ (pb)**

- **QCD Uncertainty**

- **Expected limit**

- **Observed limit**
DØ Search for GMSB diphoton plus $E_T$ events

Using 760 pb$^{-1}$ data, search for events with 2$\gamma$

\[ E_T > 25 \text{ GeV} \quad \text{and} \quad |\eta| < 1.1 \quad E_T > 45 \text{ GeV} \]

4 events with an estimated background of $2.1 \pm 0.7$ events.
DØ GMSB limit using 760 pb$^{-1}$

Expected limit on chargino mass $> 215$ GeV.
LEP2 GMSB limits

\begin{equation}
\tilde{\tau} \quad (\tilde{\tau} \text{NLSP})
\end{equation}

\[ \sqrt{s} = 189-209 \text{ GeV} \]

\[ \sigma_{\text{exp.}} \sqrt{s} = 208 \text{ GeV} \]

excluded at 95% CL for all lifetimes

\[ 86.9 \text{ GeV/c}^2 \]

Search using acoplanar diphoton plus $E_T$, and di-taus plus $E_T$. 

\[ m(\tilde{e}) = 2.0 m(\tilde{\chi}_1^0) \]

\[ m(\tilde{e}) = 1.1 m(\tilde{\chi}_1^0) \]
Anomaly mediated SUSY breaking

Randall, Sundrum; Giudice, Luty, Murayama, Rattazzi

In 4D, gravity mediation between hidden and visible sectors is always present. In extra dimensions, one can separate the two sectors geometrically. The hidden and visible sectors on separate branes. Only the gravity in the bulk communicate in between. Almost all SUSY breaking effects are suppressed.

The conformal anomaly generates loop-suppressed soft SUSY breaking. These contributions are always present. Gauginos and scalars acquire

$$M_a = \frac{\beta_a}{g_a} m_{3/2}, \quad (M^2)_i^j = -\frac{1}{2} \frac{d\gamma_i^j}{d(ln Q)} m_{3/2}$$

where $m_{3/2}$ is the gravitino mass, $\beta_a = -b_a g_a^3$, and $b_a = (-33/5, -1, 3)$, and $\gamma_i^j$ are anomalous dimensions.

The slepton masses are tachyonic.
Phenomenology

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- Note that the gravitino mass $m_{3/2}$ needs to be in $O(100)$ TeV in order to give acceptable gaugino masses.
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- Need some means to give positive slepton mass squared.
- The most special feature of the model is the gaugino mass pattern. Wino is the lightest particle.
- Note that the gravitino mass $m_{3/2}$ needs to be in $O(100)$ TeV in order to give acceptable gaugino masses.
- In this wino-LSP scenario, the charged wino forming the lightest chargino is very close in mass to the LSP:

$$\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 + (\pi^+ \text{ or } \ell^+ \nu)$$
Typical AMSB spectrum

SPS 9

SPS9: AMSB : $m_0 = 450$ GeV, $m_{3/2} = 60$ TeV, $\tan \beta = 10$, $\mu > 0$. 
Chargino Decay and detection in wino-LSP scenario

The decay of $\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 W^{+*}$ depends critically on $\Delta M \equiv M_{\tilde{\chi}_1^+} - M_{\tilde{\chi}_1^0}$.

- $\Delta M < m_\pi$:
  The only available modes are $e^+\nu_e$ and $\mu^+\nu_\mu$. The chargino will travel 1 m or so before decay, so appears as heavily charged tracks. Background free.

- $m_\pi < \Delta M < 1 \text{ GeV}$:
  The most difficult region that depends on how many layers of silicon that the chargino can travel before decays, and the momentum resolution to tell the non-pointing pion.

- $1 - 2 \text{ GeV} \lesssim \Delta M$:
  The decay is prompt. If $\Delta M$ is large enough to have energetic leptons and jets, it is easy for detection. If the decay products are too soft, have to rely on other methods. E.g.
  \[
  e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-\gamma
  \]
  Chen, Gunion, Drees.

