Constraints on Timeon Model

Takeshi Araki & Chao-Qiang Geng
National Tsing-Hua University
Outline

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Introduction

The standard model (SM) of particle physics is a very successful theory, but there are some unsolved problems and questions:

- There are a lot of free parameters in the theory. (ex). fermion masses, gauge couplings.
- The SM does not have a dark matter candidate.
- What is the origin of neutrino masses?
- etc...
- What is the origin of CP violation (CPV)?

CP violation is one of the most important ingredients of particle physics and was observed in 1964, $K_L^0 \rightarrow \pi \pi$, but its origin remains still unclear.

Moreover, it is unknown whether CP violation also occurs in the lepton sector or not.
Introduction

In the SM, CP violation appears in the CKM matrix included in the charge-changing quark current with the gauge bosons

\[
\frac{g}{\sqrt{2}} \bar{u}_L i \vec{W} (V_{CKM})_{ij} d_L j
\]

and originates from the complex Yukawa couplings, \( Y^* \neq Y \).

\[
V_{CKM} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{13}c_{23} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13}
\end{pmatrix},
\]

Even if \( Y^* = Y \), we can break CP symmetry by introducing extra Higgs doublets and by assuming that they get complex VEVs, \(< \phi_i >= |v_i|e^{i\theta_i} \).

Spontaneous CP violation (SCPV)
What is the "timeon"?

Theory of Timeon [Friedberg and Lee, arXiv:0809.3633]

Friedberg and Lee proposed a new spontaneous CP violation mechanism. They introduced a new gauge singlet pseudo-scalar $\tau(x)$ and new Yukawa type interactions, $\bar{q}\tau q$

$$\mathcal{H} = \bar{u}_i [M_u + i\gamma_5\tau_0 F]_{ij} u_j + \bar{d}_i [M_d + i\gamma_5\tau_0 F]_{ij} d_j.$$  

They assumed $M_q$ and $F$ are real, and $\det M_q = 0$.

- CP and T symmetries are spontaneously broken after the pseudo-scalar (timeon) gets the VEV, 
- the pseudo-scalar is also responsible for the up and down quark masses.

$\phi(x)Y_{ij} \rightarrow vY_{ij} = M_{ij}$

$F$ is a $3 \times 3$ real matrix and $\tau_0$ is the VEV of a new gauge singlet pseudo-scalar, $\tau(x).$

$$\bar{q}[i\gamma_5\tau(x)F]q \rightarrow \bar{q}[i\gamma_5\tau_0 F]q$$
What is the “timeon”?

Theory of Timeon [Friedberg and Lee, arXiv: 0809.3633]

Friedberg and Lee proposed the following quark mass matrix

\[ \mathcal{H} = \bar{u}_L [M_u + i \tau_0 F] u_R + \bar{d}_L [M_d + i \tau_0 F] d_R + h.c. \]

\[ M_q = \begin{pmatrix} 
  b_q \eta_q^2 (1 + \xi_q^2) & -b_q \eta_q & -b_q \xi_q \eta_q \\
  -b_q \eta_q & b_q + a_q \xi_q^2 & -a_q \xi_q \\
  -b_q \xi_q \eta_q & -a_q \xi_q & a_q + b_q 
\end{pmatrix} \quad q = u, d \]

where \( a_q, b_q, \eta_q, \xi_q \) are assumed to be real, and the mass matrix is invariant under the translational family symmetry:

\[ q_1 \rightarrow q_1 + z, \quad q_2 \rightarrow q_2 + \eta \; z, \quad q_3 \rightarrow q_3 + \eta \xi \; z. \]

\( F \) is also a \( 3 \times 3 \) matrix described by two angles and given by

\[ F = \begin{pmatrix} 
  \cos^2 \alpha & \sin \alpha \cos \alpha \cos \beta & \sin \alpha \cos \alpha \sin \beta \\
  \sin \alpha \cos \alpha \cos \beta & \sin^2 \alpha \cos^2 \beta & \sin^2 \alpha \sin \beta \cos \beta \\
  \sin \alpha \cos \alpha \sin \beta & \sin^2 \alpha \sin \beta \cos \beta & \sin^2 \alpha \sin^2 \beta 
\end{pmatrix}. \]

Since \( \det M_q = 0 \), \( m_u \) and \( m_d \) may be proportional to the VEV, \( \tau_0 \).

\( \tau_0 \) may be much smaller that the EW scale
What is the “timeon”?

