Unified dark fluid with effect on dark radiation

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7th Joint NCTS/FGCPA-LeCosPA Meeting on Dark Energy
Outline

- The dark radiation at CMB and BBN epochs
- Interacting models
- Dark radiation in interacting models
- Conclusions
What is dark radiation?

The constituent of our universe include **vacuum energy, matter, and radiation**

The relativistic particle ($T >> m$) behaves as radiation
What is dark radiation?

The constituent of our universe include vacuum energy, matter, and radiation

The relativistic particle (T >> m) behaves as radiation

Cosmic Microwave Background (CMB)

\[ T \approx eV, \quad z \approx 1100 \]

Big Bang Nucleosynthesis (BBN)

\[ T \approx 1 \text{ MeV}, \quad z \approx 10^9 \]

In standard model (SM), only photon and 3 neutrinos (m_\nu < eV) are belong to radiation components in this two epoches

\[
\rho_{\text{rad}} = \rho_\gamma + N_{\text{eff}} \rho_\nu = \rho_\gamma \left(1 + \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11}\right)^{\frac{4}{3}}\right)
\]

\[ N_{\text{eff}} = 3.046 \text{ for SM 3 neutrinos} \]

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At Big Bang Nucleosynthesis epoch (BBN) \( T \approx 1 \text{ MeV}, \ z \approx 10^9 \)

\( m_n - m_p = 1.3 \text{ MeV} \), the rate of neutron-proton conversion was freeze-out

\[ n + \nu \leftrightarrow p + e^- \ , \ n + e^+ \leftrightarrow p + \bar{\nu} \]

\[ f_{n-p} = \frac{n_n}{n_p} = \exp\left(\frac{\Delta m}{T}\right) \approx 0.15 \]

light nuclei start to be produced

Most of the neutrons exist in \(^4\text{He}\), so the primordial He mass fraction is

\[ Y_P = \frac{4n_{\text{He}}}{n_H + 4n_{\text{He}}} = \frac{2(n_n/n_p)}{1 + (n_n/n_p)} \approx 0.25 \]

\[ Y_P = 0.254 \pm 0.003 \quad \text{Izotov, Stasinska Guseva (2013)} \]
The primordial helium fraction $Y_P$ and baryon number density are sensitive to $H$

$$Y_P = \left(0.2469 \pm 0.0006\right) + 0.0016 \left[\eta_{10} + 100(S - 1) - 6\right]$$  \hspace{1cm} \text{Steigman (2012)}

$$\eta = \eta_{10} \times 10^{-10} = \frac{n_b}{n_\gamma}; \quad S = H/H_{\text{SM}}, \quad H^2 = \frac{\rho_{\text{tot}}}{3M_P^2}$$

$$\rho_{\text{tot}} = \rho_{\text{rad}} = \rho_\gamma \left(1 + \frac{7}{8} N_{\text{eff}}^{\text{BBN}} \left(\frac{4}{11}\right)^\frac{4}{3}\right)$$

From the primordial helium mass fraction $Y_P$ measurement

D and $^4\text{He}$

$$N_{\text{eff}}^{\text{SM}} = 3.046; \Delta N_{\text{eff}}^{\text{BBN}} = N_{\text{eff}}^{\text{BBN}} - N_{\text{eff}}^{\text{SM}}$$  \hspace{1cm} \text{the d.o.f. of dark radiation at BBN epoch}

$$N_{\text{eff}} = 3.50 \pm 0.20$$  \hspace{1cm} \text{Cooke, Pettini, Jorgenson, Murphy, Steidel (2013)}

$$N_{\text{eff}}^{\text{BBN}} = 3.71^{+0.47}_{-0.45}$$  \hspace{1cm} \text{Steigman (2012)}
At Cosmic Microwave Background (CMB)

$N_{\text{eff}}$ can be derived from CMB measurement for radiation-matter equality epoch $z_{\text{eq}}$

$$\frac{1}{1 + z_{\text{eq}}} = a_{\text{eq}} = \frac{\Omega_{\text{rad}}^0}{\Omega_m^0}$$

$$\rho_{\text{rad}} = \left[ 1 + N_{\text{eff}}^{\text{CMB}} \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \right] \rho_\gamma$$