- $\Delta M > \text{ a few GeV}$:
  charged lepton and jets are detectable.
DØ search for metastable charginos

Signatures for stable charged tracks, reconstructed as “muons”, but with speed and mass inconsistent with beam-produced muons.

If $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} < m_\pi$, chargino may have a long decay.
LEP2 limits on stable charginos

\[
\sigma(e^+e^-\rightarrow\tilde{\chi}^\pm\tilde{\chi}) (\text{pb})
\]

excluded 95% CL

Preliminary

ADLO \sqrt{s}=133-208 \text{ GeV}
Split Supersymmetry

(Arkani, Dimopoulos 2004)

The magic word: Landscape!!

In the vast number of vacua, there is a good chance to find some with high SUSY breaking scales.

In reality, Why Not!! These scenarios are not impossible

- All scalars are super heavy, except for a light SM-like Higgs boson
  \[ \tilde{m} \sim 10^{9-16} \text{ GeV} \]

- Gauginos and Higgsinos are \( O(\text{TeV}) \).
Split Supersymmetry

Properties:

- Gauge coupling unification
- Light SM-like Higgs boson
- Super heavy scalars $\Rightarrow$ safe FCNC, CP-violation, EDM, but there is still one possible source
- Relatively light gauginos, Higgsinos, $\mu$ parameter $\Rightarrow$ Dark Matter
- Stable gluino, gluinonium signature
Gauge Coupling Unification

\[ \frac{1}{\alpha_i(M_X^2)} = \frac{1}{\alpha_i(M_Z^2)} - \frac{\beta_i}{4\pi} \ln \left( \frac{M_X^2}{M_Z^2} \right) \]

\[ SM : (\beta) = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F + \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} N_H \]

\[ SUSY : (\beta)_{SUSY} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} F + \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} N_H \]

\[ Split - SUSY : (\beta)_{\text{split}}|<\tilde{m} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F + \begin{pmatrix} \frac{5}{10} \\ \frac{5}{6} \\ 0 \end{pmatrix} \]
Gauge Coupling Unification

SM  |  Weak Scale SUSY  |  Split SUSY

Ellis, Kelly, Nanopoulos (1991);  Arkani, Dimopoulos 2004
Consider split SUSY with SU(5) unification.

- Dim-5 operator is highly suppressed

\[ O_5 \sim \frac{QQ\tilde{Q}\tilde{Q}}{M_{Pl}} \]

- Dim-6 operators: \( p \rightarrow e^+\pi^0 \) with exchanges of \( M_X,Y \) bosons. Proton decay rate depends on \( M_{GUT} \) and \( \alpha_{GUT} \). Additional matter in complete SU(5) added to the intermediate scale will increase \( \alpha_{GUT} \).
Decay of gluino has to go through a squark:

\[ \tilde{g} \rightarrow q \tilde{q} \]  
\[ \tilde{q} \rightarrow q \tilde{\chi}^0 \]

\[ \tilde{m} \sim 10^{9-16} \text{ GeV} \Rightarrow \tau_{\tilde{g}} \gtrsim 10^{-2} - 10^{14} \text{s} \]

Gluino is stable within particle detectors.
Stable gluino-hadron

- Hadronize into a massive stable particle.
- Electrically either neutral or charged, depending on the mass spectrum.
- The heavy neutral particle will go through the detector unnoticed, very small energy loss.
- Charged particles also undergo ionization energy loss, via which it can be detected. It happens in central vertex detector and also in muon chamber.

(Kilian et al., Hewett et al., KC and Keung)
Experimentally, the massive stable charged particle will produce a track in the central tracking and/or silicon vertex system, where $dE/dx$ and $p$ can be measured.

$$\beta \gamma = \frac{p}{E} \frac{E}{M} = \frac{p}{M} \lesssim 0.85$$

The particle is required to penetrate to the outer muon chamber.

$$0.25 - 0.5 \lesssim \beta \gamma$$

c.f. CDF Coll. used a criteria: $0.26 - 0.5 \lesssim \beta \gamma \lesssim 0.86$, but it is for a particle of mass of $50 - 500 \text{ GeV}$ only.
Cross sections at the LHC.