Theory of Timeon [Friedberg and Lee, arXiv: 0809.3633]

Let us estimate the magnitude of the timeon VEV, $\tau_0$.

- CKM matrix
  
  $M_q$ can be diagonalized by the following matrix:

  $\left(\begin{array}{ccc}
  \cos \theta_q & -\sin \theta_q & 0 \\
  \sin \theta_q \cos \phi_q & \cos \theta_q \cos \phi_q & -\sin \phi_q \\
  \sin \theta_q \sin \phi_q & \cos \theta_q \sin \phi_q & \cos \phi_q
  \end{array}\right)$.  

  $\eta_q = \tan \theta_q \cos \phi_q$

  $\xi_q = \tan \phi_q$

[(Ansatz)]

$\theta_u = \theta_c \simeq 13.1^\circ$

$\phi_u = \pi + \epsilon$

$\theta_d = 0$

$\phi_d = \pi - \gamma$

$U_u \simeq \left(\begin{array}{ccc}
  \cos \theta_c & -\sin \theta_c & 0 \\
  -\sin \theta_c & -\cos \theta_c & \epsilon \\
  -\epsilon \sin \theta_c & -\epsilon \cos \theta_c & -1
  \end{array}\right)$,  

$U_d = \left(\begin{array}{ccc}
  1 & 0 & 0 \\
  0 & -\cos \gamma & -\sin \gamma \\
  0 & \sin \gamma & -\cos \gamma
  \end{array}\right)$.

$V_{CKM} = U_u^\dagger U_d + \delta U(\tau_0)$

We ignore $\delta U(\tau_0)$ term here, then $\epsilon$ and $\gamma$ can be determined.

$\epsilon \simeq 1.03^\circ$

$\gamma \simeq 1.26^\circ$

$\eta_u \simeq -0.233$  $\xi_u \simeq 0.018$

$\eta_d = 0$  $\xi_d \simeq -0.022$
What is the “timeon”?

**Theory of Timeon** [Friedberg and Lee, *arXiv: 0809.3633*]

Let us estimate the magnitude of the timeon VEV, \( \tau_0 \).

- up- and down-quark masses
  
  \( M_q \) can be diagonalized by \( U_q \) and leads to the following eigenvalues
  
  \[ \lambda_q(1) = 0, \quad \lambda_q(2) = b_q [1 + \eta_q^2 (1 + \xi_q^2)], \quad \lambda_q(3) = a_q (1 + \xi_q^2) + b_q. \]

  Quark masses are given by \( m_q(i) = \lambda_q(i) + \delta \Lambda_q^i (\tau_0) \). Here we assume
  
  \[ m_q(2) = \lambda_q(2), \quad m_q(3) = \lambda_q(3). \]

  On the other hand
  
  \[ \det [(M_q + i \tau_0 F)(M_q + i \tau_0 F)^\dagger] = m_q^2(1) \ m_q^2(2) \ m_q^2(3). \]

  Therefore the light quark masses are given as
  
  \[ m_u(1) = \tau_0 \sin^2 \alpha \cos^2 (\beta + \theta_c) + O(\tau_0^2), \]

  \[ m_d(1) = \tau_0 \sin^2 \alpha \cos^2 \beta + O(\tau_0^2). \]

  Given \( m_u(1)/m_d(1) \approx 0.5 \) and \( \theta_c \approx 13.1^\circ \)

  \[ \beta \approx 48^\circ. \]
What is the “timeon”?

Theory of Timeon [Friedberg and Lee, arXiv:0809.3633]

Let us estimate the magnitude of the timeon VEV, $\tau_0$.

- **Jarlskog invariant** [Jarlskog, PRD35 (1978)]

$$ J = \text{Im}[V_{11}V_{22}V_{12}^*V_{21}^*] $$

$$ \simeq \frac{m_d(1)}{\sin^2\alpha \cos^2\beta} \left[ \frac{\sin \alpha \cos(\beta + \theta_c)}{m_d(2)} A + \frac{\cos \alpha \sin \gamma - \sin \alpha \sin \beta \cos \gamma}{m_d(3)} B ight. $$

$$ \left. + \frac{-\cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma}{m_d(2) + m_d(3)} C \right] $$

- $A \simeq -2 \times 10^{-4}$, $B \simeq 8.8 \times 10^{-3}$, $C \simeq 1.1 \times 10^{-3}$,
  
  $m_d(2) \simeq 95$ MeV, $m_d(b) \simeq 1.25$ GeV,

- $\beta \simeq 48^\circ$, $\gamma \simeq 1.26^\circ$, $\theta_c \simeq 13.1^\circ$,

- $J^{\text{exp}} \simeq 3.08 \times 10^{-5}$.

$$ \alpha \simeq -36^\circ \quad \Rightarrow \quad \tau_0 \simeq 33 \text{ MeV} \quad \left( m_d(1) \simeq \tau_0 \sin^2 \alpha \cos^2 \beta \right) $$
What is the “timeon”?

