From Dancing WavePacket to the Frictionless Atom Cooling

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• Motivation
• Quantum friction and the classical picture
• The frictionless atom cooling
Initial problem:

\[\omega(t) = \frac{2}{mL^2(t)}\]

\[L(t) = L_0 + v t\]

\[v_e \sim \frac{1}{mL}\]

\(v \ll v_e : \text{ ADIABATIC APPROXIMATION.}\)

\(v \gg v_e : \text{ SUDDEN APPROXIMATION.}\)

\(v \approx v_e : ?\)
Simulation Method

For a linear expansion or contraction potential, the wavefunction will be

\[ H(t)\varphi_n(x, t) = i\frac{\partial}{\partial t}\varphi_n(x, t) \]

\[ \varphi_n(x, t) = N(t)\phi_n\left(\frac{x}{L(t)}\right)e^{i\frac{muv^2}{2(L_0 + vt)}} - i\int_0^t E_n(t') dt', \]

where \( L(t) = L_0 + vt \) and \( N(t) = N(0)/\sqrt{L(t)/L(0)} \)

\[ \varphi_n\left(\frac{x}{L(t)}\right) \] \( \quad \text{THE INSTANTS WAVEFUNCTION,} \)

\[ E_n(t) \] \( \quad \text{THE INSTANTS ENERGY.} \)

Simulation Result I: adiabatic limit

Set $v_e = 1/mL_0$, $v = v_e/100$.

$E(t) \approx \frac{\omega(t)}{\omega_0} E_0$
Simulation Result II: sudden limit

\[ \nu = 10 \nu_e \]

\[ E(t) \approx E_0 + \frac{1}{2} m (\omega_t^2 - \omega_0^2) \langle \Psi_0 | x^2 | \Psi_0 \rangle. \]
Simulation Result III

\[ v = 0.5v_e \]

\[ (E - E_{ad})/E_0 \]

\[ L/L_0 \]

\[ E/E_0 \]
Why the wavepacket distribution and energy oscillate during the contraction?

$v = 0.5 v_e$
Simulation Result III

\[ v = 0.5v_e \]

\[ \psi_0 = a_0 \phi_0 + a_2 \phi_2 \]
Quantum Breathing

Breathing oscillation induced by sudden change


FIG. 2 (color online). Momentum distribution of an oscillating Tonks gas with $N = 9$ and $\omega_0/\omega_1 = 10$ at different times (in units of $T = \pi/\omega_1$) indicated on the panels, from numerical solution (solid lines) and Thomas-Fermi approximation (dashed lines). The units are indicated on the axis labels.

$$\omega_0 \rightarrow \omega_1$$

$$T = \pi/\omega_1$$
Quantum Breathing: Experimental Result


\[
\omega_{x,z} \approx \sqrt{\frac{\mu}{m}} \frac{B'(i_Q)}{\sqrt{B_0(i_Q,i_{B_0})}},
\]
\[
\omega_y = \sqrt{\frac{\mu}{m}} \sqrt{B''(i_Q)}.
\]

FIG. 1. Trapping geometry (figure in the horizontal plane). Ultracold $^{87}$Rb atoms are trapped in an Ioffe-Pritchard-type magnetic trap created by current $i_Q$ running through the three QUIC coils 1, 2, and 3. An additional pair of coils (a and b) produces a homogeneous field along $y$, which allows an independent tuning of the trap minimum field $B_0$ via the current $i_{B_0}$. shortcut decompression
Quantum Breathing: Experimental Result

FIG. 3. (Color online) Vertical trap decompression: comparison between different schemes. We report in (a) and (b), respectively, the cloud’s vertical center-of-mass position $z_{cm}$ and size $\sigma_z$ versus time after decompression for four different sequences. Open circles (green): abrupt decompression; solid circles (black): linear decompression in 35 ms; stars (red): shortcut decompression in 35 ms; squares (blue): linear decompression in 6 s.

\[ E_{exc} = E_{dip} + E_{breath} \]

\[ E_{dip} = \frac{1}{2} m \omega_{fz}^2 \Delta z_{cm}^2 \]

\[ E_{breath} \approx 2 m \omega_{fz}^2 \Delta \sigma_z^2 \]