$N_{\text{eff}}^{\text{CMB}} = 3.84 \pm 0.40 \ (\text{WMAP9+BAO}+\text{H0})$

$N_{\text{eff}}^{\text{CMB}} = 3.36 \pm 0.34 \ (95\% \text{ C.L. Planck+WMAP})$

$N_{\text{eff}}^{\text{CMB}} = 3.30^{+0.51}_{-0.54} \ (95\% \text{ C.L. Planck+WMAP +BAO})$

$N_{\text{eff}}^{\text{CMB}} = 3.62 \pm 0.25 \ (\text{PLANCK+WMAP+H0})$

$$\Delta N_{\text{eff}}^{\text{CMB}} = N_{\text{eff}}^{\text{CMB}} - N_{\text{eff}}^{\text{SM}}: \text{dark radiation at CMB epoch}$$

$\Delta N_{\text{eff}}$ may have different values in BBN and CMB era

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Dark Radiation

$\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$: The number of relativistic degree of freedom beyond the three neutrinos with mass and mixing predicted in SM

$$N_{\text{eff}}^{\text{SM}} = 3.046$$

$$\rho_{\text{rad}} = \rho_\gamma + N_{\text{eff}} \rho_\nu = \rho_\gamma \left( 1 + \frac{7}{8} N_{\text{eff}} \left( \frac{4}{11} \right)^{\frac{4}{3}} \right)$$

$\Delta N_{\text{eff}}$ is enough for describing the background, while in perturbation in FRW space,

$$w \left( \delta P - \frac{\dot{P}}{\dot{\rho}} \delta P \right) = \left( c_{\text{eff}}^2 - \frac{\dot{P}}{\dot{\rho}} \right) P \left( \frac{\delta \rho}{\rho} \right)_{\text{rest}}$$

$$w \left( \dot{\pi} + 3 \frac{\dot{a}}{a} \pi \right) = 4 c_{\text{vis}}^2 \left( k \nu - \dot{H} \right)$$

$\{\Delta N_{\text{eff}}, c_{\text{vis}}, c_{\text{eff}}\}$: for sterile neutrino $c_{\text{vis}} = c_{\text{eff}} = 1/3$. The mass of DR is zero

Wayne Hu (1998)
Possible DR candidate

\( \Delta N_{\text{eff}} \) seems to exist but less than 1, and it might be departure from BBN to CMB epoch

A sterile neutrino is a simply extension to explain such an anomaly.

\[
N_{\text{eff}}^{\text{SM}} = 3.04, \text{coming from three } \nu_L 's
\]

\[
\nu_L + f \rightarrow l + f ' \text{ and } \nu_L + f \rightarrow \nu_L + f
\]

\( N_R \) can be the sterile neutrinos:

Helicity flip, proportional to \( m_\nu \nu_L \bar{N}_R \)

oscillation between \( \nu_L \) and \( N_R \) \( \text{Langacker (1989)} \)

The reaction rate proportional to

\[
\sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 t_c}{4E} \right)
\]
Particles with extra symmetry

To have light species $\chi$ at CMB epoch ($\sim$eV), additional symmetry is required to ensure $\chi$ with small mass

Nakayama, Takahashi, Yanagida (2011)

The time of decoupling of $\chi$ should be prior to BBN epoch (MeV)

Note that the conservation of entropy in comoving volume will dilute the energy density of $\chi$ during radiation dominant epoch

$$\rho_{\chi} g_{\ast \chi} = \rho_{\nu} g_{\ast \nu}, \quad g_{\ast \nu} = 43/4, \quad g_{\ast \chi} = 205/4 \text{ for } T_{\chi} > T_c$$

$T_c = 200\text{MeV}$ is the temperature related to QCD phase transition

The spin-1 gauge bosons can not thermalized before BBN,

$$\Gamma_{\gamma e \rightarrow \gamma' e} < H, \text{ for } m_\gamma < 0.1\text{eV}$$

spin-1/2 can be the candidate of DR

$$\Gamma_{\bar{e}e \rightarrow \bar{\psi} \psi} = H, \text{ for } \Lambda > 1\text{TeV}$$
Goldstone boson as Dark Radiation

Goldstone boson as the DR the d.o.f. is \((1/2)^* (8/7)=4/7\),

The decoupling condition for Goldstone boson with SM fermions

\[
g^2 m_f^2 (kT)^5 \frac{m_{pl}}{m_H^4 m_\phi^4} \approx 1
\]

Goldstone boson would miss the chances to be heated if it decoupled with SM particles too early.