$\sigma_{1\text{MCP}}, \sigma_{2\text{MCP}}$ denote requiring the detection of 1, 2 massive stable charged particles (MCP) in the final state.

<table>
<thead>
<tr>
<th>$m_{\tilde{g}}$ (TeV)</th>
<th>$\sigma_{1\text{MCP}}$ (fb)</th>
<th>$\sigma_{2\text{MCP}}$</th>
<th>$\sigma_{\geq 1\text{MCP}}$</th>
<th>$\sigma_{\geq 1\text{MCP}}$</th>
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<td>$P = 0.5$</td>
<td>$P = 0.1$</td>
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<td>0.09</td>
<td>0.39</td>
<td>0.09</td>
<td>0.0096</td>
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</table>

$P \equiv$ probability that $\tilde{g}$ fragments into charged R-hadron
Problems with $R$-hadron detection

- The probability that $\tilde{g} \rightarrow$ charged hadrons depends crucially on the bound state spectrum.
- The detected cross section depends strongly on $P$:
  \[ \sigma(m_{\tilde{g}} = 1.5\text{TeV}) = 4.6 - 0.11 \text{ fb} \quad \text{for } P = 0.5 - 0.01 \]
- More complications occur when frequent swapping between neutral and charged $R$-hadron states. E.g.,
  \[ (\tilde{g}ud) \leftrightarrow (\tilde{g}d\bar{d}) \]
- Need an unambiguous signature.
Gluinos are stable, exchange gluons to form a bound state.

\[
\text{Color: } 8 \otimes 8 = 1 + 8_S + 8_A + 10 + \overline{10} + 27
\]

S-wave, Spin: \( ^1 S_0 \) (antisymmetric), \( ^3 S_1 \) (symmetric)

Possible configurations: \( ^1 S_0 (1) \), \( ^1 S_0 (8_S) \), and \( ^3 S_1 (8_A) \)
Production of Gluonium

Replace the spinor combination \( u(P/2)\bar{v}(P/2) \) by

\[
\begin{align*}
1S_0(1) \quad & \quad u(P/2)\bar{v}(P/2) \quad \longrightarrow \quad \frac{1}{\sqrt{2}} \frac{R_1(0)}{2\sqrt{4\pi M}} \frac{1}{\sqrt{8}} \delta^{ab} \gamma^5 (\mathbb{P} + M) \\
1S_0(8_S) \quad & \quad u(P/2)\bar{v}(P/2) \quad \longrightarrow \quad \frac{1}{\sqrt{2}} \frac{R_8(0)}{2\sqrt{4\pi M}} \sqrt{\frac{3}{5}} d^{hab} \gamma^5 (\mathbb{P} + M) \\
3S_1(8_A) \quad & \quad u(P/2)\bar{v}(P/2) \quad \longrightarrow \quad \frac{1}{\sqrt{2}} \frac{R_8(0)}{2\sqrt{4\pi M}} \frac{1}{\sqrt{3}} f^{hab} \varphi(P) (\mathbb{P} + M)
\end{align*}
\]

Color factors:

\[
\begin{align*}
1 \quad & \quad \frac{1}{\sqrt{8}} \delta^{ab} \\
8_S \quad & \quad \sqrt{\frac{3}{5}} d^{hab} \\
8_A \quad & \quad \frac{1}{\sqrt{3}} f^{hab}
\end{align*}
\]
Production of Gluinionium ...

The values of the color octet and singlet wave functions at the origin are given by the 

\textit{coulombic potential} between the gluinos, with one-gluon approximation

\[ |R_8(0)|^2 = \frac{27\alpha_s^3(M)M^3}{128} \]

\[ |R_1(0)|^2 = \frac{27\alpha_s^3(M)M^3}{16} \]

There is an additional factor of $1/\sqrt{2}$ because of the 

\textit{identical gluinos} in the wave function of the gluinionium.
Hadronic Production of Gluononium $^3S_1(8A)$