Avoid quantum breathing can enhance cooling efficiency!!

induced by trap center shifting!!
Analytical analysis:
Method of time dependent eigenvectors

For
\[ \hat{H}(t) = \frac{\hat{p}^2}{2m} + m\omega^2(t)\hat{x}^2/2, \]

The time-dep eigenvector is
\[ \Psi_n(t, x) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \frac{e^{-i(n+1/2)\int_0^t dt' (\omega_0/b^2)}}{(2^n n! b)^{1/2}} \]
\[ \times e^{i(m/2\hbar)(\dot{b}/b + i\omega_0/b^2)x^2} H_n \left[ \left(\frac{m\omega_0}{\hbar}\right)^{1/2} \frac{x}{b} \right], \]

with
\[ \dot{b} + \omega^2(t)b = \omega_0^2/b^3, \quad b(0) = 1, \quad \dot{b}(0) = 0. \]

\[ b(t) = \frac{L(t)}{L(0)}. \]

Analytical analysis:
The energy change for packet in the time-dep trap I


\[ \langle \hat{H}(t) \rangle = \sum_{nl} c_n c_l^* \langle \Psi_l(t) | \hat{H}(t) | \Psi_n(t) \rangle \]

\[ = \langle \psi_0 | e^{im\hat{b}(0)\hat{x}^2/2\hbar} e^{i\hat{H}(0)/\hbar} \int_0^t dt' / b^2 \left[ \left( \frac{\hat{H}(0)}{b^2} - \frac{m}{2} \hat{b} \hat{x}^2 \right) + \frac{m}{2} b^2 \hat{x}^2 + \frac{\hat{b}}{2b} \{ \hat{x}, \hat{p} \} \right] e^{-i\hat{H}(0)/\hbar} \int_0^t dt' / b^2 e^{-im\hat{b}(0)\hat{x}^2/2\hbar} | \psi_0 \rangle, \]

|\psi_0\rangle \quad \text{initial wavepacket} \quad e^{-im\hat{b}(0)\hat{x}^2/2\hbar} \quad \text{projection operator}

\[ e^{-i\hat{H}(0)/\hbar} \int_0^t dt' / b^2 \quad \text{evolution operator} \]
Analytical analysis:
The energy change for packet in the time-dep trap II

The energy contributed from

1. Instantaneous energy for trap $\omega(t)$

\[
\left( \frac{\hat{H}(0)}{b^2} - \frac{m}{2} b\ddot{x}^2 \right)
\]

2. The kinetic energy

\[
\frac{m}{2} b^2 \dot{x}^2
\]

3. The quantum friction

\[
\frac{i}{2b} \{ \hat{x}, \hat{p} \} \propto [\hat{H}(0), \hat{H}(t)]
\]


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3. The quantum friction

$$\frac{b}{2b} \{ \hat{x}, \hat{p} \} \propto [\hat{H}(0), \hat{H}(t)]$$

The energy depend on the packet follows or resists to the trap motion!!
Analytical analysis:
The energy change for packet in the time-dep trap III

\[ \langle \hat{H}(t) \rangle = \hbar \left( \frac{\omega_0}{b^2} + \frac{1}{2\omega_0} (\dot{b}^2 - b\ddot{b}) \right) \]
\[ \times \left[ \frac{1}{2} + \langle \psi_0 | \Lambda^+ \Lambda | \psi_0 \rangle \right] \]
\[ + \hbar \text{Re} \left[ \left( \frac{1}{2\omega_0} (\dot{b}^2 - b\ddot{b}) - \frac{i}{b} \dot{b} \right) \right. \]
\[ \times \left. e^{-i2 \int_0^t dt' \omega_0 / b^2} \langle \psi_0 | \Lambda \Lambda | \psi_0 \rangle \right], \]

with \[ \Lambda = a - \frac{i\dot{b}(0)}{2\omega_0} (a^+ + a) \]
Analytical analysis:
The energy change for packet in the time-dep trap III