Theory of Timeon [Friedberg and Lee, arXiv:0809.3633]

The potential of the timeon field is given by

\[ V(\tau) = -\frac{1}{2} \lambda \tau^2 \left( \tau_0^2 - \frac{1}{2} \tau^2 \right). \]

Expanding \( V(\tau) \) around \( \tau = \tau_0 \), we have

\[ V(\tau) \sim -\frac{1}{4} \lambda \tau_0^4 + \frac{1}{2} M_T (\tau - \tau_0)^2 + \cdots, \]

where \( M_T = \sqrt{2\lambda} \tau_0 \) is the mass of a new quantum, timeon.

Since the timeon is responsible for the light quark masses, its VEV cannot be much larger than electroweak scale.

\[ m_d(1) \simeq \tau_0 \sin^2 \alpha \cos^2 \beta = 3.5 \sim 6.0 \text{ MeV} \]

For instance they estimated that \( \tau_0 \simeq 33 \text{ MeV} \), which implies the lower bound of the timeon mass

\[ M_T < 47 \text{ MeV} \quad (\text{for } \lambda < 1). \]
Summary-1

- Friedberg and Lee introduced a new gauge singlet pseudo-scalar field and named it timeon.
- Once the timeon gets the VEV, CP and T parities are spontaneously broken.
- The timeon is also responsible for the up and down quark masses.
- The mass of the timeon would be much lower than the EW scale such as $M_T < 47 \text{ MeV}$ (for $\lambda < 1$).
FCNCs

Problem

A light timeon could cause dangerous FCNC processes!!

In general, $M_q$ and $F$ cannot be diagonalized simultaneously.

$$\bar{q}_L [M_q + i \tau_0 F] q_R \rightarrow \bar{q}_L^m U^\dagger_q [M_q + i \tau_0 F] U_q^* q_R^m$$

$$\equiv \bar{q}_L^m [M_q' + i \tau_0 F^q] q_R^m$$

$$\equiv \text{diag}(m_q(1), m_q(2), m_q(3))$$

Each $M_q'$ and $F^q$ are not diagonal in general.

In fact, in the mass eigenstate basis, off diagonal elements are given by

$$F^u \equiv U^T_u F U_u = \begin{pmatrix} 0.08 & -0.14 & 0.23 \\ -0.14 & 0.25 & -0.41 \\ 0.23 & -0.41 & 0.67 \end{pmatrix}$$

$$F^d \equiv U^T_d F U_d = \begin{pmatrix} 0.15 & -0.18 & 0.31 \\ -0.18 & 0.21 & -0.36 \\ 0.31 & -0.36 & 0.64 \end{pmatrix}$$

Note

$\tau_0 = 33 \text{ MeV}$ corresponds to

$\eta_u \simeq -0.233$, $\xi_u \simeq 0.018$,

$a_d \simeq 173.76$, $b_u \simeq 1.186$,

$\eta_d = 0$, $\xi_u \simeq -0.022$,

$a_d \simeq 4.103$, $b_d \simeq 0.095$,

$\alpha \simeq -36^\circ$, $\beta \simeq 48^\circ$. 
FCNCs

The off diagonal elements of $F^q$ causes flavor changing timeon interactions such as

$$i F_{ij}^u u_L^m t u_R^m, \quad i F_{ij}^d d_L^m t d_R^m. \quad (i \neq j)$$

For example, $K^0 - \bar{K}^0$ mixing process occurs at tree level.