The lowest order process for $^3S_1(8A)$

$$q\bar{q} \rightarrow ^3 S_1(8A)$$

The next order include

$$q\bar{q} \rightarrow ^3 S_1(8A) + g$$
$$qg \rightarrow ^3 S_1(8A) + q$$
$$gg \rightarrow ^3 S_1(8A) + g$$
Hadronic Production of Gluonium $^3S_1(8_A)$ ...

\[
\hat{\sigma} = \frac{16\pi^2 \alpha_s^2}{3} \frac{|R_8(0)|^2}{M^4} \delta(\sqrt{s} - M) .
\]

After folding with the parton distribution functions:

\[
\sigma = \frac{32\pi^2 \alpha_s^2}{3s} \frac{|R_8(0)|^2}{M^3} \int f_{q/p}(x) f_{\bar{q}/p}(M^2/sx) \frac{dx}{x} ,
\]

Decay width into $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, $t\bar{t}$:

\[
\Gamma (^3S_1(8_A)) = \sum_{Q=u,d,s,c,b,t} \alpha_s^2 \frac{|R_8(0)|^2}{M^4} (M^2 + 2m_Q^2) \sqrt{1 - 4m_Q^2/M^2}
\]

Each mode $\sim \frac{1}{6}$ for heavy gluonium.
Hadronic Production of Gluonium ...

(a) Tevatron

(b) LHC

Total cross section (pb)

Gluonium mass M (GeV)

$^{3}\!S_{1}(8_{A})$

$^{1}\!S_{0}(1)+^{1}\!S_{0}(8_{S})$

$^{2}\!S_{1}(8_{A})$

$^{1}\!S_{0}(1)+^{1}\!S_{0}(8_{S})$
Detection & Background analysis

\[ {}^1S_0(1, 8_S) \rightarrow gg \]
\[ {}^3S_1(8_A) \rightarrow q\bar{q} \]

buried under huge QCD background.

\[ {}^3S_1(8_A) \rightarrow t\bar{t}, b\bar{b} \]

have the potential feasibility for observation.

Irreducible background comes from QCD \( t\bar{t} \) or \( b\bar{b} \)
Detection & Background analysis ...

- Gluinonium annihilates into $t$ and $\bar{t}$ with a large $p_T$

$$p_T \sim O(M_{\tilde{g}})$$

We impose

$$p_T(t), p_T(\bar{t}) > \frac{3}{4} m_{\tilde{g}} \quad \text{for } M \geq 1 \text{ TeV}$$
$$p_T(t), p_T(\bar{t}) > 100 \text{ GeV} \quad \text{for } M < 1 \text{ TeV}$$

- Invariant mass $M_{tt}$ forms a peak right at gluinonium mass, width determined by experimental resolution.

$$\delta E/E = 50\%/\sqrt{E}$$

We can then calculate the SM background right under the peak.
Cross sections at the LHC for the gluonium signal into $t\bar{t}$ with mass $M$ and the continuum $t\bar{t}$ background between $M - 50$ GeV and $M + 50$ GeV.

$$M = 2m_{\tilde{g}} \quad \sigma(3S_1(8_A)) \quad t\bar{t} \text{ bkgd (fb)} \quad S/\sqrt{B}$$

<table>
<thead>
<tr>
<th>$M$ (TeV)</th>
<th>$\sigma(3S_1(8_A))$ (fb)</th>
<th>$t\bar{t}$ bkgd (fb)</th>
<th>$S/\sqrt{B}$ for $L = 100$ fb$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
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<td>83000</td>
<td>4.2</td>
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<tr>
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<td>1.4</td>
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<tr>
<td>1.5</td>
<td>0.78</td>
<td>97</td>
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<tr>
<td>2.0</td>
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<td>14</td>
<td>0.45</td>
</tr>
<tr>
<td>3.0</td>
<td>0.014</td>
<td>0.67</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Including $b\bar{b}$ would increase $S/\sqrt{B}$ by $\sqrt{2}$. 
DØ search for stopped Gluinos

The long-lived gluino can hadronize into charged $R$-hadron. It may lose all its K.E. by ionization and stopped inside the detector. Then after a “while” it decays into a gluon and LSP. The signature is a jet plus $E_T$.