\[ \langle \hat{H}(t) \rangle = \hbar \left( \frac{\omega_0}{b^2} + \frac{1}{2\omega_0} (\dot{b}^2 - \ddot{b}) \right) \times \left[ \frac{1}{2} + \langle \psi_0 | \Lambda^+ \Lambda | \psi_0 \rangle \right] \]

\[ + \hbar \text{Re} \left[ \left( \frac{1}{2\omega_0} (\dot{b}^2 - \ddot{b}) - i \frac{\dot{b}}{b} \right) \times e^{-i 2 \int_0^t dt' \omega_0 / b^2} \langle \psi_0 | \Lambda \Lambda | \psi_0 \rangle \right], \]

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The energy change for packet in the time-dep trap III


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\[ + \hbar \text{Re} \left[ \left( \frac{1}{2\omega_0} (\dot{b}^2 - b\ddot{b}) - i \frac{\dot{b}}{b} \right) \right. \]
\[ \times e^{-i2 \int_0^t dt' \omega_0 / b^2} \langle \psi_0 | \Lambda \Lambda | \psi_0 \rangle \left. \right] , \]

Quantum breathing term

with

\[ \Lambda = a - i \frac{\dot{b}(0)}{2\omega_0} (a^+ + a) \]
Quantum breathing during the contraction


The period is determined by

\[
\int_0^T dt' \frac{\omega_0}{b^2(t)} = \pi,
\]

\[
T = \frac{b\pi}{\omega_0}
\]

Figure 1: (color online). (a) The average energies of the wave packet for different contraction processes: linear contraction for $\dot{b} = -\omega_0/4$ and $\dot{b} = 0$ (solid line), adiabatic process for $E(b) = E_0/b^2$ (dashed line). (b) The solid line is the difference between linear contraction and adiabatic energies. The dashed line is the expectation value for $E_0\{\dot{x}, \dot{p}\}/\hbar$ and the gray area denote the region of wave-packet diffusion.
Corresponding effect in classical field

The harmonic trap can be constructed by electromagnetic field

\[ \vec{A} = \frac{1}{2} B(-y, x, 0), \]
\[ \hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \frac{e^2 B^2}{8mc^2}(x^2 + y^2) + \hat{H}_z \]

We chose

\[ \frac{eB}{c} = 2m\omega = \frac{4}{L^2(t)}, \]
\[ \vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}, \]

The electron in the classical field

Figure 2: (color online). The electron with initial conditions \( x = 0, y = 0, v_x = \omega_0 L_0/2, \) and \( v_y = 0 \) in the EM fields. (a) The moving trajectory with the lengthscale \( L_0 = (2\hbar/m\omega_0)^{1/2}. \) (b) The energy change.
Why the packet dancing in the time-dependent harmonic trap

- The oscillation appear as the projecting from the initial state to the eigenvectors.
- The harmonic trap decide the breathing frequency induced by the quantum friction.
- The quantum breathing modify the energy from the friction term in the trap.

Conclusion I

• The general solution which include sudden and adiabatic approximation is found.

• The quantum breathing will modify the energy in the time-dep trap.
Part II

Frictionless Cooling in the time-dependent harmonic trap
Fast Optimal Cooling between two states

Frictionless Cooling in the trap
Our Strategy

The energy change equivalent to the adiabatic result:

\[ E_f = \frac{\omega_f}{\omega_0} E_0 \]
Frictionless Cooling in the trap
Our Strategy II


Figure 4: (color online). Cooling in $t_f = 2\,\text{ms}$. (a) Examples of ansatz for $b$. The simple polynomial ansatz for our result (dashed line) and for Ref. [1] (dotted line). The exponentials of a polynomial for our result (solid line) and Ref. [1] (dash-dotted line). (b) The corresponding squared frequency $\omega^2(t)$. $\omega(0) = 250 \times 2\pi$ Hz, $\gamma = 10$.

Besides the original condition:

$$b(0) = 1, \quad \ddot{b}(0) = 0.$$ 

We add conditions

$$\dot{b}(0) = 0, \quad \dot{b}(t_f) = 0$$

to avoid Quantum Breathing & Kinetic Energy !!