The conservation of entropy per comoving volume during muon annihilation heat the neutrinos

\[
\Delta (Ta) = \left( \frac{57}{43} \right)^{1/3}
\]

Ex: Majoron as the DR

Chang, Ng, and Wu (2013)
The Planck hints that there might exist dark radiation, and we don't know if its amount are the same in BBN and in CMB epoch.

The dark radiation component should decouple from SM particle before the neutrino decoupling epoch, which cannot explain the *varying* $N_{\text{eff}}$ *with time*.

If such a new species is produced non-thermally, then the giving $\Delta N_{\text{eff}}$ is not of order one in general.

If the dark radiation not couple to SM particle, but with other dark sector, ex: dark matter or dark energy?
DR from heavy particle decaying

DR component could only couple to particles beyond SM

A thermalize heavy particle X decaying into light particle

\[
\frac{d \rho_X}{dt} + 3H \rho_X = -\frac{\rho_X}{\tau_X}
\]

\[
\frac{d \rho_{dr}}{dt} + 3H(1+1/3) \rho_{dr} = \frac{\rho_X}{\tau_X}
\]

\[
\rho_M = \rho_{M0} a^{-3} e^{-t/\tau_X}
\]

\[\tau_X\] is the life time of X, and assuming the hundred percent decaying into light particle.

\[
\Delta N_{\text{eff}} = \frac{\rho_X}{\rho_\nu}
\]

\[
\rho_{\text{DR}}(\tau_X) = N_{\text{eff}} \left( \frac{4}{11} \right)^{4/3} \rho_Y = \rho_X(\tau_X) = s(\tau_X) M_X Y_X
\]

\[\Gamma\] is the interacting rate between \(\rho_{\text{dm}}\) and \(\rho_{\text{dr}}\)

\[
\Delta N_{\text{eff}} \approx 7.1 \left( \frac{\tau_X}{\text{sec}} \right)^{1/2} \left( \frac{M_X Y_X}{1\,\text{MeV}} \right)
\]

\[\Delta N_{\text{eff}}\] vary with time

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Consider a fluid with the equation of state

\[ P = P_0 + \alpha \rho \]  

Quercellini, Bruni, Balbi, Pietrobon(2008)

Substituting into the energy conservation equation

\[ \frac{d\rho}{dt} + 3H(1 + \alpha)\rho = -3HP_0 \]

shift \( \rho \rightarrow \rho - 3P_0 \)

\[ \frac{d\rho_M}{dt} + 3H\rho_M(1 + \alpha) = 0 \]

\[ \frac{d\rho_\Lambda}{dt} = 0, \text{ with } \rho_\Lambda = -3P_0 \]

For the case \( \alpha = 0 \) The model will go back to \( \Lambda \) CDM

\[ c_{\text{eff}}^2 = \alpha, \alpha = (8 \pm 11) \times 10^{-4}, \Omega_\Lambda = 0.76 \pm 0.04 \]  

Pietrobon, Balbi, Bruni, and Quercellini(2008)
Generalize Chaplygin Gas (GCG)

Consider a fluid with the equation of state

\[ P = -A \rho^{-\alpha} \]

Substituting into the energy conservation equation

\[ \rho = \left( A + Ba^{-3(1+\alpha)} \right)^{1/(1+\alpha)} \]

The fluid can be decomposed into two fluids

\[
\frac{d\rho_M}{dt} + 3H \rho_M = -Q \\
\frac{d\rho_\Lambda}{dt} = Q,
\]

\[ Q = 3H \alpha \frac{\rho_M \rho_\Lambda}{\rho_M + \rho_\Lambda} \]

\[ \rho_{dm} = Ba^{-3(1+\alpha)} \left( Aa^{-3(1+\alpha)} + B \right)^{-\alpha/(1+\alpha)} \]

\[ \rho_{dr} = A \left( Aa^{-3(1+\alpha)} + B \right)^{-\alpha/(1+\alpha)} \]
DR from Unified Model

Chao-Qiang Geng, LHT and Xin-Zhang (arXiv: 1306.1910)