However, FCNC processes are strongly constrained by experiments.
**FCNCs**

For instance, the contribution from the timeon to the mass mixing parameter in the neutral K meson system can be estimated as

\[
\Delta M_{K}^{timeon} = 2 \frac{2F_{ds}(F_{sd}^d)^*}{M_T^2} < K^0 |\bar{d}_L s_R \bar{d}_R s_L^\beta | \bar{K}^0 > \\
+ \frac{(F_{ds})^2 + (F_{sd})^2}{M_T^2} < K^0 |\bar{d}_L^\alpha s_R^\alpha \bar{d}^\beta L s_R^\beta | \bar{K}^0 > \\
\simeq \frac{1.76 \times 10^{-3} \text{ GeV}^3}{M_T^2}.
\]

From \( \Delta M_K^{exp} \simeq 3.5 \times 10^{-15} \text{ GeV} \), we find the lower limit of \( M_T \) as

\[
M_T \geq 7 \times 10^5 \text{ GeV}.
\]

This bound conflicts with the previous result \( M_T < 47 \text{ MeV} \) (for \( \lambda < 1 \)).

We do not consider the strongly interacting coupling, i.e., \( \lambda \gg 1 \).

\( ( M_T = \sqrt{2\lambda} \tau_0 ) \)
**FCNCs**

In order to avoid the problem, we introduce a small dimensionless parameter $\epsilon$ for the timeon term

$$\bar{q}[i\gamma_5\tau_0 F]q \rightarrow \bar{q}[i\gamma_5 \epsilon \tau_0 F]q.$$ 

### FCNC

- $M_T \geq 7 \times 10^5$ GeV
- $\frac{M_T}{\epsilon} \geq 7 \times 10^5$ GeV

### Light quarks

- $\tau_0 \simeq 33$ MeV since $m_u, m_d \propto \tau_0$.
- $\epsilon \tau_0 \simeq 33$ MeV

If $\epsilon$ is very small, both the constraints from FCNC and light quark masses can be relaxed.

Here, we assume $\tau_0 \sim M_T \ (\lambda \sim 0.5)$, then we obtain

$$M_T > 151 \text{ GeV}, \quad \epsilon < 0.22 \times 10^{-3}. \quad (M_T = \sqrt{2\lambda} \tau_0)$$
FCNCs

By assuming the lowest value of $M_T$: $M_T = 151$ GeV and $\epsilon = 0.22 \times 10^{-3}$, we can also compute the mass parameters in $B_s$, $B_d$ and $D$ systems.

\[
\Delta M_{B_s}^{\text{timeon}} \sim 0.365 \times 10^{-13} \text{ GeV} \quad < \Delta M_{B_s}^{\text{exp}} \simeq 1.17 \times 10^{-11} \text{ GeV}
\]

\[
\Delta M_{B_d}^{\text{timeon}} \sim 0.178 \times 10^{-13} \text{ GeV} \quad < \Delta M_{B_d}^{\text{exp}} \simeq 3.34 \times 10^{-13} \text{ GeV}
\]

\[
\Delta M_{D}^{\text{timeon}} \sim 0.205 \times 10^{-14} \text{ GeV} \quad < \Delta M_{D}^{\text{exp}} \simeq 1.40 \times 10^{-14} \text{ GeV}
\]

All the FCNC processes are sufficiently suppressed.

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<td>$f_K$</td>
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<tr>
<td>$m_d$</td>
<td>$5 \times 10^{-3}$ GeV</td>
<td>$f_{B_s}$</td>
<td>0.24 GeV</td>
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<td>$M_D$</td>
<td>1.865 GeV</td>
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Summary-2

- The original timeon model suggests a light timeon, but it conflicts with the constraints from FCNCs.
- In order to avoid the problem, we introduce a small dimensionless parameter $\epsilon$.
- We calculate the lower bound of the timeon mass with the small parameter, and find that
  \[ M_T > 151 \text{ GeV}, \quad \epsilon < 0.22 \times 10^{-3}, \]
  form the neutral K meson system.
- We also find that the lowest value, $M_T = 151$ GeV, is large enough to satisfy the constraints form other neutral meson systems.
**Lepton sector**

**Extension to the lepton sector**

We extend the timeon model to the lepton sector

\[
\mathcal{H}_l = \bar{\ell}_i [M_\ell + i\gamma_5 \epsilon \tau_0 F_l]_{ij} \ell_j + \bar{\nu}_i^{(c)} [M_\nu + i\gamma_5 \epsilon_\nu \tau_0 F_l]_{ij} \nu_j.
\]

We put the same parameter as that of the quark sector.

\[
\epsilon \simeq 0.22 \times 10^{-3}
\]

( \( \tau_0 = 151 \text{ GeV} \) )