Backgrounds:

- **Cosmic muons** that fake the signal by initiating a high-energy shower within the detector (identified by a muon in or out of the muon system).

- **Beam-halo muons** are those traveling parallel to the beam, very narrow in $\phi$.

A gluino-induced shower would be wide and contain no muon.
DØ results for stopped gluinos

A simulated signal of $m\tilde{g} = 400$ GeV. 95% C.L. upper limits on $\sigma(\tilde{g}) \times B(\tilde{g} \rightarrow j + E_T)$ for 3 LSP masses of 50, 90, 200 GeV.
Neutralinos and Charginos

(KC and J. Song, hep-ph/0507113)

- Production and decay via intermediate $\tilde{f}$ disappear.
- Direct production via Drell-Yan-like processes ($\gamma, Z^*, W^*$).
- Neutralino decays via

$$
\tilde{\chi}_j^0 \rightarrow \tilde{\chi}_i^0 Z^* \rightarrow \tilde{\chi}_i^0 f \bar{f} \\
\tilde{\chi}_j^0 \rightarrow \tilde{\chi}_i^{\pm} W^* \rightarrow \tilde{\chi}_i^0 f \bar{f}' \\
\tilde{\chi}_j^0 \rightarrow \tilde{\chi}_i^0 h^* \rightarrow \tilde{\chi}_i^0 b \bar{b} \\
\tilde{\chi}_j^0 \xleftarrow{\chi^+ W^- \text{ loop}} \tilde{\chi}_i^0 \gamma
$$

- Chargino decays via

$$
\tilde{\chi}_j^+ \rightarrow \tilde{\chi}_i^0 W^* \rightarrow \tilde{\chi}_i^0 f \bar{f}'
$$
Aim: to enhance the $\tilde{\chi}_j^0 \to \tilde{\chi}_i^0 + \gamma$ by suppressing the $Z - \tilde{\chi}_j^0 - \tilde{\chi}_i^0$ couplings:

$$O''_{ij}^L = -O''_{ij}^R = \frac{1}{2} (N_{i4}^* N_{j4} - N_{i3} N_{j3}^*)$$

It is possible but needs fine tuned relation between $\mu$ and $M_1$. Specifically, need

$$M_1 \sim |\mu|$$

Potentially, observe mono-photon plus missing energy signature at hadron colliders.
\[ \tilde{\chi}_2^0 \text{ decay widths, } M_1 = 200 \text{ GeV} \]
$\widetilde{\chi}_3^0$ decay widths, $M_1 = 200$ GeV
$\tilde{\chi}^0_{3,2}$ radiative decay branching ratios
Production of Neutralinos and Charginos

\begin{align*}
q + \overline{q} & \xrightarrow{Z^*} \tilde{\chi}_i^0 + \tilde{\chi}_j^0 \\
q + \overline{q} & \xrightarrow{\gamma, Z^*} \tilde{\chi}_i^- + \tilde{\chi}_j^- \\
q + \overline{q'} & \xrightarrow{W^*} \tilde{\chi}_i^\pm + \tilde{\chi}_j^0 \\
e^-e^+ & \xrightarrow{\gamma, Z^*} \tilde{\chi}_i^+ + \tilde{\chi}_j^- \\
e^-e^+ & \xrightarrow{Z^*} \tilde{\chi}_i^0 + \tilde{\chi}_j^0
\end{align*}

(Zhu; Kilian et al.; KC and Song)
$\tilde{\chi}_3^0$ production, $M_1 = 200$ GeV
Production at 0.5 TeV $e^+e^-$ ILC

$M_1 = 200$ GeV, $M_2 = 400$ GeV, $\tan \beta = 10$
Production at 1 TeV $e^+e^-$ ILC

$M_1 = 200$ GeV, $M_2 = 400$ GeV, $\tan\beta = 10$
Topics that cannot be covered in this lecture but equally interesting

- SUSY breaking by 5d boundary conditions (Scherk-Schwarz SUSY breaking).
- Gluino LSP scenarios (Baer et al. hep-ph/9806361).
- Next-to-minimal supersymmetric model (NMSSM).