Explain the DM and DR by an unified fluid

\[ \rho_{\text{dark}} = (Aa^{-3(1+\alpha)} + Ba^{-4(1+\alpha)})^{1/(1+\alpha)} \]

which is a special case \((w_1=0 \text{ and } w_2=1/3)\) of “new generalize chaplygin gas“

Xin Zhang, Feng-Quan Wu, Jingfei Zhang(2006)

The equation of state \(w\) is the function of \(a\)

\[ w_{\text{dark}} = \frac{P_{\text{dark}}}{\rho_{\text{dark}}} = \frac{Aa^{-4(1+\alpha)}}{3(Aa^{-4(1+\alpha)} + Ba^{-3(1+\alpha)})} \]

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Decompose the unified fluids into

\[ \rho_{\text{dm}} = K^{1/(1+\alpha)} \left( 1 - \frac{A}{K} a^{-4(1+\alpha)} \right), \quad \rho_{\text{dr}} = K^{1/(1+\alpha)} \frac{A}{K} a^{-4(1+\alpha)} \]

\[ K = A a^{-3(1+\alpha)} + B a^{-4(1+\alpha)} \]

A, B can be determined by the initial condition

\[ Q = 3 \alpha H \frac{P_{\text{dark}}}{\rho_{\text{dark}}} \left( \rho_{\text{dark}} - 3 P_{\text{dark}} \right) = 3 \alpha H \frac{\rho_{\text{dm}} \rho_{\text{dr}}}{\rho_{\text{dm}} + \rho_{\text{dr}}} \]

In the limit \( \rho_{\text{dr}} \gg \rho_{\text{dm}} \), \( Q = 3 \alpha H \rho_{\text{dm}} \)

In the limit \( \rho_{\text{dm}} \gg \rho_{\text{dr}} \), \( Q = 3 \alpha H \rho_{\text{dr}} \) in radiation dominant era

\( H \sim T^2 \)
The extra radiation energy density is shown as

$$\Delta \rho_v = \Delta N_{\text{eff}} \frac{7}{8} \rho^0_\gamma \left( \frac{T_v}{T_\gamma} \right)^4 a^{-4} \quad \left( \frac{T_v}{T_\gamma} \right)^4 = \left( \frac{4}{11} \right)^{\frac{1}{3}}$$

$$\rho_{\text{dr}} + \rho_{\text{dm}} = \rho^0_{\text{dark}} \left( r a^{-4(1+\alpha)} + (1-r) a^{-3(1+\alpha)} \right)^{1/(1+\alpha)}$$

$$= \Delta N_{\text{eff}}(a) \frac{7}{8} \rho^0_\gamma \left( \frac{T_v}{T_\gamma} \right)^4 + \rho^0_{\text{dm}} f(a) a^{-3}$$

It leads to

$$\rho_{\text{dr}} = \Delta N_{\text{eff}}(a) \frac{7}{8} \rho^0_\gamma \left( \frac{T_v}{T_\gamma} \right)^4$$

Note that $r$ is arbitrary, we take is to be $r = 10^{-6} \sim 10^{-5}$ as illustration.
Negative value of $\Delta N_{\text{eff}}$ is possible if $r$ is negative.
Comparison of $H$ and effective equation of state with $\Lambda$CDM

![Graph comparing $H/H_{SC}$ and $w_{eff}$ with different values of $\alpha$.]
Effect on density perturbation

We can define the perturbation variables for the unified fluid

\[
\delta_{\text{dark}}' = -(1 + w_{\text{dark}}) \left( \theta_{\text{dark}} + \frac{h_s'}{2} \right) - 3 \frac{a'}{a} \left( c_{\text{dark}}^2 - w_{\text{dark}} \right) \delta_{\text{dark}}
\]

\[
\theta_{\text{dark}}' = -\frac{a'}{a} \left( 1 - 3 w_{\text{dark}} \right) \theta_{\text{dark}} - \frac{w_{\text{dark}}'}{1 + w_{\text{dark}}} \theta_{\text{dark}} + \frac{c_{\text{dark}}^2}{1 + w_{\text{dark}}} k^2 \delta_{\text{dark}} - k^2 \Theta_{\text{dark}}
\]

The effective sound speed is given by

\[
c_{\text{dark}}^2 = \frac{\delta P}{\delta \rho} = \frac{1}{3} \frac{w_{\text{dark}}}{1 + w_{\text{dark}}} \left[ 4 + \alpha \left( 1 - 3 w_{\text{dark}} \right) \right]
\]

\[
w_{\text{dark}}' = \frac{a'}{a} \left( 1 + \alpha \right) r \left( 1 - r \right) a^{-4(1+\alpha)} \left[ -4 a^{-4(1+\alpha)} + a^{-3(1+\alpha)} \right] \left[ r a^{-4(1+\alpha)} + (1 - r) a^{-3(1+\alpha)} \right]^2
\]
The deviation between $\Lambda$CDM and our model sensitively depends on the size of $r$. 