**Simple model**

The matrix \( F_l \) takes the same form as that of the quark sector

\[
F_l = \begin{pmatrix}
\cos^2 \alpha_l & \sin \alpha_l \cos \alpha_l \cos \beta_l & \sin \alpha_l \cos \alpha_l \sin \beta_l \\
\sin \alpha_l \cos \alpha_l \cos \beta_l & \sin^2 \alpha_l \cos^2 \beta_l & \sin^2 \alpha_l \sin \beta_l \cos \beta_l \\
\sin \alpha_l \cos \alpha_l \sin \beta_l & \sin^2 \alpha_l \sin \beta_l \cos \beta_l & \sin^2 \alpha_l \sin^2 \beta_l
\end{pmatrix},
\]

but described by different angles: \( \alpha_l \) and \( \beta_l \)
Lepton sector

Simple model

\[ \mathcal{H}_l = \bar{\ell}_i [M_\ell + i \gamma_5 \epsilon \tau_0 F_l]_{ij} \ell_j + \bar{\nu}^{(c)}_i [M_\nu + i \gamma_5 \epsilon \tau_0 F_l]_{ij} \nu_j. \]

We assume the following simple mass matrices.

\[
\begin{pmatrix}
    b_i \eta_i^2 (1 + \xi_i^2) & -b_i \eta_i & -b_i \xi_i \eta_i \\
    -b_i \eta_i & b_i + a_i \xi_i^2 & -a_i \xi_i \\
    -b_i \xi_i \eta_i & -a_i \xi_i & a_i + b_i \\
\end{pmatrix}_l = \ell, \nu
\]

\[
\begin{pmatrix}
    \eta_\ell = 0 \\
    \xi_\ell = -1 \\
\end{pmatrix}
\]

\[
M_\ell = \begin{pmatrix}
    0 & 0 & 0 \\
    0 & b_\ell + a_\ell & a_\ell \\
    0 & a_\ell & a_\ell + b_\ell \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \eta_\nu = -\sqrt{1/2} \\
    \xi_\nu = 0 \\
\end{pmatrix}
\]

\[
M_\nu = \begin{pmatrix}
    \frac{1}{2} b_\nu & \sqrt{\frac{1}{2}} b_\nu & 0 \\
    \sqrt{\frac{1}{2}} b_\nu & b_\nu & 0 \\
    0 & 0 & a_\nu + b_\nu \\
\end{pmatrix}
\]

These matrices lead to the exact tri-bi-maximal (TBM) mixing.

\[
V_{TB} = \frac{1}{\sqrt{6}} \begin{pmatrix}
    2 & \sqrt{2} & 0 \\
    -1 & \sqrt{2} & -\sqrt{3} \\
    -1 & \sqrt{2} & \sqrt{3} \\
\end{pmatrix}
\]

\[
\sin^2 \theta_{23} = 1/2 \\
\sin^2 \theta_{12} = 1/3 \\
\sin^2 \theta_{13} = 0
\]

After the timeon gets the VEV, the mixing matrix deviates form TBM.
Lepton sector

Comments

- After the timeon gets the VEV, the mixing matrix deviates from the exact TBM pattern.

- $\alpha_l = 0$

  $F_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

  No flavor changing timeon couplings in the charged lepton sector and
  $\sin \theta_{13} = 0$.

- $\alpha_l \neq 0$

  Flavor changing timeon couplings in the charged lepton sector
  are induced, and it leads to $\sin \theta_{13} \neq 0$. 
**Lepton sector**

**Parameter space**

The model has seven controllable parameters:

\[ a_\ell, \ b_\ell, \ a_\nu, \ b_\nu, \ \alpha_\ell, \ \beta_\ell, \ \epsilon_\nu, \]

which can be fixed by seven physical quantities:

\[
m_e = 0.511 \text{ MeV}, \quad m_\mu = 105.658 \text{ MeV}, \quad m_\tau = 1776.84 \pm 0.17 \text{ MeV},
\]

\[
\sin^2 \theta_{12} = 0.288 \sim 0.326, \quad \sin^2 \theta_{23} = 0.44 \sim 0.57,
\]

\[
\Delta m_{21}^2 = (7.45 - 7.88) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = (2.29 - 2.52) \times 10^{-3} \text{ eV}^2.
\]

From the right figure we find that

\[ \sin \theta_{13} \text{ must be small but } \sin \theta_{13} \neq 0. \]

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**Non-zero flavor changing timeon couplings**
Lepton sector

Leptonic processes

Now that all the parameters included in the model are determined we can calculate some leptonic processes.

\[ \mu \rightarrow e + \gamma \]  \hspace{1cm} \[ \ell^- \rightarrow \ell^- + \ell^+ + \ell^- \]

\[
Br(\mu \rightarrow e\gamma) = \frac{\alpha_{em} \tau_\mu}{2^{10}\pi^4} \frac{m_\mu^3 m_\tau^2}{M_T^4} \left( \epsilon F_{\mu\tau} \epsilon F_{e\tau}^\ell \right)^2 \ln \left| \frac{m_\tau^2}{M_T^2} + \frac{3}{2} \right|^2
\]

\[
Br(\ell^- \rightarrow \ell^- \ell^+ \ell^-) = \frac{5}{3} \frac{\tau_\ell}{2^{11}\pi^3} \frac{m_\ell^5}{M_T^4} \left( \epsilon F_{\ell\ell}^\ell \epsilon F_{\ell\ell}^\ell \right)^2
\]
Lepton sector

Leptonic processes

Now that, all the parameters included in the model are determined we can calculate some leptonic processes.

Lepton Flavor Violating decays

\[
\begin{align*}
Br(\mu \to e\gamma) &\simeq 10^{-16} \sim 10^{-21} & Br^{\exp}(\mu \to e\gamma) &< 1.2 \times 10^{-11} \\
Br(\mu^- \to e^-e^+e^-) &\simeq 10^{-21} \sim 10^{-26} & Br^{\exp}(\mu^- \to e^-e^+e^-) &< 1.0 \times 10^{-12} \\
Br(\tau^- \to e^-e^+e^-) &\simeq 3 \times 10^{-21} & Br^{\exp}(\tau^- \to e^-e^+e^-) &< 3.6 \times 10^{-8} \\
Br(\tau^- \to \mu^-\mu^+\mu^-) &\simeq 10^{-19} \sim 10^{-35} & Br^{\exp}(\tau^- \to \mu^-\mu^+\mu^-) &< 3.2 \times 10^{-8}
\end{align*}
\]

Muon Anomalous Magnetic Moment

\[
\Delta a_\mu \simeq 10^{-16} - 10^{-21} \\
a^{exp}_\mu - a^{SM}_\mu \simeq 10^{-11}
\]

All results are consistent with the experimental data, but the contributions from the timeon are too small to be measured in the near future.
Summary-3

- We extend the timeon model to the lepton sector, and use the same small parameter $\epsilon$ for the charged lepton sector, whereas a new parameter $\epsilon_\nu$ for the neutrino sector.
- We propose a simple model which is consistent with the neutrino oscillation data.
- The model predicts non-zero $\sin \theta_{13}$ and flavor changing timeon couplings.
- Unfortunately, all contributions to the leptonic processes are not measurable in the near future.
Summary

• We have investigated the timeon model.
• The original timeon model predicts the small timeon mass, but it conflicts with the constraints of FCNCs.
• We have introduced a small parameter $\epsilon$ and shown that the parameter can relax the constraints.
• We have found that $M_T > 151$ GeV or $\epsilon < 0.22 \times 10^{-3}$.
• We have also extended the timeon model to the lepton sector and found that our model predicts non-zero flavor changing timeon couplings.
• Unfortunately, all contributions to the leptonic processes are not measurable in the near future.
fin
**Origin of $\epsilon$**

Since the timeon is a gauge singlet scalar, it cannot be incorporated into the (renormalizable) SM.

\[ F_{ij} \bar{\psi}_L \tau \psi_R j \text{ is not invariant under the } SU(2)_L \times U(1)_Y. \]

That is, the timeon model is an effective theory after EWSB. If we consider a non-renormalizable interaction such as

\[ \frac{F_{ij}}{\Lambda} \bar{\psi}_L \tau \psi_R j \Phi, \quad \Phi : SU(2)_L \text{ doublet scalar} \]

\[ \Lambda : \text{typical energy scale} \]

the timeon term appears after $\Phi$ gets the VEV, $< \Phi >$. In this case, $< \Phi > / \Lambda$ might be able to explain the existence of $\epsilon$.

\[ \frac{F_{ij}}{\Lambda} \bar{\psi}_L \tau \psi_R j \Phi \rightarrow \frac{< \Phi >}{\Lambda} F_{ij} \bar{\psi}_L \tau \psi_R j \equiv \epsilon F_{ij} \bar{\psi}_L \tau \psi_R j ? \]