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\[ Q' = \frac{|Q|}{\rho_t H} \] approach zero in the early and future is better

**Blue curve:** \( \alpha = 0.1, r = 5 \times 10^{-6} \) (solid); \( \alpha = -0.1, r = 3 \times 10^{-5} \) (dashed)

**Brown curve:** \( Q = \alpha_1 H \rho_{dm}, \alpha_1 = 0.03 \) (solid) and 0.01 (dashed)

**Magenta curve:** \( Q = \rho_{dm}/\tau_{dm}, \tau_{dm} = 2000 \text{ s} \) (solid) and 500 s (dashed)
Conclusion

- DR evidence still exists after the Planck results.
- DR with interaction with other components in the universe can explain the different values of $N_{\text{eff}}$ at different time.
- We proposed a new generalized Chaplygin interacting model to explain the difference of $\Delta N_{\text{eff}}$ between CMB and BBN epochs.
BACKUP
The results after Planck

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<th>Planck</th>
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<td>$\Omega_m h^2$</td>
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Planck results

\[ w = w_0 + w_a (1 - a) \]

\[ P_R(k) = A_s \left( \frac{A_k}{A_{k_0}} \right)^{n_s - 1} \]

\[ r = \frac{P_h}{P_R} \text{ at } k = 0.05 \text{ Mpc}^{-1} \]

\[ \sum m_\nu = 0.23 \text{ eV} \]

\[ f_{NL} = 2.7 \pm 5.8 \]

\[ \sum m_\nu = 0.23 \text{ eV} \]
Brown dashed line: $< \sigma v > \equiv \frac{Q_2}{a^2} m_{dm}$, where $Q_2 = 10^{-43} \text{ cm}^2 \text{ MeV}^{-1}$

$$Q_B = \frac{Q_2 H}{H_0} \left( \rho_{dr}^2 - \rho_{dm}^2 \right)$$

LEP constrain the number of neutrinos from Z decay: $N_\nu = 2.92 \pm 0.05$
Interacting Models

The balance equation for DM and vacuum energy (dark energy)

\[
\frac{d\rho_M}{dt} + 3H (\rho_M + P_M) = -Q \\
\frac{d\rho_x}{dt} + 3H (\rho_x + P_x) = Q
\]

\[Q = (\lambda_x \rho_x + \lambda_M \rho_M), \quad \lambda_1 \rho_x^\alpha \rho_M^\beta, \quad \ldots\]

Coupling dark matter, dark energy, and dark radiation together

\[Q = (Q_x, \ Q_m, \ Q_r), \text{ with} \]

\[(1 + w_x)Q_x + (1 + w_m)Q_m + (1 + w_r)Q_r = 0\]

One asymptotic solution \(\gamma = \frac{\gamma_x \rho_x + \gamma_x \rho_m + \gamma_r \rho_r}{\rho_x + \rho_m + \rho_r}\), \(\gamma \to \gamma_s\)

\(\gamma_s\) of order \(10^{-3}, -4\) is allowed
DR in Interacting Models

We can consider the scenarios of dark radiation interacting with dark matter.

An intuitive interaction term can be expressed in the form

\[ Q = \Gamma \rho_{dm} \]

\( \Gamma \) is the interacting rate between \( \rho_{dm} \) and \( \rho_{dr} \)

Bjaelde, Das and Moss (2012)

A simple case is to make \( \Gamma = \alpha_1 H \)

\[ Q = \alpha_1 H \rho_{dm} \]

Another way is to fix the \( <\sigma v> = Q_2 / a^2 \) which is equivalent

Diamanti, Giusarma, Mena, Archidiacono, Melchiorri (2012)

\[ Q = Q_2 \left( \rho_{dm}^2 - \rho_{dr}^2 \right) \frac{H}{H_0} \]